

SOFT BI-IDEALS RELATED TO GENERALIZED FUZZY BI-IDEALS IN SEMIGROUPS

O. Kazancı^{*†}, Ş. Yılmaz^{*}

Received 25:05:2010 : Accepted 01:11:2011

Abstract

In this paper, the concepts of \in -soft set and q -soft set are introduced and some interesting properties are investigated. Using the notion of generalized fuzzy bi-ideals in a semigroup, characterizations for an \in -soft set and a q -soft set to be bi-idealistic soft semigroups are established.

Keywords: Semigroup, Bi-ideal, Soft semigroup, Bi-idealistic soft semigroup, Belong to, Quasi-coincident with, $(\in, \in \vee q)$ -fuzzy bi-ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$ -fuzzy bi-ideals.

2000 AMS Classification: 20N25, 20M12.

1. Introduction

The theory of fuzzy sets, which was introduced by Zadeh [28], has been applied to many mathematical branches. Rosenfeld [25] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy subgroup. Das [6] characterized fuzzy subgroups by their level subgroups. The concept of a fuzzy ideal in semigroups was developed by Kuroki [14]–[18]. He studied fuzzy ideals, fuzzy bi-ideals and fuzzy semiprime ideals in semigroups. Fuzzy ideals and Green's relations in semigroups were investigated by McLean and Kummer [21]. Dib and Galhum [7] introduced definitions of a fuzzy groupoid and a fuzzy semigroup, studied fuzzy ideals and fuzzy bi-ideals of a fuzzy semigroup. Murali [24] proposed a definition of a fuzzy point belonging to a fuzzy subset under a natural equivalence on a fuzzy set. The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [22], played a vital role in generating some different types of fuzzy subgroup. A new type of fuzzy subgroup (viz, a $(\in, \in \vee q)$ -fuzzy subgroup) was introduced in an earlier paper Bhakat and Das [3, 4] by using the combined notions of “belongingness” and “quasi-coincidence” of fuzzy points and fuzzy sets. In fact, a

^{*}Department of Mathematics, Karadeniz Technical University, 61080 Trabzon, Turkey.
E-mail: (O. Kazancı) kazancio@yahoo.com (Ş. Yılmaz) serifeyilmaz@ktu.edu.tr

[†]Corresponding Author.

$(\in, \in \vee q)$ -fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup.

It is now natural to investigate similar types of generalizations of the existing fuzzy subsystems of other algebraic structures. With this objective in view, Kazancı *et al.* [11] discussed a kind of generalized fuzzy bi-ideal of semigroups. Recently, several new results have been obtained about characterizations of fuzzy semigroups by fuzzy ideals [12].

In 1992, Molodtsov [23] introduced the concept of soft set, which can be seen as a new mathematical tool for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applicable to many different fields. For example, the study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement and so on. At present, works on soft set theory are progressing rapidly. Maji *et al.* [20] described an application of soft set theory to a decision making problem. Regarding theoretical aspects, Maji *et al.* [19] defined several operations on the theory of soft sets. Chen *et al.* [5] presented a new definition of soft set parameterization reduction, and compared this definition to the related concept of attribute reduction in rough set theory. Some results on an application of fuzzy-soft-sets in decision making problem has been given by Roy *et al.* [26]. Up to now, the algebraic structure of soft sets has been investigated by some authors (see [1, 2, 8, 9, 10, 13, 27, 29]).

In this paper, based on [11], we continue to study the characterization of semigroups. We deal with soft semigroups based on fuzzy sets by means of \in -soft sets and q -soft sets. Using the notion of generalized fuzzy bi-ideals in semigroups, we provide characterizations for an \in -soft set and a q -soft set to be bi-idealistic soft semigroups.

2. Preliminaries

We first recall some of the basic definitions proposed by pioneers in this field ([14]–[18]). Let S be a semigroup. By a *subsemigroup* of S we mean a non-empty subset A of S such that $A^2 \subseteq A$, and by a *left (right) ideal* of S a non-empty set A such that $SA \subseteq A$ ($AS \subseteq A$). By a *two sided ideal*, we mean a non-empty subset of S which is both a left and right ideal of S . A subsemigroup A of a semigroup S is called a *bi-ideal* of S if $ASA \subseteq A$.

