Hacettepe Journal of Mathematics and Statistics
Volume 41 (5) (2012), 715 – 721

SOME OSCILLATION RESULTS FOR SECOND-ORDER NEUTRAL DYNAMIC EQUATIONS

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Received 30 : 06 : 2011 : Accepted 24 : 05 : 2012

Abstract

This paper is concerned with the oscillation of certain second-order neutral dynamic equations on a time scale. Four new oscillation criteria are presented that supplement those results given in Arun K. Tripathy (*Some oscillation results for second order nonlinear dynamic equations of neutral type*, Nonlinear Anal. **71**, 1727–1735, 2009).

Keywords: Oscillation, Neutral dynamic equation, Second-order equation.

2000 AMS Classification: 34 K 11, 34 N 05, 39 A 10, 39 A 12, 39 A 13, 39 A 21.

1. Introduction

In [15], the author studied the oscillatory behavior of second-order neutral dynamic equations of the form

$$\left(r(t)((x(t)+p(t)x(t-\tau))^{\Delta})^{\alpha}\right)^{\Delta}+q(t)x^{\alpha}(t-\delta)=0,\ t\in\mathbb{T}, \tag{1.1}$$

where $0 \le p(t) \le p_0 < \infty$ and $\alpha > 0$ is a quotient of odd positive integers. Here it is assumed that τ, δ are such that $t-\tau, t-\delta \in \mathbb{T}$ for all $t \in \mathbb{T}$, which is satisfied only for certain time scales such as $\mathbb{T}=\mathbb{R}$, $\mathbb{T}=\mathbb{Z}$, or $\mathbb{T}=h\mathbb{Z}$ for $h>0$. Therefore, we will use different methods to derive four new oscillation criteria on an arbitrary time scale $\mathbb{T}$ with $\sup\mathbb{T}=\infty$ for the second-order neutral dynamic equation

$$\left(r(z^{\Delta})^{\alpha}\right)^{\Delta}(t)+q(t)x^{\alpha}(\delta(t))=0, \tag{1.2}$$

where we assume the following:

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(A_1) The time scale $\mathbb{T} \subset \mathbb{R}$ satisfies $\sup \mathbb{T} = \infty$, $\alpha > 0$ is the ratio of odd positive integers, $r, p, q \in \mathrm{C_{rd}}(\mathbb{T}, (0, \infty))$, $p(t) \geq 1$, $p(t) \not\equiv 1$ eventually,

$z(t) := x(t) + p(t)x(\tau(t))$,

$\tau, \delta : \mathbb{T} \to \mathbb{T}$, τ is strictly increasing, and

$\lim_{t\to\infty} \tau(t) = \lim_{t\to\infty} \delta(t) = \infty$.

We define a time scale interval by $[t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$. By a solution of equation (1.2) we mean a nontrivial function $x \in \mathrm{C_{rd}}([T_x, \infty), \mathbb{R})$, where $T_x \in [t_0, \infty)_{\mathbb{T}}$, which satisfies (1.2) on $[T_x, \infty)_{\mathbb{T}}$. We consider only those solutions x of (1.2) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \in [T_x, \infty)_{\mathbb{T}}$. A solution of equation (1.2) is called *oscillatory* if it has arbitrarily large zeros on $[T_x, \infty)_{\mathbb{T}}$, and otherwise, it is said to be *nonoscillatory.* Equation (1.2) is said to be oscillatory if all its solutions are oscillatory.

The analogue for (1.2) in case $\mathbb{T} = \mathbb{R}$ has been studied in [12] (see also [4, 13]). Similar results for $\mathbb{T} = \mathbb{Z}$ are contained in [2]. For related results in the general time scales case, we refer the reader to [1, 3, 5, 7, 10, 11, 14].

In the next section, we shall establish four new oscillation criteria for equation (1.2). The last section contains some remarks concerning further study and some examples that illustrate the main results. Throughout, we use the following notation.

(A_2) τ^{-1} is the inverse function of τ,

$$(\eta^\Delta(t))_+ := \max\{0, \eta^\Delta(t)\},$$

$$\phi(t) := \frac{m(t)}{m(\sigma(t))}, \quad \beta(t) := \begin{cases} \phi(t) & \text{if } \alpha < 1, \\ \phi^\alpha(t) & \text{if } \alpha \geq 1, \end{cases}$$

$$p^*(t) := \frac{1}{p(\tau^{-1}(t))}\left(1 - \frac{1}{p(\tau^{-1}(\tau^{-1}(t)))}\right) > 0,$$

$$p_*(t) := \frac{1}{p(\tau^{-1}(t))}\left(1 - \frac{1}{p(\tau^{-1}(\tau^{-1}(t)))}\frac{m(\tau^{-1}(\tau^{-1}(t)))}{m(\tau^{-1}(t))}\right) > 0,$$

for all sufficiently large t, where m will be specified later.

