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CLASSIFICATION OF CUBIC EDGE-TRANSITIVE GRAPHS OF ORDER $14p^2$

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Abstract

A graph is called *edge-transitive* if its automorphism group acts transitively on its set of edges. In this paper we classify all connected cubic edge-transitive graphs of order $14p^2$, where p is a prime.

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1. Introduction

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For a graph X, we denote by $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}(X)$ the *vertex set*, the *edge set*, the *arc set* and the *full automorphism group* of X, respectively. For the group-theoretic concepts and notations not defined here we refer to [3, 4, 14, 19, 24].

Let G be a finite group and S a subset of G such that $1 \notin S$ and $S = S^{-1}$. The Cayley graph $X = \mathrm{Cay}(G, S)$ on G with respect to S is defined to have vertex set $V(X) = G$ and edge set $E(X) = \{(g, sg) | g \in G, s \in S\}$. The Cayley graph $X = \mathrm{Cay}(G, S)$ is said to be *normal* if $G \trianglelefteq \mathrm{Aut}(X)$. By definition, $\mathrm{Cay}(G, S)$ is connected if and only if S generates the group G.

An s-arc of a graph X is an ordered $(s + 1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \le i \le s$ and $v_{i-1} \ne v_{i+1}$ for $1 \le i < s$. A graph X is said to be *s-arc-transitive* if $\mathrm{Aut}(X)$ acts transitively on the set of its s-arcs. In particular, 0-arc-transitive means vertex-transitive, and 1-arc-transitive means *arc-transitive* or *symmetric*. X is said to be *s-regular* if $\mathrm{Aut}(X)$ acts regularly on the set of its s-arcs. Tutte [20] showed that every finite connected cubic symmetric graph is s-regular for $1 \le s \le 5$. A subgroup of $\mathrm{Aut}(X)$ is said to be s-regular if it acts regularly

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on the set of s-arcs of X. If a subgroup G of $\mathrm{Aut}(X)$ acts transitively on $V(X)$ and $E(X)$, we say that X is *G-vertex-transitive* and *G-edge-transitive*, respectively. In the special case, when $G = \mathrm{Aut}(X)$, we say that X is vertex-transitive and edge-transitive, respectively.

It can be shown that a G-edge-transitive but not G-vertex-transitive graph X is necessarily bipartite, where the two parts of the bipartition are orbits of $G \leq \mathrm{Aut}(X)$. Moreover, if X is regular then these two parts have the same cardinality. A regular G-edge-transitive but not G-vertex-transitive graph X will be referred to as a *G-semisymmetric graph*. In particular if $G = \mathrm{Aut}(X)$, X is said to be *semisymmetric*.

The classification of cubic symmetric graphs of different orders is given in many papers. In [2, 3], the cubic s-regular graphs up to order 2048 are classified. Throughout this paper, p and q are prime numbers. The s-regular cubic graphs of some orders such as $2p^2$, $4p^2$, $6p^2$, $10p^2$ were classified in [8-11]. Recently cubic s-regular graphs of order $2pq$ were classified in [25].

The study of semisymmetric graphs was initiated by Folkman [13]. For example, cubic semisymmetric graphs of orders $6p^2$, $8p^2$ and $2pq$ were classified in [15, 1, 7]. In this paper we classify cubic edge-transitive (symmetric or semisymmetric) graphs of order $14p^2$.

1.1. Theorem. *Let p be a prime and X a connected cubic edge-transitive graph of order $14p^2$. Then X is isomorphic either to the semisymmetric graph $S126$ or to one s-regular graph, where $1 \leq s \leq 3$. Furthermore,*

(1) *X is 1-regular if and only if X is isomorphic to one of the graphs $F56A$, $F126$, $F350$, $F686A$, $F686C$, $F1694$, $EF14p^2$, where $p \geq 13$, or to $Cay(G,S)$, where $G =< a, b \mid a^2 = b^{7p^2} = 1, aba = b^{-1} > \cong D_{14p^2}$, $S = \{a, ba, b^{t+1}a\}$, $t^2+t+1 = 0$ $(mod 7p^2)$, $p \geq 13$ and $3|(p-1)$.*