Now we recall some structures commonly used in fuzzy set theory. In 1965, Zadeh [28] introduced the notion of a *fuzzy subset* A of a non-empty set X as a membership function $\mu_A : X \rightarrow [0, 1]$ which associates each point $x \in X$ with its “degree of membership” $\mu_A(x) \in [0, 1]$. The *complement* of A , denoted by A^c , is the fuzzy subset given by $\mu_{A^c}(x) = 1 - \mu_A(x)$ for all $x \in X$. In 1971, Rosenfeld [25] applied the concept of fuzzy sets to the theory of groups and studied fuzzy subgroups of a group.

The concept of a fuzzy ideal in semigroups was developed by Kuroki [14]–[18]. He studied fuzzy ideals, fuzzy bi-ideals and fuzzy semiprime ideals in semigroups.

2.1. Definition. [14] Let μ be a fuzzy subset of S .

(i) μ is called a *fuzzy subsemigroup* of S if

$$\mu(xy) \geq \mu(x) \wedge \mu(y) \text{ for all } x, y \in S.$$

(ii) A fuzzy subsemigroup μ is called a *fuzzy ideal* of S if

$$\mu(xy) \geq \mu(x) \vee \mu(y) \text{ for all } x, y \in S.$$

(iii) A fuzzy subsemigroup μ is called a *fuzzy bi-ideal* of S if

$$\mu(xay) \geq \mu(x) \wedge \mu(y) \text{ for all } x, a, y \in S.$$

2.2. Definition. [22] A fuzzy subset μ in a set S of the form

$$\mu(y) = \begin{cases} t \neq 0 & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

is said to be a *fuzzy point with support x and value t* , and is denoted x_t . A fuzzy point x_t is said to be *belong to* (resp. *be quasi-coincident with*) a fuzzy set μ , written as $x_t \in \mu$ (resp. $x_t q\mu$) if $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t q\mu$, then we write $x_t \in \vee q\mu$. The symbol $\overline{\in \vee q}$ means neither \in nor q hold.

2.3. Definition. [11] A fuzzy subset μ of a semigroup S is called an $(\in, \in \vee q)$ -fuzzy bi-ideal of S if for all $t, r \in (0, 1]$, $x, a, y \in S$,

- (i) $x_t, y_r \in \mu$ implies $(xy)_{t \wedge r} \in \vee q\mu$;
- (ii) $x_t, y_r \in \mu$ and $a \in S$ implies $(xay)_{t \wedge r} \in \vee q\mu$.

2.4. Theorem. [11] Let μ be a fuzzy subset of a semigroup S . Then μ is an $(\in, \in \vee q)$ -fuzzy bi-ideal of S if and only if the following conditions are hold.

- (i) $\mu(xy) \geq \mu(x) \wedge \mu(y) \wedge 0.5$;
- (ii) $\mu(xay) \geq \mu(x) \wedge \mu(y) \wedge 0.5$

for all $x, a, y \in S$. □

2.5. Definition. [11] A fuzzy subset μ of a semigroup S is said to be an $(\overline{\in}, \overline{\in} \vee \overline{q})$ -fuzzy bi-ideal of S if for all $t, r \in (0, 1]$, $x, a, y \in S$,

- (i) $(xy)_{t \wedge r} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q}\mu$, or $y_r \overline{\in} \vee \overline{q}\mu$;
- (ii) $(xay)_{t \wedge r} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q}\mu$, or $y_r \overline{\in} \vee \overline{q}\mu$.

2.6. Theorem. [11] Let μ be a fuzzy subset of a semigroup S . Then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ -fuzzy bi-ideal of S if and only if the following conditions are hold.

- (i) $\mu(xy) \vee 0.5 \geq \mu(x) \wedge \mu(y)$;
- (ii) $\mu(xay) \vee 0.5 \geq \mu(x) \wedge \mu(y)$,

for all $x, a, y \in S$. □

2.7. Definition. [11] Let $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$. Let μ be a fuzzy subset of a semigroup S . Then μ is called a *fuzzy bi-ideal with thresholds* of S if it satisfies the following conditions:

- (i) $\mu(xy) \vee \alpha \geq \mu(x) \wedge \mu(y) \wedge \beta$;
- (ii) $\mu(xay) \vee \alpha \geq \mu(x) \wedge \mu(y) \wedge \beta$,

for all $x, y, a \in S$.

3. Bi-idealistic soft semigroups

Molodtsov [23] defined the notion of a soft set in the following way: Let U be an initial universe set and E a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E .

