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THE REVISED EDGE SZEGED INDEX OF BRIDGE GRAPHS

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Abstract

The revised edge Szeged index of a connected graph G is defined as

$$Sz_e^*(G) = \sum_{e=uv\in E(G)} \Big(m_u(e|G) + \frac{m_0(e|G)}{2}\Big)\Big(m_v(e|G) + \frac{m_0(e|G)}{2}\Big),$$

where $E(G)$ is the edge set of G, $m_u(e|G)$ is the number of edges closer to vertex u than to vertex v in G, $m_v(e|G)$ is the number of edges closer to vertex v than to vertex u in G, and $m_0(e|G)$ is the number of edges equidistant from both ends of e. We give a formula for the revised edge Szeged index of a bridge graph, from which the revised edge Szeged indices for several classes of graphs are calculated.

Keywords: Szeged index, Revised edge Szeged index, Revised Szeged index, Bridge graphs, Distance.

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1. Introduction

Topological indices are used in theoretical chemistry for the design of chemical compounds with given physicochemical properties or given pharmacologic and biological activities [10]. The Wiener index is one of the oldest and the most thoroughly studied topological index. Motivated by the original definition of the Wiener index of a tree, Gutman [3] introduced the Szeged index, which coincides with the Wiener index for a tree. It found applications in quantitative structure-property-activity-toxicity modeling, see [5]. There are some variants of the Szeged index. Randić [9] introduced the revised Szeged index, which shows to be a 'better descriptor for structure-property relationships for cyclic molecules'. Recently, the edge Szeged index and the revised edge Szeged index were proposed in [4] and [2], respectively.

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Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. For $u, v \in V(G)$, $d_G(u, v)$ denotes the distance between u and v in G.

Let $e = uv \in E(G)$, $w \in V(G)$. The distance between e and w in G is defined as $d_G(e, w) = \min\{d_G(u, w), d_G(v, w)\}$. Let $m_u(e|G)$ be the number of edges closer to vertex u than to vertex v in G and $m_v(e|G)$ the number of edges closer to vertex v than to vertex u in G, i.e.,

$$m_u(e|G) = |\{f \in E(G) : d_G(f, u) < d_G(f, v)\}|,$$
$$m_v(e|G) = |\{f \in E(G) : d_G(f, v) < d_G(f, u)\}|.$$

The edge Szeged index of G is defined as [4]

$$Sz_e(G) = \sum_{e=uv\in E(G)} m_u(e|G)m_v(e|G).$$

Some basic properties of the edge Szeged index have been established, see [1, 4, 6, 11].

Let $m_0(e|G)$ be the number of edges equidistant from both ends of $e = uv \in E(G)$, i.e.,

$$m_0(e|G) = |\{f \in E(G) : d_G(f, u) = d_G(f, v)\}|.$$

The revised edge Szeged index of G is defined in [2] as

$$Sz_e^*(G) = \sum_{e=uv\in E(G)} \left(m_u(e|G) + \frac{m_0(e|G)}{2}\right)\left(m_v(e|G) + \frac{m_0(e|G)}{2}\right).$$

Let $\{G_i\}_{i=1}^d$ be a set of finite pairwise vertex-disjoint connected graphs with $v_i \in V(G_i)$. The *bridge graph* $B(G_1, G_2, \ldots, G_d) = B(G_1, G_2, \ldots, G_d;\, v_1, v_2, \ldots, v_d)$ of $\{G_i\}_{i=1}^d$ with respect to the vertices $\{v_i\}_{i=1}^d$ is the graph obtained from the graphs $G_1, G_2, \ldots, G_d$ by connecting the vertices v_i and v_{i+1} by an edge for all $i = 1, 2, \ldots, d-1$.

Formulae for the Szeged index, edge Szeged index, revised Szeged index of bridge graphs have been given in [7, 13, 8], respectively. Here we give a formula for the revised edge Szeged index of a bridge graph, from which the revised edge Szeged indices for several classes of graphs are calculated.

2. Results

For a connected graph G with $w \in V(G)$, let $L_{G,w}$ and $R_{G,w}$ be respectively the sets of edges $e = uv$ in $E(G)$ such that $d_G(u, w) < d_G(v, w)$ and $d_G(u, w) > d_G(v, w)$, and $Q_{G,w}$ the set of edges $e = uv$ in $E(G)$ such that $d_G(u, w) = d_G(v, w)$. To make this well-defined, we choose an arbitrary direction on the edges of G (which is fixed for all the following computations); the results do not depend on the direction chosen.

