APPROXIMATION OF FIXED POINTS OF
ASYMPTOTICALLY $\kappa$-STRICT PSEUDO-
CONTRACTIONS IN A BANACH SPACE

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Abstract

In this paper, weak convergence theorems of a finite family of asymptotically $k$-strict pseudo-contractions are established in the framework of 2-uniformly smooth and uniformly convex Banach spaces.

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1. Introduction and Preliminaries

Let $E$ be an arbitrary real Banach space and $J_q (q > 1)$ denotes the generalized duality mapping from $E$ into $2^{E^*}$ give by

$$J_q(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in E,$$

where $E^*$ denotes the dual space of $E$ and $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. In particular, $J_2$ is called the normalized duality mapping which is usually denoted by $J$. In this paper, we use $j$ to denote the single-valued normalized duality mapping. It is well known (see, for example, [14]) that $J_q(x) = \|x\|^q J(x)$ if $x \neq 0$. If $E$ is a Hilbert space, then $J = I$, where $I$ denotes the identity mapping.

Let $U_E = \{x \in E : \|x\| = 1\}$. $E$ is said to uniformly convex if, for any $\epsilon \in (0, 2]$, there exists $\delta > 0$ such that

$$\|x - y\| \geq \epsilon \quad \text{implies} \quad \left\| \frac{x + y}{2} \right\| \leq 1 - \delta, \quad \forall x, y \in U_E.$$

A Banach space $E$ is said to be smooth if the limit

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t}$$


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