3.1. Definition. [23] If $F : A \rightarrow \mathcal{P}(U)$ is a mapping the pair (F, A) is called a *soft set over U* . In other words, a soft set over U is a parameterized family of subsets of the universe U . For $x \in A$, $F(x)$ may be considered as the set of x -approximate elements of the soft set (F, A) . Clearly, a soft set is not merely a set.

In what follows, unless otherwise specified, S and A will be a semigroup and a nonempty set, respectively, and R an arbitrary binary relation between the elements of A and those of S , that is, R is a subset of $A \times S$. The equalities $F(x) = \{y \in S \mid (x, y) \in R\}$, $x \in A$, define a set-valued function $F : A \rightarrow \mathcal{P}(S)$ and the pair (F, A) is then a soft set over S which is derived from the relation R .

For a soft set (F, A) , the set $\text{Supp}(F, A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the *support* of the soft set (F, A) and the soft set (F, A) is called *non-null* if $\text{Supp}(F, A) \neq \emptyset$ [8].

3.2. Definition. Let (F, A) be a non-null soft set over S . Then (F, A) is called a *soft semigroup* over S if $F(x)$ is a *subsemigroup* of S for all $x \in \text{Supp}(F, A)$.

3.3. Example. Let $S = \{a, b, c, d, e\}$ be a semigroup with the following Cayley table.

·	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

(i) Let (F, A) be a soft set over S , where $A = S$ and $F : A \rightarrow \mathcal{P}(S)$ is the set-valued function defined by

$$F(x) = \{y \in A \mid y \cdot (y \cdot x) = a\}$$

for all $x \in A$. Then $\text{Supp}(F, A) = A$, and $F(a) = F(b) = S$, $F(c) = F(d) = F(e) = \{a, b\}$ are all subsemigroups of S . Therefore (F, A) is a soft semigroup over S .

(ii) Let (F, A) be a soft set over S , where $A = \{c, d, e\}$ and $F : A \rightarrow \mathcal{P}(S)$ is the set-valued function defined by

$$F(x) = \begin{cases} \{y \in S \mid y \cdot (y \cdot x) = a\}, & \text{if } x \in \{c, d\}, \\ \emptyset, & \text{if } x = e. \end{cases}$$

Then $\text{Supp}(F, A) = \{c, d\} \subset A$ and $F(c) = \{a, b\}$, $F(d) = \{a, b\}$ are both subsemigroups of S . This example shows that $\text{Supp}(F, A)$ can be a proper subset of A .

3.4. Definition. Let (F, A) be a non-null soft set over S . Then (F, A) is called an *idealistic soft semigroup* over S if $F(x)$ is an *ideal* of S for all $x \in \text{Supp}(F, A)$.

3.5. Example. Let $S = \{a, b, c, d, e\}$ be a semigroup with the following Cayley table.

·	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

Let (F, A) be a soft set over S , where $A = \{a, b, c\}$ and $F : A \rightarrow \mathcal{P}(S)$ is the set-valued function defined by

$$F(x) = \{y \in S \mid y \cdot x \in \{a, e\}\}$$

for all $x \in A$. Then $\text{Supp}(F, A) = A$, and $F(a) = F(b) = S$, $F(c) = \{a, b\}$ are all ideals of S . Therefore (F, A) is an idealistic soft semigroup over S .

3.6. Definition. Let (F, A) be a non-null soft set over S . Then (F, A) is called a *bi-idealistic soft semigroup* over S if $F(x)$ is a *bi-ideal* of S for all $x \in \text{Supp}(F, A)$.

3.7. Example. Let $S = \{0, 1, 2, 3, 4\}$ be a semigroup with the following Cayley table.

·	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	2
3	0	0	0	2	3
4	0	1	2	3	4

Let (F, A) be a soft set over S , where $A = \{0, 1, 2\}$ and $F : A \rightarrow \mathcal{P}(S)$ is the set-valued function defined by

$$F(x) = \{y \in S \mid y \cdot x \in \{0, 2, 3\}\}$$

for all $x \in A$. Then $\text{Supp}(F, A) = A$, and $F(0) = F(2) = S$, $F(1) = \{0, 1, 2, 3\}$ are all bi-ideals of S . Therefore (F, A) is a bi-idealistic soft semigroup over S .

Note that, for convenience, the empty set \emptyset is regarded as a bi-ideal of a semigroup S .