In a bridge graph $B(G_1, G_2, \ldots, G_d)$, for $i = 1, 2, \ldots, d$, let $L_i = L_{G_i,v_i}$, $R_i = R_{G_i,v_i}$ and $Q_i = Q_{G_i,v_i}$.

2.1. Lemma. [13] *The edge Szeged index of the bridge graph* $G = B(G_1, G_2, \ldots, G_d)$ *of* $\{G_i\}_{i=1}^d$ *with respect to the vertices* $\{v_i\}_{i=1}^d$ *is given by*

$$\begin{aligned} Sz_e(G) = \sum_{i=1}^{d} Sz_e(G_i) \\ + \sum_{i=1}^{d}(|E(G)| - |E(G_i)|)\Big(\sum_{e=uv\in L_i} m_v(e|G_i) + \sum_{e=uv\in R_i} m_u(e|G_i)\Big) \\ + \sum_{i=1}^{d-1}\alpha_i(|E(G)| - \alpha_i - 1), \end{aligned}$$

where $\alpha_i = \sum_{j=1}^{i}|E(G_j)| + i - 1$ *for all* $i = 1, 2, \ldots, d.$ □

2.2. Theorem. *The revised edge Szeged index of the bridge graph* $G = B(G_1, G_2, \ldots, G_d)$ *of* $\{G_i\}_{i=1}^d$ *with respect to the vertices* $\{v_i\}_{i=1}^d$ *is given by*

$$\begin{aligned} Sz_e^*(G) = \sum_{i=1}^{d} Sz_e^*(G_i) + \frac{1}{4}\sum_{i=1}^{d}(|E(G)|^2 - |E(G_i)|^2)|Q_i| \\ + \sum_{i=1}^{d}(|E(G)| - |E(G_i)|)(l_i + r_i) + \sum_{i=1}^{d-1}\alpha_i(|E(G)| - \alpha_i - 1) \\ + \frac{1}{2}(d-1)\left(|E(G)| - \frac{1}{2}\right), \end{aligned}$$

where

$$\begin{aligned} l_i &= \sum_{e=uv\in L_i}\left(m_v(e|G_i) + \frac{1}{2}m_0(e|G_i)\right), \\ r_i &= \sum_{e=uv\in R_i}\left(m_u(e|G_i) + \frac{1}{2}m_0(e|G_i)\right), \\ \alpha_i &= \sum_{j=1}^{i}|E(G_j)| + i - 1 \end{aligned}$$

for all $i = 1, 2, \ldots, d.$

Proof. Let $|E(G)| = m$ and $|E(G_i)| = m_i$. Obviously, $m_u(e|G)+m_v(e|G)+m_0(e|G) = m$ and $m_u(e|G_i) + m_v(e|G_i) + m_0(e|G_i) = m_i$. Note that $E(G_i) = Q_i \cup L_i \cup R_i$ for $i = 1, 2, \ldots, d$. From the definition of the revised edge Szeged index, we have

$$\begin{aligned} Sz_e^*(G) &= \sum_{e=uv\in E(G)} m_u(e|G)m_v(e|G) \\ &\quad + \sum_{e=uv\in E(G)}\left[\frac{m_0(e|G)}{2}(m_u(e|G) + m_v(e|G)) + \frac{m_0^2(e|G)}{4}\right] \\ &= Sz_e(G) + \sum_{e=uv\in E(G)}\left[\frac{m_0(e|G)}{2}(m - m_0(e|G)) + \frac{m_0^2(e|G)}{4}\right] \\ &= Sz_e(G) + \frac{1}{2}\sum_{e=uv\in E(G)} m_0(e|G)\left(m - \frac{m_0(e|G)}{2}\right) \end{aligned}$$

$$= Sz_e(G) + \frac{1}{2}\sum_{i=1}^{d}\sum_{e=uv\in Q_i} m_0(e|G)\left(m - \frac{m_0(e|G)}{2}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G)\left(m - \frac{m_0(e|G)}{2}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{d-1} m_0(v_iv_{i+1}|G)\left(m - \frac{m_0(v_iv_{i+1}|G)}{2}\right).$$

