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ON WEAKLY COMMUTING MAPS AND COMMON FIXED POINT RESULTS FOR FOUR MAPS IN G-METRIC SPACES

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Abstract

In this paper, we introduce the concept of weakly commuting maps in G-metric spaces and prove a common fixed point theorem for four self maps in the setting of generalized metric spaces. We also present an example to support our result.

Keywords: Common fixed Point, Weakly Commuting Maps, Generalized Metric Spaces.

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1. Introduction

The notion of G-metric space was introduced by Z. Mustafa and B. Sims [10] as a generalization of the notion of metric spaces. Mustafa *et al.* studied many fixed point results in G- metric spaces (see [8, 9, 10, 11, 12]). The study of common fixed point theorems in generalized metric spaces was initiated by Abbas and Rhoades [2], while, Saddati *et al.* [13] studied some fixed points in generalized partially ordered G-metric spaces. Shatanawi [15] obtained fixed points of Φ-maps in G-metric spaces. Also, Shatanawi [16] obtained a coupled coincidence fixed point theorem in the setting of a generalized metric spaces for two mapping F and g under certain conditions with an assumption of G-continuity of one of the mapping involved therein, see also [3, 17, 1, 4, 18, 5], while Chugh *et al.* [6] obtained some fixed point results for maps satisfying property p in a G-metric space. In the present paper, we introduce the concept of weakly commuting maps in G-metric spaces and prove a common fixed point theorem for four self maps in the setting of generalized metric spaces.

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2. Preliminaries.

The following definition was introduced by Mustafa and Sims [10].

2.1. Definition. [10] Let X be a nonempty set and $G : X \times X \times X \to \mathbf{R}^+$ a function satisfying the following properties:

(G_1) $G(x, y, z) = 0$ if $x = y = z$,
(G_2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,
(G_3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, symmetry in all three variables,
(G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a *generalized metric*, or, more specifically, a G-metric on X, and the pair (X, G) is called a G-metric space.

2.2. Definition. [10] Let (X, G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the *limit of the sequence* $\{x_n\}$, if

$$lim_{n,m\to+\infty} G(x, x_n, x_m) = 0,$$

and we say that the sequence $\{x_n\}$ is *G-convergent to x* or $\{x_n\}$ *G-converges to x*.

Thus, $x_n \to x$ in a G-metric space (X, G) if for any $\varepsilon > 0$, there exists $k \in \mathbf{N}$ such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq k$.

2.3. Proposition. [10] *Let (X, G) be a G-metric space. Then the following are equivalent:*

(1) *$\{x_n\}$ is G-convergent to x.*
(2) *$G(x_n, x_n, x) \to 0$ as $n \to +\infty$.*
(3) *$G(x_n, x, x) \to 0$ as $n \to +\infty$.*
(4) *$G(x_n, x_m, x) \to 0$ as $n, m \to +\infty$.* □

2.4. Definition. [10] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called *G-Cauchy* if for every $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq k$; that is $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

2.5. Proposition. [10] *Let (X, G) be a G- metric space. Then the following are equivalent:*

(1) *The sequence $\{x_n\}$ is G-Cauchy.*
(2) *For every $\epsilon > 0$, there is $k \in \mathbf{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq k$* □

2.6. Definition. [10] Let (X, G) and (X', G') be G-metric spaces, and let $f : (X, G) \to (X', G')$ be a function. Then f is said to be *G-continuous at a point $a \in X$* if and only if for every $\varepsilon > 0$, there is $\delta > 0$ such that $x, y \in X$ and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \varepsilon$. A function f is *G-continuous on X* if and only if it is G-continuous at all $a \in X$.

2.7. Proposition. [10] *Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.* □

Every G-metric on X defines a metric d_G on X by

$$d_G(x, y) = G(x, y, y) + G(y, x, x), \text{ for all } x, y \in X.$$

For a symmetric G-metric space

$$d_G(x, y) = 2G(x, y, y), \text{ for all } x, y \in X.$$

However, if G is not symmetric, then the following inequality holds:

$$\tfrac{3}{2} G(x, y, y) \leq d_G(x, y) \leq 3G(x, y, y), \text{ for all } x, y \in X.$$

The following are examples of G-metric spaces.

2.8. Example. [10] Let $(\mathbb{R}, d)$ be the usual metric space. Define G_s by

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z)$$

for all $x, y, z \in \mathbb{R}$. Then it is clear that $(\mathbb{R}, G_s)$ is a G-metric space.