(2) *X is 2-regular if and only if X is isomorphic to one of the graphs $F56B$ and $F686B$.*

(3) *X is 3-regular if and only if X is isomorphic to $F56C$.*

2. Preliminaries

Let X be a graph and N a subgroup of $\mathrm{Aut}(X)$. For u, $v \in V(X)$, denote by $\{u, v\}$ the edge incident to u and v in X, and by $N_X(u)$ the set of vertices adjacent to u in X. The *quotient graph* X_N (also denoted by X/N) induced by N is defined as the graph such that the set Σ of N-orbits in $V(X)$ is the vertex set of X_N, and $B, C \in \Sigma$ are adjacent if and only if there exist $u \in B$ and $v \in C$ such that $\{u, v\} \in E(X)$.

A graph $\widetilde{X}$ is called a *covering* of a graph X with projection $\wp : \widetilde{X} \to X$ if there is a surjection $\wp : V(\widetilde{X}) \to V(X)$ such that $\wp|_{N_{\widetilde{X}}(\tilde{v})} : N_{\widetilde{X}}(\tilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\tilde{v} \in \wp^{-1}(v)$. A covering graph $\widetilde{X}$ of X with projection $\wp$ is said to be *regular* (or a *K-covering*) if there is a semiregular subgroup K of the automorphism group $\mathrm{Aut}(\widetilde{X})$ such that the graph X is isomorphic to the quotient graph $\widetilde{X}_K$, say by h, and the quotient map $\widetilde{X} \to \widetilde{X}_K$ is the composition $\wp h$ of $\wp$ and h. The fibre of an edge or a vertex is its preimage under $\wp$.

The group of automorphisms of $\widetilde{X}$ mapping fibres to fibres is called the fibre-preserving subgroup of $\mathrm{Aut}(\widetilde{X})$.

Let X be a graph and let K be a finite group. By a^{-1} we mean the reverse arc to an arc a. A *voltage assignment* (or, *a K-voltage assignment*) of X is a function $\phi : A(X) \to K$ with the property that $\phi(a^{-1}) = \phi(a)^{-1}$ for each arc $a \in A(X)$. The values of ϕ are called *voltages*, and K is the *voltage group*. The graph $X \times_\phi K$ derived from a voltage assignment $\phi : A(X) \to K$ has vertex set $V(X) \times K$ and edge set $E(X) \times K$, so that the edge (e, g) of $X \times_\phi K$ joins the vertex (u, g) to $(v, \phi(a)g)$ for $a = (u, v) \in A(X)$ and $g \in K$, where $e = u, v$.

Clearly, the derived graph $X \times_\phi K$ is a covering of X; the first coordinate projection $\wp : X \times_\phi K \to X$ is called the *natural projection*. By defining $(u, g')^g = (u, g'g)$ for any $g \in K$ and $(u, g') \in V(X \times_\phi K)$, K becomes a subgroup of $\mathrm{Aut}(X \times_\phi K)$ which acts semiregularly on $V(X \times_\phi K)$. Therefore, $X \times_\phi K$ can be viewed as a *K-covering*. For each $u \in V(X)$ and $u, v \in E(X)$, the vertex set $\{(u, g) \mid g \in K\}$ is the fibre of u and the edge set $\{(u, g)(v, \phi(a)g) \mid g \in K\}$ is the fibre of u, v, where $a = (u, v)$. Conversely, each regular covering $\widetilde{X}$ of X with a covering transformation group K can be derived from a K-voltage assignment.