For $i = 1, 2, \ldots, d$, if $e = uv \in Q_i$, then all the edges in $E(G)\backslash E(G_i)$ are equidistant from both ends of the edge $e = uv$, and thus $m_0(e|G) = m_0(e|G_i) + m - m_i$. Then

$$\sum_{i=1}^{d}\sum_{e=uv\in Q_i} m_0(e|G)\left(m - \frac{m_0(e|G)}{2}\right)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in Q_i} (m_0(e|G_i) + m - m_i)\left(m - \frac{m_0(e|G_i) + m - m_i}{2}\right)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in Q_i} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{d}\sum_{e=uv\in Q_i} (m^2 - m_i^2)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in Q_i} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{d}(m^2 - m_i^2)|Q_i|.$$

For $i = 1, 2, \ldots, d$, if $e = uv \in L_i \cup R_i$, then there is no edge in $E(G)\backslash E(G_i)$ which is equidistant from both ends of the edge $e = uv$, and thus $m_0(e|G) = m_0(e|G_i)$. Then

$$\sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G)\left(m - \frac{m_0(e|G)}{2}\right)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i)\left(m - \frac{m_0(e|G_i)}{2}\right)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)$$
$$+ \sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i)(m - m_i)$$
$$= \sum_{i=1}^{d}\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)$$
$$+ \sum_{i=1}^{d}(m - m_i)\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i).$$

For $i = 1, 2, \ldots, d$, it is obvious that $m_0(v_iv_{i+1}|G) = 1$. Thus

$$\sum_{i=1}^{d-1} m_0(v_iv_{i+1}|G)\left(m - \frac{m_0(v_iv_{i+1}|G)}{2}\right) = (d-1)\left(m - \frac{1}{2}\right).$$

It follows that

$$\begin{aligned}Sz_e^*(G) = Sz_e(G) &+ \frac{1}{2}\sum_{i=1}^{d}\sum_{e=uv\in E(G_i)} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)\\ &+ \frac{1}{4}\sum_{i=1}^{d}(m^2 - m_i^2)|Q_i| + \frac{1}{2}\sum_{i=1}^{d}(m-m_i)\sum_{e=uv\in L_i\cup R_i} m_0(e|G_i)\\ &+ \frac{1}{2}(d-1)\left(m - \frac{1}{2}\right).\end{aligned}$$

By Lemma 2.1,

$$\begin{aligned}&Sz_e^*(G)\\ &= \sum_{i=1}^{d} Sz_e(G_i) + \sum_{i=1}^{d}(m-m_i)\left(\sum_{e=uv\in L_i} m_v(e|G_i) + \sum_{e=uv\in R_i} m_u(e|G_i)\right)\\ &\quad + \sum_{i=1}^{d-1}\alpha_i(m-\alpha_i-1) + \frac{1}{2}\sum_{i=1}^{d}\sum_{e=uv\in E(G_i)} m_0(e|G_i)\left(m_i - \frac{m_0(e|G_i)}{2}\right)\\ &\quad + \frac{1}{4}\sum_{i=1}^{d}(m^2-m_i^2)|Q_i| + \sum_{i=1}^{d}(m-m_i)\sum_{e=uv\in L_i\cup R_i}\frac{m_0(e|G_i)}{2}\\ &\quad + \frac{1}{2}(d-1)\left(m-\frac{1}{2}\right)\\ &= \sum_{i=1}^{d} Sz_e^*(G_i) + \frac{1}{4}\sum_{i=1}^{d}(m^2-m_i^2)|Q_i| + \sum_{i=1}^{d}(m-m_i)(l_i+r_i)\\ &\quad + \sum_{i=1}^{d-1}\alpha_i(m-\alpha_i-1) + \frac{1}{2}(d-1)\left(m-\frac{1}{2}\right),\end{aligned}$$

as desired. □

For a connected graph H with vertex w, let $G_d(H,w)$ be the bridge graph

$$B(G_1, G_2, \ldots, G_d) = B(G_1, G_2, \ldots, G_d; v_1, v_2, \ldots, v_d)$$

with $G_1 = G_2 = \cdots = G_d = H$ and $v_1 = v_2 = \cdots = v_d = w$, i.e.,

$$G_d(H,w) = B(\underbrace{H, \ldots, H}_{d\,\text{times}}; \underbrace{w, \ldots, w}_{d\,\text{times}}).$$