Given a fuzzy subset μ of a semigroup S and $A \subseteq [0, 1]$, consider the set-valued functions $F : A \rightarrow \mathcal{P}(S)$ and $F_q : A \rightarrow \mathcal{P}(S)$ over S defined by

$$F(t) = \{x \in S \mid x_t \in \mu\}, \quad F_q(t) = \{x \in S \mid x_t q \mu\},$$

for all $t \in A$, respectively. Then (F, A) and (F_q, A) are soft sets over S called the \in -soft set and the q -soft set over S , respectively, determined by μ .

3.8. Theorem. *Let μ be a fuzzy set of S and (F, A) the corresponding \in -soft set over S with $A = (0, 1]$. Then (F, A) is a bi-idealistic soft semigroup over S if and only if μ is a fuzzy bi-ideal of S .*

Proof. Suppose that (F, A) is a bi-idealistic soft semigroup over S . If there exist $a, b \in S$ such that $\mu(ab) < \mu(a) \wedge \mu(b)$, then we can choose $t \in A$ such that $\mu(ab) < t \leq \mu(a) \wedge \mu(b)$. Then $a_t \in \mu$ and $b_t \in \mu$, but $(ab)_t \notin \mu$, that is, $a_t \in F(t)$ and $b_t \in F(t)$, but $(ab)_t \notin F(t)$. This is a contradiction. Therefore $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in S$, that is, μ is a fuzzy subsemigroup of S .

If there exist $u, v, s \in S$ such that $\mu(usv) < \mu(u) \wedge \mu(v)$, then we can choose $t \in A$ such that $\mu(usv) < t \leq \mu(u) \wedge \mu(v)$. Then $u_t \in \mu$ and $v_t \in \mu$, but $(usv)_t \notin \mu$, that is, $u_t \in F(t)$ and $v_t \in F(t)$, but $(usv)_t \notin F(t)$. This is a contradiction. Therefore $\mu(xay) \geq \mu(x) \wedge \mu(y)$ for all $x, y, a \in S$. Hence μ is a fuzzy bi-ideal of S .

Conversely, suppose that μ is a fuzzy bi-ideal of S . Let $t \in A$ and $x, y \in F(t)$. Then $x_t \in \mu$ and $y_t \in \mu$, that is, $\mu(x) \geq t$ and $\mu(y) \geq t$. Since μ is a fuzzy subsemigroup of S , it follows that $\mu(xy) \geq \mu(x) \wedge \mu(y) \geq t \wedge t = t$ and so $(xy)_t \in \mu$. Hence $xy \in F(t)$. Thus $F(t)$ is a subsemigroup of S , i.e., (F, A) is a soft semigroup over S .

Let $t \in A$, $x, y \in F(t)$ and $a \in S$. Then $x_t \in \mu$ and $y_t \in \mu$, that is, $\mu(x) \geq t$, $\mu(y) \geq t$. Since μ is a fuzzy bi-ideal of S , we obtain $\mu(xay) \geq \mu(x) \wedge \mu(y) \geq t \wedge t = t$ and so $(xay)_t \in \mu$. Hence $xay \in F(t)$. Thus $F(t)$ is a bi-ideal of S , i.e., (F, A) is a bi-idealistic soft semigroup over S . □

3.9. Theorem. *Let μ be a fuzzy set of S and (F_q, A) an q -soft set over S with $A = (0, 1]$. Then the following conditions are equivalent:*

- (i) μ is a fuzzy bi-ideal of S ;
- (ii) If $F_q(t) \neq \emptyset$, then $F_q(t)$ is a bi-ideal of S for all $t \in A$.

Proof. Let μ be a fuzzy bi-ideal of S . Let $t \in A$ and $x, y \in F_q(t)$. Then $x_t q \mu$ and $y_t q \mu$, that is $\mu(x) + t > 1$ and $\mu(y) + t > 1$. Since μ is a fuzzy subsemigroup of S , we get $\mu(xy) + t \geq \mu(x) \wedge \mu(y) + t = (\mu(x) + t) \wedge (\mu(y) + t) > 1$ and so $(xy)_t q \mu$, that is $xy \in F_q(t)$.

Now for every $x, y \in F_q(t)$, $t \in A$ and $a \in S$, we have $x_t q \mu$, $y_t q \mu$ that is $\mu(x) + t > 1$, $\mu(y) + t > 1$. Since μ is a fuzzy bi-ideal of S , we get $\mu(xay) + t \geq \mu(x) \wedge \mu(y) + t = (\mu(x) + t) \wedge (\mu(y) + t) > 1$, and so $(xay)_t q \mu$ i.e., $xay \in F_q(t)$. Therefore $F_q(t)$ is a bi-ideal of S for all $t \in A$.