2.9. Example. [10] Let $X = \{a, b\}$. Define G on $X \times X \times X$ by

$$G(a, a, a) = G(b, b, b) = 0,$$
$$G(a, a, b) = 1, \; G(a, b, b) = 2$$

and extend G to $X \times X \times X$ by using the symmetry in the variables. Then it is clear that (X, G) is a G-metric space.

2.10. Definition ([10])**.** A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

3. Main Results.

In 1982, Sessa [14] introduced the concept of weakly commuting maps in metric spaces as follows

3.1. Definition. Let (X, d) be a metric space and f, g be two self mappings of X. Then f and g are called *weakly commuting* if

$$d(fgx, gfx) \leq d(fx, gx)$$

holds for all $x \in X$.

Following Sessa [14], the concept of weakly commuting maps in G-metric space is defined as:

3.2. Definition. Let (X, G) be a G-metric space and f, g two self mappings of X. Then the pair $\{f, g\}$ is called *weakly commuting* if

$$G(fgx, gfx, gfx) \leq G(fx, gx, gx)$$

holds for all $x \in X$.

Now, we study a common fixed point for four maps satisfying a set of conditions in a G-metric space; in addition we introduce an example of our main result.

3.3. Theorem. *Let X be a complete G-metric space, and let $A, B, S, T : X \to X$ be mappings satisfying:*

$$G(Sx, Ty, Ty) \leq pG(Ax, By, By) + qG(Sx, Sx, Ax) + rG(Ty, Ty, By) \tag{3.1}$$

and

$$G(Sx, Sx, Ty) \leq pG(Ax, Ax, By) + qG(Sx, Sx, Ax) + rG(Ty, Ty, By). \tag{3.2}$$

Assume the maps A, B, S and T satisfy the following conditions:

1. *$TX \subseteq AX$ and $SX \subseteq BX$,*
2. *The mappings A and B are sequentially continuous, and*
3. *The pairs $\{A, S\}$ and $\{B, T\}$ are weakly commuting.*

If $p, q, r \geq 0$ with $p+q+r \in [0, 1)$, then A, B, S and T have a unique common fixed point.

Proof. If X is a symmetric G-metric space, then by adding the above two inequalities we obtain

$$\begin{aligned} G(Sx,Ty,Ty)+G(Sx,Sx,Ty) &\le p[G(Ax,By,By)+G(Ax,Ax,By)] \\ &\quad + 2q[G(Sx,Sx,Ax)] + 2r[G(Ty,Ty,By)], \end{aligned}$$

which further implies that

$$d_G(Sx,Ty) \le p d_G(Ax,By) + q d_G(Sx,Ax) + r d_G(Ty,By),$$

for all $x, y \in X$ with $0 \le p+q+r < 1$ and the fixed point of A, B, S and T follows from the result for metric spaces, see [14].

Now if X is not a symmetric G-metric space then by the definition of the metric (X, d_G) and Inequalities (3.1) and (3.2), we obtain

$$\begin{aligned} d_G(Sx,Ty) &= G(Sx,Ty,Ty)+G(Sx,Sx,Ty) \\ &\le p[G(Ax,By,By)+G(Ax,Ax,By)] \\ &\qquad + q[G(Sx,Sx,Ax)+G(Sx,Sx,Ax)] \\ &\qquad\qquad + r[G(Ty,Ty,By)+G(Ty,Ty,By)] \\ &\le p d_G(Ax,By) + \tfrac{4}{3} q d_G(Sx,Ax) + \tfrac{4}{3} r d_G(Ty,By). \end{aligned}$$

for all $x \in X$. Here, the contractivity factor $p + \frac{4}{3}(q+r)$ may not be less than 1. Therefore the metric gives no information. In this case, for given $x_0 \in X$, choose $x_1 \in X$ such that $Ax_1 = Tx_0$, choose $x_2 \in X$ such that $Sx_1 = Bx_2$, choose $x_3 \in X$ such that $Ax_3 = Tx_2$. Continuing the above process, we can construct a sequence $\{x_n\}$ in X such that $Ax_{2n+1} = Tx_{2n}$, $n \in \mathbf{N} \cup \{\mathbf{0}\}$ and $Bx_{2n+2} = Sx_{2n+1}$, $n \in \mathbf{N} \cup \{\mathbf{0}\}$. Let