2. Oscillation criteria

All functional inequalities considered in this section are assumed to hold eventually, i.e., they are satisfied for all t large enough.

Before stating the main results, we begin with the following lemma.

2.1. Lemma. *Assume* (A_1) *and let x be an eventually positive solution of* (1.2)*. If*

$$\int_{t_0}^{\infty} r^{-\frac{1}{\alpha}}(t)\Delta t = \infty, \tag{2.1}$$

then eventually

$$z > 0, \quad z^\Delta > 0, \quad (r(z^\Delta)^\alpha)^\Delta < 0.$$

Proof. The proof is simple and so is omitted. □

2.2. Theorem. *Assume* (A_1), (A_2), (2.1)*, and let*

$\tau(t) > t$ *and* $\tau(\sigma(t)) \geq \delta(t)$ *for all* $t \in [t_0, \infty)_{\mathbb{T}}$.

If there exist functions $\eta, m \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}},(0,\infty))$ *such that*

$$\frac{m(t)}{r^{\frac{1}{\alpha}}(t)\int_{t_1}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s} - m^{\Delta}(t) \le 0 \tag{2.2}$$

for all sufficiently large t_1*, and for some* $t_2 \in [t_1,\infty)_{\mathbb{T}}$*, one has*

$$\begin{aligned}\limsup_{t\to\infty}\int_{t_2}^{t}\Bigg[&\eta(\sigma(s))q(s)(p^*(\delta(s)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(s)))}{m(\sigma(s))}\right)^{\alpha}\\ &-\frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(s)((\eta^{\Delta}(s))_+)^{\alpha+1}}{(\eta(\sigma(s))\beta(s))^{\alpha}}\Bigg]\Delta s=\infty,\end{aligned} \tag{2.3}$$

then (1.2) *is oscillatory.*

Proof. Let x be a nonoscillatory solution of (1.2). Without loss of generality, we may assume that there exists $t_1 \in [t_0,\infty)_{\mathbb{T}}$ such that $x(t)>0$, $x(\tau(t))>0$, and $x(\delta(t))>0$ for $t\in[t_1,\infty)_{\mathbb{T}}$. Then $z^{\Delta}(t)>0$ for $t\in[t_1,\infty)_{\mathbb{T}}$ due to Lemma 2.1. It follows (see also [6, (8.6)]) that

$$\begin{aligned}x(t) &= \frac{1}{p(\tau^{-1}(t))}\big(z(\tau^{-1}(t)) - x(\tau^{-1}(t))\big)\\ &= \frac{z(\tau^{-1}(t))}{p(\tau^{-1}(t))} - \frac{1}{p(\tau^{-1}(t))}\left(\frac{z(\tau^{-1}(\tau^{-1}(t)))}{p(\tau^{-1}(\tau^{-1}(t)))} - \frac{x(\tau^{-1}(\tau^{-1}(t)))}{p(\tau^{-1}(\tau^{-1}(t)))}\right)\\ &\ge \frac{z(\tau^{-1}(t))}{p(\tau^{-1}(t))} - \frac{z(\tau^{-1}(\tau^{-1}(t)))}{p(\tau^{-1}(t))p(\tau^{-1}(\tau^{-1}(t)))}\\ &\ge \frac{1}{p(\tau^{-1}(t))}\left(1-\frac{1}{p(\tau^{-1}(\tau^{-1}(t)))}\right)z(\tau^{-1}(t))\\ &= p^*(t)z(\tau^{-1}(t)).\end{aligned}$$

From this and (1.2), we have

$$(r(z^{\Delta})^{\alpha})^{\Delta}(t) + q(t)(p^*(\delta(t)))^{\alpha}(z(\tau^{-1}(\delta(t))))^{\alpha} \le 0. \tag{2.4}$$

Define

$$\omega(t) := \eta(t)\frac{r(t)(z^{\Delta}(t))^{\alpha}}{z^{\alpha}(t)},\ t\in[t_1,\infty)_{\mathbb{T}}. \tag{2.5}$$