Conversely, Let μ be a fuzzy subset in S such that the set $F_q(t)$ is a bi-ideal of S for all $t \in A$. If there exist $a, b \in S$ such that $\mu(ab) < \mu(a) \wedge \mu(b)$, then we can choose $t \in A$ such that $\mu(ab) + t \leq 1 < (\mu(a) \wedge \mu(b)) + t = (\mu(a) + t) \wedge (\mu(b) + t)$. Then $a_t q \mu$ and $b_t q \mu$, that is, $a \in F_q(t)$ and $b \in F_q(t)$.

Since $F_q(t) (\neq \emptyset)$ is a subsemigroup of S , we get $(ab) \in F_q(t)$, that is, $(ab)_t q \mu$ or equivalently $\mu(ab) + t > 1$. This is a contradiction, and so $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in S$. Therefore μ is a fuzzy subsemigroup of S . If there exist $u, v, s \in S$ such that $\mu(usb) < \mu(u) \wedge \mu(v)$, then we can choose $t \in A$ such that $\mu(usb) + t \leq 1 < (\mu(u) \wedge \mu(v)) + t = (\mu(u) + t) \wedge (\mu(v) + t)$. Then $u_t q \mu$ and $v_t q \mu$ i.e., $u \in F_q(t)$ and $v \in F_q(t)$. Since $F_q(t)$ is a bi-ideal of S , we have $(usb) \in F_q(t)$, that is, $(usb)_t q \mu$, and so $\mu(usb) + t \geq 1$ contradiction. Hence $\mu(xay) \geq \mu(x) \wedge \mu(y)$ for all $x, a, y \in S$. Therefore μ is a fuzzy bi-ideal of S . \square

3.10. Theorem. Let μ be a fuzzy set of S and (F, A) the corresponding \in -soft set over S with $A = (0, 0.5]$. Then the following conditions are equivalent:

- (i) μ is an $(\in, \in \vee q)$ fuzzy bi-ideal of S ;
- (ii) (F, A) is a bi-idealistic soft semigroup over S .

Proof. Let μ is an $(\in, \in \vee q)$ fuzzy bi-ideal of S . For any $t \in A$ and $x, y \in F(t)$, we have $x_t \in \mu$ and $y_t \in \mu$, that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.4 (i), we obtain $\mu(xy) \geq \mu(x) \wedge \mu(y) \wedge 0.5 \geq t \wedge t \wedge 0.5 = t$, and so $\mu(xy) \geq t$, that is, $(xy)_t \in \mu$, i.e., $xy \in F(t)$. Hence $F(t)$ is a subsemigroup of S for all $t \in A$.

For any $t \in A$, $x, y \in F(t)$ and $a \in S$. Then $x_t \in \mu$, $y_t \in \mu$ that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.4 (ii), we obtain $\mu(xay) \geq \mu(x) \wedge \mu(y) \wedge 0.5 \geq t \wedge 0.5 = t$, and so $\mu(xay) \geq t$, that is, $(xay)_t \in \mu$, i.e., $xay \in F(t)$ for all $t \in A$. Therefore (F, A) is a bi-idealistic soft semigroup over S .

Assume that (F, A) is a bi-idealistic soft semigroup over S . If there exist $a, b \in S$ such that $\mu(ab) < \mu(a) \wedge \mu(b) \wedge 0.5$, then we can choose $t \in A$ such that $\mu(ab) < t \leq \mu(a) \wedge \mu(b) \wedge 0.5$, which implies $a_t \in \mu$ and $b_t \in \mu$, but $(ab)_t \notin \mu$, that is, $a_t \in F(t)$ and $b_t \in F(t)$, but $(ab)_t \notin F(t)$. This is a contradiction, and so $\mu(xy) \geq \mu(x) \wedge \mu(y) \wedge 0.5$ for all $x, y \in S$. Hence μ is an $(\in, \in \vee q)$ fuzzy subsemigroup of S .

If there exist $u, v, s \in S$ such that $\mu(usb) < \mu(u) \wedge \mu(v) \wedge 0.5$, then we can choose $t \in A$ such that $\mu(usb) < t \leq \mu(u) \wedge \mu(v) \wedge 0.5$. Then $u_t \in \mu$, $v_t \in \mu$ but $(usb)_t \notin \mu$, that is, $u_t \in F(t)$, $v_t \in F(t)$ but $(usb)_t \notin F(t)$. This is a contradiction, and so $\mu(xay) \geq \mu(x) \wedge \mu(y) \wedge 0.5$ for all $x, a, y \in S$. Therefore, μ is an $(\in, \in \vee q)$ fuzzy bi-ideal of S . \square

3.11. Theorem. Let μ be a fuzzy set of S and (F, A) the corresponding \in -soft set over S with $A = (0.5, 1]$. Then the following conditions are equivalent:

- (i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy bi-ideal of S ;
- (ii) (F, A) is a bi-idealistic soft semigroup over S .