$$y_{2n} = Ax_{2n+1} = Tx_{2n}, \ n \in \mathbb{N} \cup \{0\}$$

and

$$y_{2n+1} = Bx_{2n+2} = Sx_{2n+1}, \ n \in \mathbb{N} \cup \{0\}.$$

Take $n \in \mathbb{N}$. If n is even, then $n = 2k$ for some $k \in \mathbb{N}$. Then from (3.2), we have

$$\begin{aligned} G(y_n,y_{n+1},y_{n+1}) &= G(y_{2k},y_{2k+1},y_{2k+1}) \\ &= G(Tx_{2k},Sx_{2k+1},Sx_{2k+1}) \\ &= G(Sx_{2k+1},Sx_{2k+1},Tx_{2k}) \\ &\le pG(Ax_{2k+1},Ax_{2k+1},Bx_{2k}) + qG(Sx_{2k+1},Sx_{2k+1},Ax_{2k+1}) \\ &\qquad + rG(Tx_{2k},Tx_{2k},Bx_{2k}) \\ &= pG(y_{2k},y_{2k},y_{2k-1}) + qG(y_{2k+1},y_{2k+1},y_{2k}) \\ &\qquad + rG(y_{2k},y_{2k},y_{2k-1}) \\ &= pG(y_n,y_n,y_{n-1}) + qG(y_{n+1},y_{n+1},y_n) + rG(y_n,y_n,y_{n-1}), \end{aligned}$$

which further implies that

$$(1-q)G(y_n,y_{n+1},y_{n+1}) \le (p+r)G(y_{n-1},y_n,y_n).$$

Hence

$$G(y_n,y_{n+1},y_{n+1}) \le \frac{p+r}{1-q} G(y_{n-1},y_n,y_n),$$

or $G(y_n,y_{n+1},y_{n+1}) \le \lambda_1 G(y_{n-1},y_n,y_n)$, where $\lambda_1 = \frac{p+r}{1-q} < 1$.

If n is odd, then $n = 2k+1$ for some $k \in \mathbb{N}$. Again, from (3.1),

$$\begin{aligned} G(y_n, y_{n+1}, y_{n+1}) &= G(y_{2k+1}, y_{2k+2}, y_{2k+2}) = G(Sx_{2k+1}, Tx_{2k+2}, Tx_{2k+2}) \\ &\leq pG(Ax_{2k+1}, Bx_{2k+2}, Bx_{2k+2}) \\ &\quad + qG(Sx_{2k+1}, Sx_{2k+1}, Ax_{2k+1}) \\ &\quad\quad + rG(Tx_{2k+2}, Tx_{2k+2}, Bx_{2k+2}) \\ &= pG(y_{2k}, y_{2k+1}, y_{2k+1}) + qG(y_{2k+1}, y_{2k+1}, y_{2k}) \\ &\quad + rG(y_{2k+2}, y_{2k+2}, y_{2k+1}) \\ &= pG(y_{n-1}, y_n, y_n) + qG(y_n, y_n, y_{n-1}) + rG(y_{n+1}, y_{n+1}, y_n), \end{aligned}$$

that is

$$G(y_n, y_{n+1}, y_{n+1}) \leq \frac{(p+q)}{1-r} G(y_{n-1}, y_n, y_n),$$

or $G(y_n, y_{n+1}, y_{n+1}) \leq \lambda_2 G(y_{n-1}, y_n, y_n)$, where $\lambda_2 = \frac{p+q}{1-r} < 1$. Choose $\lambda = \max\{\lambda_1, \lambda_2\}$. Thus, for each $n \in \mathbb{N}$, we have

(3.3) $$G(y_n, y_{n+1}, y_{n+1}) \leq \lambda^n G(y_0, y_1, y_1).$$

Thus, if $y_0 = y_1$, we get $G(y_n, y_{n+1}, y_{n+1}) = 0$ for each $n \in \mathbb{N}$. Hence $y_n = y_0$ for each $n \in \mathbb{N}$. Therefore $\{y_n\}$ is G-Cauchy. So we may assume that $y_0 \neq y_1$. Let $n, m \in \mathbb{N}$ with $m > n$. By axiom (G_5) of the definition of a G-metric space, we have

$$G(y_n, y_m, y_m) \leq G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \cdots + G(y_{m-1}, y_m, y_m).$$

By Equation (3.3), we get

$$\begin{aligned} G(y_n, y_m, y_m) &\leq \lambda^n G(y_0, y_1, y_1) + \lambda^{n+1} G(y_0, y_1, y_1) + \ldots + \lambda^{m-1} G(y_0, y_1, y_1) \\ &= \lambda^n \sum_{i=0}^{m-1-n} q^i G(y_0, y_1, y_1) \leq \frac{\lambda^n}{1-\lambda} G(y_0, y_1, y_1). \end{aligned}$$