Then $\omega(t)>0$ and, using the quotient rule, (2.4), and (2.5),

$$\begin{aligned}\omega^{\Delta}(t) &= \eta^{\Delta}(t)\frac{r(t)(z^{\Delta}(t))^{\alpha}}{z^{\alpha}(t)} + \eta(\sigma(t))\left(\frac{r(z^{\Delta})^{\alpha}}{z^{\alpha}}\right)^{\Delta}(t)\\ &= \eta^{\Delta}(t)\frac{r(t)(z^{\Delta}(t))^{\alpha}}{z^{\alpha}(t)}\\ &\qquad + \eta(\sigma(t))\frac{(r(z^{\Delta})^{\alpha})^{\Delta}(t)z^{\alpha}(t) - r(t)(z^{\Delta}(t))^{\alpha}(z^{\alpha})^{\Delta}(t)}{z^{\alpha}(t)z^{\alpha}(\sigma(t))}\\ &\le \frac{(\eta^{\Delta}(t))_+}{\eta(t)}\omega(t) - \eta(\sigma(t))q(t)(p^*(\delta(t)))^{\alpha}\left(\frac{z(\tau^{-1}(\delta(t)))}{z(\sigma(t))}\right)^{\alpha}\\ &\qquad - \eta(\sigma(t))\frac{r(t)(z^{\Delta}(t))^{\alpha}(z^{\alpha})^{\Delta}(t)}{z^{\alpha}(t)z^{\alpha}(\sigma(t))}.\end{aligned} \tag{2.6}$$

On the other hand, we have

$$z(t) = z(t_1) + \int_{t_1}^{t}\frac{\big(r(s)(z^{\Delta}(s))^{\alpha}\big)^{\frac{1}{\alpha}}}{r^{\frac{1}{\alpha}}(s)}\Delta s \ge \left(r^{\frac{1}{\alpha}}(t)\int_{t_1}^{t}\frac{1}{r^{\frac{1}{\alpha}}(s)}\Delta s\right)z^{\Delta}(t),$$

which yields, using the quotient rule and (2.2),

$$\left(\frac{z}{m}\right)^{\Delta}(t) = \frac{z^{\Delta}(t)m(t) - z(t)m^{\Delta}(t)}{m(t)m(\sigma(t))} \leq \frac{z(t)}{m(t)m(\sigma(t))}\left(\frac{m(t)}{r^{\frac{1}{\alpha}}(t)\int_{t_1}^{t} r^{-\frac{1}{\alpha}}(s)\Delta s} - m^{\Delta}(t)\right) \leq 0,$$

and thus z/m is nonincreasing. Since $\tau^{-1}(\delta(t)) \leq \sigma(t)$ and $t \leq \sigma(t)$, we obtain

$$\frac{z(\tau^{-1}(\delta(t)))}{z(\sigma(t))} \geq \frac{m(\tau^{-1}(\delta(t)))}{m(\sigma(t))}, \quad \frac{z(t)}{z(\sigma(t))} \geq \frac{m(t)}{m(\sigma(t))}. \tag{2.7}$$

If $\alpha \geq 1$, then we get

$$(z^{\alpha})^{\Delta}(t) \geq \alpha z^{\alpha-1}(t)z^{\Delta}(t) \tag{2.8}$$

due to [8, Theorem 1.90]. By (2.5), (2.6), (2.7), and (2.8), we see that

$$\begin{aligned} \omega^{\Delta}(t) \leq& -\eta(\sigma(t))q(t)(p^*(\delta(t)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(t)))}{m(\sigma(t))}\right)^{\alpha} + \frac{(\eta^{\Delta}(t))_+}{\eta(t)}\omega(t) \\ & - \alpha\frac{\eta(\sigma(t))}{r^{\frac{1}{\alpha}}(t)\eta^{\frac{\alpha+1}{\alpha}}(t)}\left(\frac{m(t)}{m(\sigma(t))}\right)^{\alpha}\omega^{\frac{\alpha+1}{\alpha}}(t). \end{aligned} \tag{2.9}$$

If $\alpha < 1$, then we have

$$(z^{\alpha})^{\Delta}(t) \geq \alpha z^{\alpha-1}(\sigma(t))z^{\Delta}(t) \tag{2.10}$$

due to [8, Theorem 1.90]. By (2.5), (2.6), (2.7), and (2.10), we see that

$$\begin{aligned} \omega^{\Delta}(t) \leq& -\eta(\sigma(t))q(t)(p^*(\delta(t)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(t)))}{m(\sigma(t))}\right)^{\alpha} + \frac{(\eta^{\Delta}(t))_+}{\eta(t)}\omega(t) \\ & - \alpha\frac{\eta(\sigma(t))}{r^{\frac{1}{\alpha}}(t)\eta^{\frac{\alpha+1}{\alpha}}(t)}\frac{m(t)}{m(\sigma(t))}\omega^{\frac{\alpha+1}{\alpha}}(t). \end{aligned} \tag{2.11}$$