Proof. Let μ be an $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideal of S . For any $t \in A$ and $x, y \in F(t)$, we have $x_t \in \mu$, $y_t \in \mu$, that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.6 (i), we obtain $t \leq \mu(x) \wedge \mu(y) \leq \mu(xy) \vee 0.5 = \mu(xy)$, and so $\mu(xy) \geq t$, that is, $(xy)_t \in \mu$, i.e., $xy \in F(t)$.

Let $x, y \in F(t)$ and $a \in S$. Then $x_t \in \mu$, $y_t \in \mu$ that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.6 (ii), we obtain $t \leq \mu(x) \wedge \mu(y) \leq \mu(xay) \vee 0.5 = \mu(xay)$, and so $\mu(xay) \geq t$, that is, $(xay)_t \in \mu$, i.e., $xay \in F(t)$. Thus, (F, A) is a bi-idealistic soft semigroup over S .

Let (F, A) be a bi-idealistic soft semigroup over S . If there exist $a, b \in S$ such that $\mu(ab) \vee 0.5 < \mu(a) \wedge \mu(b)$, then we can choose $t \in A$ such that $\mu(ab) \vee 0.5 < t \leq \mu(a) \wedge \mu(b)$, which implies $a_t \in \mu$ and $b_t \in \mu$, but $(ab)_t \notin \mu$, that is, $a_t \in F(t)$ and $b_t \in F(t)$, but $(ab)_t \notin F(t)$. This is a contradiction. Therefore $\mu(xy) \vee 0.5 \geq \mu(x) \wedge \mu(y)$ for all $x, y \in S$, that is, μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy subsemigroup of S .

If there exist $u, v, s \in S$ such that $\mu(usv) \vee 0.5 < \mu(u) \wedge \mu(v)$, then we can choose $t \in A$ such that $\mu(usv) \vee 0.5 < t \leq \mu(u) \wedge \mu(v)$. Then $u_t \in \mu$, $v_t \in \mu$ but $(usv)_t \notin \mu$, that is, $u_t \in F(t)$, $v_t \in F(t)$ but $(usv)_t \notin F(t)$. This is a contradiction, and so $\mu(xay) \vee 0.5 \geq \mu(x) \wedge \mu(y)$ for all $x, a, y \in S$. Therefore μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy bi-ideal of S . \square

3.12. Theorem. Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, let μ be a fuzzy set of S and (F, A) an \in -soft set over S with $A = (\alpha, \beta]$. Then the following conditions are equivalent:

- (i) μ is a fuzzy bi-ideal with thresholds $(\alpha, \beta]$ of S ;
- (ii) (F, A) is a bi-idealistic soft semigroup over S .

Proof. Let μ be a fuzzy bi-ideal with thresholds $(\alpha, \beta]$ of S . For any $t \in A$ and $x, y \in F(t)$ then $\mu(x) \geq t$, $\mu(y) \geq t$. By Definition 2.7 (i), we have

$$\alpha < t = t \wedge \beta \leq \mu(x) \wedge \mu(y) \wedge \beta \leq \mu(xy) \vee \alpha,$$

which implies that $\mu(xy) \geq t$, i.e. $(xy)_t \in \mu$ and so $xy \in F(t)$.

Now, if $x, y \in F(t)$, $a \in S$, then $\mu(x) \geq t$, $\mu(y) \geq t$. By Definition 2.7 (ii), we obtain

$$\alpha < t = t \wedge \beta \leq \mu(x) \wedge \mu(y) \wedge \beta \leq \mu(xay) \vee \alpha,$$

so we have $\alpha < t \leq \mu(xay) \vee \alpha$, which implies that $\mu(xay) \geq t$, i.e., $(xay)_t \in \mu$ and so $xay \in F(t)$. Hence $F(t)$ is a bi-ideal of S for all $t \in (\alpha, \beta]$. Therefore (F, A) is a bi-idealistic soft semigroup over S .