On taking limit $m, n \to \infty$, we have

$$\lim_{m,n\to\infty} G(y_n, y_m, y_m) = 0.$$

So we conclude that $\{y_n\}$ is a G-Cauchy sequence in X. Since X is G-complete, then it yields that $\{y_n\}$ and hence any subsequence of $\{y_n\}$ converges to some $z \in X$. So that, the subsequences $\{Ax_{2n+1}\}$, $\{Bx_{2n+2}\}$, $\{Sx_{2n+1}\}$ and $\{Tx_{2n}\}$ converge to z. First suppose that A is sequentially continuous, so that

$$\lim_{n\to\infty} A^2 x_{2n+1} = Az \text{ and } \lim_{n\to\infty} ASx_{2n+1} = Az.$$

Since $\{A, S\}$ is weakly commuting, we have

$$G(SAx_{2n+1}, ASx_{2n+1}, ASx_{2n+1}) \leq G(Sx_{2n+1}, Ax_{2n+1}, Ax_{2n+1}).$$

On taking the limit as $n \to \infty$, we get that $G(SAx_{2n+1}, Az, Az) \to 0$. Thus, we have

$$\lim_{n\to\infty} SAx_{2n+1} = Az.$$

Assume $Az \neq z$, we get

$$\begin{aligned} &G(SAx_{2n+1}, Tx_{2n}, Tx_{2n}) \\ &\quad \leq pG(AAx_{2n+1}, Bx_{2n}, Bx_{2n}) + qG(SAx_{2n+1}, SAx_{2n+1}, AAx_{2n+1}) \\ &\quad\quad + rG(Tx_{2n}, Tx_{2n}, Bx_{2n}). \end{aligned}$$

On letting $n \to \infty$, we have

$$G(Az, z, z) \leq pG(Az, z, z) + qG(Az, Az, Az) + rG(z, z, z).$$

Since $p < 1$, we conclude that

$$G(Az, z, z) < G(Az, z, z),$$

which is a contradiction. So $Az = z$. Also,

$$G(Sz, Sz, Tx_{2n}) \leq pG(Az, Az, Bx_{2n}) + qG(Sz, Sz, Az) + rG(Tx_{2n}, Tx_{2n}, Bx_{2n}).$$

By taking the limit as $n \to \infty$, we have

$$G(Sz, Sz, z) \leq pG(Az, Az, z) + qG(Sz, Sz, Az) + rG(z, z, z) \leq qG(Sz, Sz, z).$$

Since $q < 1$, we get $G(Sz, Sz, z) = 0$. So $Sz = z$. Suppose B is sequentially continuous, then

$$\lim_{n\to\infty} B(Bx_{2n}) = Bz \text{ and } \lim_{n\to\infty} B(Tx_{2n}) = Bz.$$

Since the pair $\{B, T\}$ is weakly commuting, we have

$$G(TBx_{2n}, BTx_{2n}, BTx_{2n}) \leq G(Tx_{2n}, Bx_{2n}, Bx_{2n}).$$

Taking the limit as $n \to +\infty$, we get $G(TBx_{2n}, Bz, Bz) \to 0$. Thus

$$\lim_{n\to\infty} T(Bx_{2n}) = Bz.$$

Assume $Bz \neq z$. Since

$$\begin{aligned} &G(Sx_{2n+1}, TBx_{2n}, TBx_{2n}) \\ &\qquad \leq pG(Ax_{2n+1}, BBx_{2n}, BBx_{2n}) + qG(Sx_{2n+1}, Sx_{2n+1}, Ax_{2n+1}) \\ &\qquad\qquad + rG(TBx_{2n}, TBx_{2n}, BBx_{2n}), \end{aligned}$$

Again taking the limit as $n \to \infty$, implies

$$G(z, Bz, Bz) \leq pG(z, Bz, Bz) + qG(z, z, z) + rG(Bz, Bz, Bz) < G(z, Bz, Bz),$$

which is a contradiction. Hence $Bz = z$. Since

$$\begin{aligned} &G(Sx_{2n+1}, Tz, Tz) \\ &\quad \leq pG(Ax_{2n+1}, Bz, Bz) + qG(Sx_{2n+1}, Sx_{2n+1}, Ax_{2n+1}) + rG(Tz, Tz, Bz), \end{aligned}$$

on taking the limit as $n \to \infty$, we get

$$\begin{aligned} G(z, Tz, Tz) &\leq pG(z, Bz, Bz) + qG(z, z, z) + rG(Tz, Tz, Bz) \\ &\leq rG(z, Tz, Tz). \end{aligned}$$