2.3. Corollary. *Let H be a connected graph with m edges. Then the revised edge Szeged index of the bridge graph $G_d(H,w)$ is given by*

$$\begin{aligned}Sz_e^*(G_d(H,w)) = dSz_e^*(H) &+ \frac{1}{4}d(d-1)(m+1)(dm+d+m-1)|Q_{H,w}|\\ &+ d(d-1)(m+1)(l_H + r_H)\\ &+ (d-1)\left(\frac{(m+1)d(md+m+d-2)}{6} + \frac{1}{4}\right),\end{aligned}$$

where

$$l_H = \sum_{e=uv \in L_{H,w}} \Big(m_v(e|H) + \frac{1}{2} m_0(e|H) \Big),$$

$$r_H = \sum_{e=uv \in R_{H,w}} \Big(m_u(e|H) + \frac{1}{2} m_0(e|H) \Big).$$

Proof. Since $|E(H)| = m$, we have $|E(G_d(H,w))| = dm + d - 1$ and $\alpha_i = \sum_{j=1}^{i} |E(H)| + i - 1 = (m+1)i - 1$. Thus

$$\begin{aligned}
&\sum_{i=1}^{d-1} \alpha_i (|E(G_d(H,w))| - \alpha_i - 1) \\
&\qquad = \sum_{i=1}^{d-1} [(m+1)i - 1][(dm + d - 1) - ((m+1)i - 1) - 1] \\
&\qquad = \sum_{i=1}^{d-1} [-(m+1)^2 i^2 + (m+1)^2 di - (m+1)d + 1] \\
&\qquad = (d-1)\Big(\frac{(m+1)d(md + m + d - 5)}{6} + 1 \Big).
\end{aligned}$$

By Theorem 2.1, it is easily seen that

$$\begin{aligned}
Sz_e^*(G_d(H,w)) &= \sum_{i=1}^{d} Sz_e^*(H) + \frac{1}{4} \sum_{i=1}^{d} (|E(G_d(H,w))|^2 - |E(H)|^2) |Q_{H,w}| \\
&\quad + \sum_{i=1}^{d} (|E(G_d(H,w))| - |E(H)|)(l_H + r_H) \\
&\quad + \sum_{i=1}^{d-1} \alpha_i (|E(G_d(H,w))| - \alpha_i - 1) \\
&\quad + \frac{1}{2}(d-1) \left(|E(G_d(H,w))| - \frac{1}{2} \right) \\
&= dSz_e^*(H) + \frac{1}{4} \sum_{i=1}^{d} ((dm + d - 1)^2 - m^2) |Q_{H,w}| \\
&\quad + \sum_{i=1}^{d} (dm + d - 1 - m)(l_H + r_H) \\
&\quad + (d-1) \left(\frac{(m+1)d(md + m + d - 5)}{6} + 1 \right) \\
&\quad + \frac{1}{2}(d-1) \left(dm + d - 1 - \frac{1}{2} \right) \\
&= dSz_e^*(H) + \frac{1}{4} d(d-1)(m+1)(dm + d + m - 1) |Q_{H,w}| \\
&\quad + d(d-1)(m+1)(l_H + r_H) \\
&\quad + (d-1) \left(\frac{(m+1)d(md + m + d - 2)}{6} + \frac{1}{4} \right),
\end{aligned}$$

as desired. □

Let $P_n = u_1u_2\dots u_n$ be the path on n vertices. Obviously, $P_n = G_n(P_1, u_1)$. By Corollary 2.3,

$$Sz_e^*(P_n) = (n-1)\left(\frac{n(n-2)}{6} + \frac{1}{4}\right) = \binom{n}{3} + \frac{1}{4}(n-1).$$

2.4. Corollary. *The revised edge Szeged index of the bridge graph $A_{d,n} = G_d(P_n, u_1)$ is given by*

$$Sz_e^*(A_{d,n}) = d(3d-2)\binom{n}{3} + 2\binom{d}{2}\binom{n}{2} + n^2\binom{d+1}{3} - \frac{1}{4}(2nd^2 - 3nd + 1).$$