Let μ be a fuzzy subset of S such that (F, A) is a bi-idealistic soft semigroup over S . If there exist $a, b \in S$ such that $\mu(ab) \vee \alpha < \mu(a) \wedge \mu(b) \wedge \beta$, then we can choose $t \in A$ such that $\mu(ab) \vee \alpha < t \leq \mu(a) \wedge \mu(b) \wedge \beta$, which implies $a_t \in \mu$ and $b_t \in \mu$, but $(ab)_t \notin \mu$, that is, $a_t \in F(t)$ and $b_t \in F(t)$, but $(ab)_t \notin F(t)$. This is a contradiction. Therefore $\mu(xy) \vee \alpha \geq \mu(x) \wedge \mu(y) \wedge \beta$ for all $x, y \in S$, that is, μ is a fuzzy subsemigroup with thresholds $(\alpha, \beta]$ of S .

If there exist $u, s, v \in S$ such that $\mu(usv) \vee \alpha < \mu(u) \wedge \mu(v) \wedge \beta$, then we can choose $t \in (\alpha, \beta]$ such that $\mu(usv) \vee \alpha < t \leq \mu(u) \wedge \mu(v) \wedge \beta$. Then $u_t \in \mu$, $v_t \in \mu$ but $(usv)_t \notin \mu$, that is, $u_t \in F(t)$, $v_t \in F(t)$, but $(usv)_t \notin F(t)$. This is a contradiction, and so $\mu(xay) \vee \alpha \geq \mu(x) \wedge \mu(y) \wedge \beta$ for all $x, a, y \in S$. Therefore, μ is a fuzzy bi-ideal with thresholds $(\alpha, \beta]$ of S . \square

3.13. Corollary. Let μ be a fuzzy subset of semigroup S and (F, A) the corresponding \in -soft set over S with $A = (0, 1]$. Then the following conditions are equivalent:

- (i) μ is an $(\in, \in \vee q)$ -fuzzy bi-ideal of S if and only if the \in -soft set (F, A) is a bi-idealistic soft semigroup over S for all $t \in (0, 0.5]$.
- (ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ -fuzzy bi-ideal of S if and only if the \in -soft set (F, A) is a bi-idealistic soft semigroup over S for all $t \in (0.5, 1]$. \square

3.14. Remark. The following example shows that there exist a set of parameters A and a fuzzy set μ in S such that

- (i) μ is neither a fuzzy bi-ideal nor an $(\in, \in \vee q)$ -fuzzy bi-ideal of S ;
- (ii) An \in -soft set (F, A) over S is a bi-idealistic soft semigroup over S .

3.15. Example. Let $S = \{0, 1, 2, 3, 4\}$ be a semigroup with the following Cayley table.

·	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	2
3	0	0	0	2	3
4	0	1	2	3	4

Let μ be the fuzzy subset in S defined by $\mu(0) = 0.8$, $\mu(1) = 1$, $\mu(2) = 0.9$, $\mu(3) = 0.4$, and $\mu(4) = 0.5$. Let (F, A) be an \in -soft set over S where $A = (0.3, 0.9]$. Then

$$F(t) = \begin{cases} \{0, 1, 2, 4\} & \text{if } 0.3 < t \leq 0.5, \\ \{0, 1, 2\} & \text{if } 0.5 < t \leq 0.8, \\ \{2\} & \text{if } 0.8 < t \leq 0.9, \end{cases}$$

which are bi-ideals of S . Hence (F, A) is a bi-idealistic soft semigroup over S . But μ ,

- is not a fuzzy bi-ideal of S ,
- is not an $(\in, \in \vee q)$ -fuzzy bi-ideal of S ,

since $\mu(1.2) = \mu(0) = 0.8 \not\geq 0.9 = \mu(1) \wedge \mu(2)$ and $4_{0.5} \in \mu$, but $(4.3.4)_{0.5} = 3_{0.5} \notin \mu$.

4. Conclusion

In this paper, the concepts soft semigroup are presented. Using the notion of generalized fuzzy bi-ideals in semigroups, we provide characterizations for \in -soft sets and q -soft sets to be bi-idealistic soft semigroups. This may lead to new directions of research in different algebraic structures.

Acknowledgements

The authors are thankful to the referees and Professor Yücel Tıraş, Editor-in-Chief, for their valuable comments and suggestions.