Since $r < 1$, we get $G(z, Tz, Tz) = 0$. Hence $Tz = z$. So z is a common fixed point for A, B, S and T. To prove that z is the unique common fixed point let w be a common fixed point for A, B, S and T with $w \neq z$. Then

$$\begin{aligned} G(z, w, w) &= G(Sz, Tw, Tw) \\ &\leq pG(Az, Bw, Bw) + qG(Sz, Sz, Az) + rG(Tw, Tw, Bw) \\ &= pG(z, w, w) + qG(z, z, z) + rG(w, w, w) = pG(z, w, w) \\ &< G(z, w, w), \end{aligned}$$

which is a contradiction. So $z = w$. □

3.4. Corollary. *Let X be a complete G-metric space, and let $A, B, S, T : X \to X$ be mappings satisfying:*

$$G(Sx, Ty, Ty) \leq hG(Ax, By, By)$$

and

$$G(Sx, Sx, Ty) \leq hG(Ax, Ax, By).$$

Assume the maps A, B, S and T satisfy the following conditions:

(1) $TX \subseteq AX$ *and* $SX \subseteq BX$,
(2) *The mappings A and B are sequentially continuous, and*
(3) *The pairs $\{A, S\}$ and $\{B, T\}$ are weakly commuting.*

If $h \in [0, 1)$, then A, B, S and T have a unique common fixed point. □

3.5. Corollary. *Let X be a complete G-metric space and let $A, S : X \to X$ be mappings satisfying:*

$$G(Sx, Sy, Sy) \leq kG(Ax, Ay, Ay)$$

for all $x, y \in X$. Assume the maps A and S satisfy the following conditions:

(1) $SX \subseteq AX$,
(2) *The map A is sequentially continuous, and*
(3) *The pair $\{A, S\}$ is weakly commuting.*

If $k \in [0, 1)$, then A and S have a unique common fixed point.

Proof. Define $B : X \to X$ by $Bx = Ax$ and define $T : X \to X$ by $Tx = Sx$. Then the four maps A, B, S and T satisfy all the hypothesis of Corollary 3.4. So, the result follows from Corollary 3.4. □

3.6. Corollary. *Let X be a complete G-metric space and let $S : X \to X$ be a mapping satisfying:*

$$G(Sx, Sy, Sy) \leq qG(x, y, y)$$

for all $x, y \in X$. If $q \in [0, 1)$, then S has a unique fixed point.

Proof. Follows from Corollary 3.5 by taking $A = B = I$ and $S = T$. □

Now, we introduce an example of Theorem 3.3.

3.7. Example. Let $X = [0, 1]$, Define $A, B, S, T : X \to X$ by $Ax = \frac{1}{2}x$, $Bx = \frac{1}{4}x$, $Sx = \frac{1}{8}x$, and $Tx = \frac{1}{16}x$. Then $TX \subseteq AX$, $SX \subseteq BX$. Note that the pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible. Define $G : X \times X \times X \to \mathbb{R}^+$ by

$$G(x, y, z) = |x - y| + |x - z| + |y - z|.$$

Then (X, G) is a complete G-metric. Also

$$G(Sx, Ty, Ty) = 2|Sx - Ty| = \tfrac{1}{8}|2x - y|,$$
$$G(Ax, By, By) = 2|Ax - by| = \tfrac{1}{2}|2x - y|,$$
$$G(Sx, SX, Ty) = 2|Sx - Ty| = \tfrac{1}{8}|2x - y|,$$

and

$$G(Ax, Ax, By) = 2|Ax - By| = \tfrac{1}{2}|2x - y|.$$

So

$$G(Sx, Ty, Ty)) \leq \frac{1}{2}G(Ax, By, By)$$

and

$$G(Sx, Sx, Ty)) \leq \frac{1}{2}G(Ax, Ax, By).$$

Since $AS = SA$ and $BT = TB$, we conclude that the pairs $\{A, S\}$ and $\{B, T\}$ are weakly commuting. Note that A, B, S and T satisfy the hypothesis of Theorem 3.3. Here, 0 is the unique common fixed point of A, B, S and T.

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