Proof. Note that $Sz_e^*(P_n) = \binom{n}{3} + \frac{1}{4}(n-1)$ and

$$\begin{aligned} l_{P_n} + r_{P_n} &= \sum_{e=uv\in L_{P_n,u_1}} \left(m_v(e|P_n) + \frac{1}{2}\right) + \sum_{e=uv\in R_{P_n,u_1}} \left(m_u(e|P_n) + \frac{1}{2}\right) \\ &= \binom{n-1}{2} + \frac{1}{2}(n-1). \end{aligned}$$

By Corollary 2.3, we have

$$\begin{aligned} &Sz_e^*(A_{d,n}) \\ &\quad = dSz_e^*(P_n) + \frac{1}{4}d(d-1)n(d(n-1) + d + (n-1) - 1)\cdot 0 \\ &\qquad + d(d-1)n(l_{P_n} + r_{P_n}) + (d-1)\left(\frac{nd((n-1)d + (n-1) + d - 2)}{6} + \frac{1}{4}\right) \\ &\quad = d\left[\binom{n}{3} + \frac{1}{4}(n-1)\right] + d(d-1)n\left[\binom{n-1}{2} + \frac{1}{2}(n-1)\right] \\ &\qquad + (d-1)\left[\frac{n^2d(d+1)}{6} - \frac{1}{4}(2nd-1)\right] \\ &\quad = d\left[\binom{n}{3} + \frac{1}{4}(n-1)\right] + 2\binom{d}{2}\left[3\binom{n}{3} + \binom{n}{2}\right] \\ &\qquad + n^2\binom{d+1}{3} - \frac{1}{4}(d-1)(2nd-1) \\ &\quad = d(3d-2)\binom{n}{3} + 2\binom{d}{2}\binom{n}{2} + n^2\binom{d+1}{3} - \frac{1}{4}(2nd^2 - 3nd + 1), \end{aligned}$$

as desired. □

Let $C_n = u_1u_2\cdots u_nu_1$ be the cycle on n vertices.

2.5. Corollary. *The revised edge Szeged index of the bridge graph $T_{d,n} = G_d(C_n, u_1)$ is given by*

$$Sz_e^*(T_{d,n}) = \begin{cases} \frac{1}{4}dn^3 + \frac{1}{4}d(d-1)(n+1)(dn + d + 2n^2 - n - 1) \\ \quad + (d-1)\Big(\frac{(n+1)d(nd+n+d-2)}{6} + \frac{1}{4}\Big) & \text{if } n \text{ is odd,} \\ \frac{1}{4}dn^3 + \frac{1}{2}dn^2(d-1)(n+1) \\ \quad + (d-1)\Big(\frac{(n+1)d(nd+n+d-2)}{6} + \frac{1}{4}\Big) & \text{if } n \text{ is even.} \end{cases}$$

Proof. By direct calculation, we have $Sz_e^*(C_n) = \frac{n^3}{4}$. If n is odd, then $|Q_{C_n,u_1}| = 1$ and

$$\begin{aligned} l_{C_n} + r_{C_n} &= \sum_{e=uv\in L_{C_n,u_1}} \left(m_v(e|C_n) + \frac{1}{2} \right) + \sum_{e=uv\in R_{C_n,u_1}} \left(m_u(e|C_n) + \frac{1}{2} \right) \\ &= \frac{1}{2} n(n-1). \end{aligned}$$

By Corollary 2.3, we have

$$\begin{aligned} Sz_e^*(T_{d,n}) &= d \cdot \frac{n^3}{4} + \frac{1}{4} d(d-1)(n+1)(dn+d+n-1) \cdot 1 \\ &\quad + d(d-1)(n+1)\frac{1}{2}n(n-1) \\ &\quad\quad + (d-1)\left(\frac{(n+1)d(nd+n+d-2)}{6} + \frac{1}{4} \right) \\ &= \frac{1}{4}dn^3 + \frac{1}{4}d(d-1)(n+1)(dn+d+2n^2-n-1) \\ &\quad + (d-1)\left(\frac{(n+1)d(nd+n+d-2)}{6} + \frac{1}{4} \right). \end{aligned}$$

If n is even, then $|Q_{C_n,u_1}| = 0$, $l_{C_n} + r_{C_n} = \frac{n^2}{2}$, and arguing as above, we have the desired expression for $Sz_e^*(T_{d,n})$. □

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