References

- [1] Aktaş, H. and Çağman, N. *Soft sets and soft groups*, Information Sciences **177**, 2726–2735, 2007.
- [2] Ali, M. I., Feng, F., Liu, X., Minc, W. K. and Shabir, M. *On some new operations in soft set theory*, Computers and Mathematics with Applications **57**, 1547–1553, 2009.
- [3] Bhakat, S. K. and Das, P. *On the definition of a fuzzy subgroup*, Fuzzy Sets and Systems **51**, 235–241, 1992.
- [4] Bhakat, S. K. and Das, P. *$(\in, \in \vee q)$ -fuzzy subgroup*, Fuzzy Sets and Systems **80**, 359–368, 1996.
- [5] Chen, D., Tsang, E. C. C., Yeung, D. S. and Wang, X. *The parameterization reduction of soft sets and its applications*, Computers and Mathematics with Applications **49**, 757–763, 2005.
- [6] Das, P. S. *Fuzzy groups and level subgroups*, J. Math. Anal. Appl. **85**, 264–269, 1981.
- [7] Dib, K. A. and Galhum, N. *Fuzzy ideals and fuzzy bi-ideals in fuzzy semigroups*, Fuzzy Sets and Systems **92**, 103–111, 1992.
- [8] Feng, F., Jun, Y. B. and Zhao, X. *Soft semirings*, Computers and Mathematics with Applications **56**, 2621–2628, 2008.
- [9] Jun, Y. B. *Soft BCK/BCI-algebras*, Computers and Mathematics with Applications **56**, 1408–1413, 2008.
- [10] Jun, Y. B. and Park, C. H. *Applications of soft sets in ideal theory of BCK/BCI-algebras*, Information Sciences **178**, 2466–2475, 2008.
- [11] Kazancı, O. and Yamak, S. *Generalized fuzzy bi-ideals of semigroup*, Soft Computing **12**, 1119–1124, 2008.

- [12] Kazancı, O. and Yamak, S. *Fuzzy ideals and semiprime fuzzy ideals in semigroups*, Information Sciences **179**, 3720–3731, 2009.
- [13] Kazancı, O., Yılmaz, Ş. and Yamak, S. *Soft sets and soft BCH-algebras*, Hacet. J. Math. Stat. **39** (2), 205–217, 2010.
- [14] Kuroki, N. *On fuzzy ideals and fuzzy bi-ideals in semigroups*, Fuzzy Sets and Systems **5**, 203–215, 1981.
- [15] Kuroki, N. *Fuzzy semiprime ideals in semigroups*, Fuzzy Sets and Systems **8**, 71–79, 1982.
- [16] Kuroki, N. *On fuzzy semigroups*, Information Sciences **53**, 203–236, 1991.
- [17] Kuroki, N. *Fuzzy generalized bi-ideals in semigroups*, Information Sciences **66**, 235–243, 1992.
- [18] Kuroki, N. *Fuzzy semiprime quasi-ideals in semigroups*, Information Sciences **75**, 201–211, 1993.
- [19] Maji, P. K., Biswas, R. and Roy, A. R. *Soft set theory*, Computers and Mathematics with Applications **45**, 555–562, 2003.
- [20] Maji, P. K., Roy, A. R. and Biswas, R. *An application of soft sets in a decision making problem*, Comput. Math. Appl. **44**, 1077–1083, 2002.
- [21] McLean, R. G. and Kummer, H. *Fuzzy ideals in semigroups*, Fuzzy Sets and Systems **48**, 137–140, 1992.
- [22] Ming, P. P. and Ming, Y. M. *Fuzzy topology I, Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. **76**, 571–599, 1980.
- [23] Molodtsov, D. *Soft set theory - first results*, Computers and Mathematics with Applications **37**, 19–31, 1999.
- [24] Murali, V. *Fuzzy points of equivalent fuzzy subsets*, Information Sciences **158**, 277–288, 2004.
- [25] Rosenfeld, A. *Fuzzy groups*, J. Math. Anal. Appl. **35**, 512–517, 1971.
- [26] Roy, A. R. and Maji, P. K. *A fuzzy soft set theoretic approach to decision making problems*, Journal of Computational and Applied Mathematics **203**, 412–418, 2007.
- [27] Shabir, M. and Ali, M. I. *Soft ideals and generalized fuzzy ideals in semigroups*, New Mathematics and Natural Computation **5** (3), 599–615, 2009.
- [28] Zadeh, L. A. **Fuzzy sets**, Information Control **8**, 338–353, 1965.
- [29] Zhan, J. and Jun, Y. B. *Soft BL-algebras based on fuzzy sets*, Computers and Mathematics with Applications **59**, 2037–2046, 2010.