If follows from (2.9), (2.11), and the definition of β that

$$\begin{aligned} \omega^{\Delta}(t) \leq& -\eta(\sigma(t))q(t)(p^*(\delta(t)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(t)))}{m(\sigma(t))}\right)^{\alpha} + \frac{(\eta^{\Delta}(t))_+}{\eta(t)}\omega(t) \\ & - \alpha\frac{\eta(\sigma(t))}{r^{\frac{1}{\alpha}}(t)\eta^{\frac{\alpha+1}{\alpha}}(t)}\beta(t)\omega^{\frac{\alpha+1}{\alpha}}(t) \end{aligned} \tag{2.12}$$

holds when $\alpha > 0$. Now set

$$y := \omega(t), \quad A := \alpha\frac{\eta(\sigma(t))}{r^{\frac{1}{\alpha}}(t)\eta^{\frac{\alpha+1}{\alpha}}(t)}\beta(t), \quad B := \frac{(\eta^{\Delta}(t))_+}{\eta(t)}.$$

Using the inequality

$$By - Ay^{\frac{\alpha+1}{\alpha}} \leq \frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}\frac{B^{\alpha+1}}{A^{\alpha}},$$

we obtain

$$\frac{(\eta^{\Delta}(t))_+}{\eta(t)}\omega(t) - \alpha\frac{\eta(\sigma(t))}{r^{\frac{1}{\alpha}}(t)\eta^{\frac{\alpha+1}{\alpha}}(t)}\beta(t)\omega^{\frac{\alpha+1}{\alpha}}(t) \leq \frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(t)((\eta^{\Delta}(t))_+)^{\alpha+1}}{(\eta(\sigma(t))\beta(t))^{\alpha}}.$$

Hence by (2.12), we have

$$\omega^{\Delta}(t) \leq -\eta(\sigma(t))q(t)(p^*(\delta(t)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(t)))}{m(\sigma(t))}\right)^{\alpha} + \frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(t)((\eta^{\Delta}(t))_+)^{\alpha+1}}{(\eta(\sigma(t))\beta(t))^{\alpha}}.$$

Integrating this inequality from $t_2 \in [t_1,\infty)_{\mathbb{T}}$ to t gives

$$\int_{t_2}^{t}\left[\eta(\sigma(s))q(s)(p^*(\delta(s)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(s)))}{m(\sigma(s))}\right)^{\alpha} - \frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(s)((\eta^{\Delta}(s))_+)^{\alpha+1}}{(\eta(\sigma(s))\beta(s))^{\alpha}}\right]\Delta s \leq \omega(t_2),$$

which contradicts (2.3). This completes the proof. □

Similar to the proof of Theorem 2.2, we can get the following result.

2.3. Theorem. *Assume* (A$_1$), (A$_2$), (2.1), *and let*

$$\tau(t) > t \text{ and } \tau(\sigma(t)) \leq \delta(t) \text{ for all } t \in [t_0,\infty)_{\mathbb{T}}.$$

If there exist functions $\eta, m \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}},(0,\infty))$ *such that* (2.2) *holds for all sufficiently large* t_1, *and for some* $t_2 \in [t_1,\infty)_{\mathbb{T}}$, *one has*

$$\limsup_{t\to\infty}\int_{t_2}^{t}\left[\eta(\sigma(s))q(s)(p^*(\delta(s)))^{\alpha} - \frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(s)((\eta^{\Delta}(s))_+)^{\alpha+1}}{(\eta(\sigma(s))\beta(s))^{\alpha}}\right]\Delta s = \infty, \tag{2.13}$$

then (1.2) *is oscillatory.* □

Note that Theorem 2.2 and Theorem 2.3 focus on the oscillation of equation (1.2) under the assumption $\tau(t) > t$. Now we will establish some oscillation results for (1.2) under the assumption $\tau(t) < t$.

2.4. Theorem. *Assume* (A$_1$), (A$_2$), (2.1), *and let*

$$\tau(t) < t \text{ and } \tau(\sigma(t)) \geq \delta(t) \text{ for all } t \in [t_0,\infty)_{\mathbb{T}}.$$

If there exist functions $\eta, m \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}},(0,\infty))$ *such that* (2.2) *holds for all sufficiently large* t_1, *and for some* $t_2 \in [t_1,\infty)_{\mathbb{T}}$, *one has*

$$\limsup_{t\to\infty}\int_{t_2}^{t}\left[\eta(\sigma(s))q(s)(p_*(\delta(s)))^{\alpha}\left(\frac{m(\tau^{-1}(\delta(s)))}{m(\sigma(s))}\right)^{\alpha} - \frac{1}{(\alpha+1)^{\alpha+1}}\frac{r(s)((\eta^{\Delta}(s))_+)^{\alpha+1}}{(\eta(\sigma(s))\beta(s))^{\alpha}}\right]\Delta s = \infty, \tag{2.14}$$

then (1.2) *is oscillatory.*

Proof. Let x be a nonoscillatory solution of (1.2). Without loss of generality, we may assume that there exists $t_1 \in [t_0,\infty)_{\mathbb{T}}$ such that $x(t) > 0$, $x(\tau(t)) > 0$, and $x(\delta(t)) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$. Then $z^{\Delta}(t) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$ due to Lemma 2.1. Similarly to the first calculation of the proof of Theorem 2.2 (see also [6, (8.6)]), we have

$$\begin{aligned} x(t) &= \frac{1}{p(\tau^{-1}(t))}\left(z(\tau^{-1}(t)) - x(\tau^{-1}(t))\right) \\ &\geq \frac{z(\tau^{-1}(t))}{p(\tau^{-1}(t))} - \frac{z(\tau^{-1}(\tau^{-1}(t)))}{p(\tau^{-1}(t))p(\tau^{-1}(\tau^{-1}(t)))}. \end{aligned} \tag{2.15}$$

From $\tau^{-1}(\tau^{-1}(t)) \geq \tau^{-1}(t)$, (1.2), and (2.15), we get

$$(r(z^{\Delta})^{\alpha})^{\Delta}(t) + q(t)(p_*(\delta(t)))^{\alpha}(z(\tau^{-1}(\delta(t))))^{\alpha} \leq 0. \tag{2.16}$$

Similar to the proof of Theorem 2.2, we see that z/m is nonincreasing. The remainder of the proof is similar to that of Theorem 2.2 and hence is omitted. This completes the proof. □

Similar to the proof of Theorem 2.4, we can derive the following criterion.

2.5. Theorem. *Assume* (A_1), (A_2), (2.1), *and let*

$$\tau(t) < t \text{ and } \tau(\sigma(t)) \le \delta(t) \text{ for all } t \in [t_0, \infty)_{\mathbb{T}}.$$

If there exist functions $\eta, m \in C^1_{rd}([t_0,\infty)_{\mathbb{T}}, (0,\infty))$ *such that* (2.2) *holds for all sufficiently large* t_1, *and for some* $t_2 \in [t_1, \infty)_{\mathbb{T}}$, *one has*

$$\limsup_{t\to\infty} \int_{t_2}^{t} \left[\eta(\sigma(s))q(s)(p_*(\delta(s)))^{\alpha} - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{r(s)((\eta^{\Delta}(s))_+)^{\alpha+1}}{(\eta(\sigma(s))\beta(s))^{\alpha}} \right] \Delta s = \infty, \tag{2.17}$$

then (1.2) *is oscillatory.*

3. Examples and remarks

In this section, we give some remarks and two examples in order to illustrate the main results.

3.1. Remark. From Theorem 2.2, Theorem 2.3, Theorem 2.4, and Theorem 2.5, we can obtain various oscillation criteria for equation (1.2), e.g., by letting

$$m(t) = \int_{t_1}^{t} \frac{1}{r^{\frac{1}{\alpha}}(s)} \Delta s \text{ and } \eta(t) = t.$$

The details are left to the reader.

3.2. Remark. By employing methods given in this note, we can obtain Philos-type oscillation criteria for equation (1.2). The details are left to the reader.

3.3. Example. Consider the equation

$$\big(x(t) + 2x(\tau(t))\big)^{\Delta\Delta} + \frac{\sigma(t)}{t^2} x(\tau(t)) = 0, \; t \in [1,\infty)_{\mathbb{T}}, \tag{3.1}$$

where τ is strictly increasing to ∞ and $\tau(t) > t$. Let $m(t) = t - t_1$ and $\eta(t) = 1$. Using [9, Theorem 5.68], we can see that equation (3.1) is oscillatory due to Theorem 2.2.

3.4. Example. Consider the equation

$$\big(x(t) + tx(\tau(t))\big)^{\Delta\Delta} + \frac{\sigma(t)}{t} x(\tau(t)) = 0, \; t \in [1,\infty)_{\mathbb{T}}, \tag{3.2}$$

where τ is strictly increasing to ∞ and $\tau(t) < t$. Let $m(t) = t - t_1$ and $\eta(t) = 1$. Using [9, Theorem 5.68], we can verify that equation (3.2) is oscillatory due to Theorem 2.4.

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