Let $\widetilde{X}$ be a K-covering of X with a projection $\wp$. If $\alpha \in \mathrm{Aut}(X)$ and $\tilde{\alpha} \in \mathrm{Aut}(\widetilde{X})$ satisfy $\tilde{\alpha}\wp = \wp\alpha$, we call $\tilde{\alpha}$ a *lift* of α, and α the *projection* of $\tilde{\alpha}$. Concepts such as a lift of a subgroup of $\mathrm{Aut}(X)$ and the projection of a subgroup of $\mathrm{Aut}(\widetilde{X})$ are self-explanatory. The lifts and the projections of such subgroups are of course subgroups in $\mathrm{Aut}(\widetilde{X})$ and $\mathrm{Aut}(X)$, respectively. In particular, if the covering graph $\widetilde{X}$ is connected, then the covering transformation group K is the lift of the trivial group, that is $K = \{\tilde{\alpha} \in \mathrm{Aut}(\widetilde{X}) : \wp = \tilde{\alpha}\wp\}$.

Clearly, if $\tilde{\alpha}$ is a lift of α, then $K\tilde{\alpha}$ are all the lifts of α. The projection $\wp$ is called *vertex-transitive (edge-transitive)* if some vertex-transitive (edge-transitive) subgroup of $\mathrm{Aut}(X)$ lifts along $\wp$, and *semisymmetric* if it is edge- but not vertex-transitive.

The next proposition is a special case of [22, Proposition 2.5].

2.1. Proposition. *Let X be a G-semisymmetric cubic graph with bipartition sets $U(X)$ and $W(X)$, where $G \leq A := Aut(X)$. Moreover, suppose that N is a normal subgroup of G. Then,*

(1) *If N is intransitive on bipartition sets, then N acts semiregularly on both $U(X)$ and $W(X)$, and X is a regular N-covering of the G/N-semisymmetric graph X_N.*
(2) *If 3 does not divide $|A/N|$, then N is semisymmetric on X.* □

2.2. Proposition. [17, Proposition 2.4] *The vertex stabilizers of a connected G-semi-symmetric cubic graph X have order $2^r \cdot 3$, where $0 \leq r \leq 7$. Moreover, if u and v are two adjacent vertices, then the edge stabilizer $G_u \cap G_v$ is a common Sylow 2-subgroup of G_u and G_v.* □

2.3. Proposition. [19, pp.236] *Let G be a finite group and let p be a prime. If G has an abelian Sylow p-subgroup, then p does not divide $|G' \bigcap Z(G)|$.* □

2.4. Proposition. [24, Proposition 4.4] *Every transitive abelian group G on a set Ω is regular, and the centralizer of G in the symmetric group on Ω is G.* □

2.5. Proposition. [12, Theorem 9] *Let X be a connected symmetric graph of prime valency and let G be an s-regular subgroup of $Aut(X)$ for some $s \geq 1$. If a normal subgroup N of G has more than two orbits, then it is semiregular and G/N is an s-regular subgroup of $Aut(X_N)$, where X_N is the quotient graph of X corresponding to the orbits of N. Furthermore, X is a regular N-covering of X_N.* □

The next proposition is a special case of [23, Theorem 1.1].

2.6. Proposition. *Let X be a connected edge-transitive Z_n-cover of the Heawood graph $F14$. Then $n = 3^k p_1^{e_1} \cdots p_t^{e_t}$, $k = 0$ or 1, $t \geq 1$, the primes p_i, $i = 1, \ldots, t$, are different primes with $p_i = 1$ (mod 3), and X is symmetric and isomorphic to a normal Cayley graph $Cay(G, S)$ for some group G with respect to a generating set S. Furthermore, if 7* *is coprime to n, then* $G = < a, b \mid a^2 = b^{7n} = 1,\ aba = b^{-1} > \cong D_{14n}$, $S = \{a, ba, b^{t+1}a\}$, $t^2 + t + 1 = 0$ *(mod 7n), and X is 1-regular.* □

3. Main results

Let p be a prime and let X be a cubic edge-transitive graph of order $14p^2$. By [21], every cubic edge and vertex-transitive graph is arc-transitive and consequently, X is either symmetric or semisymmetric.

For a prime $p \geq 13$, denote by $EF14p^2$ the $Z_p \times Z_p$-covering of the Heawood graph $F14$ with voltage assignment $(2,0), (-1,1), (1,-1), (1,1), (-1,-1), (1,1), (0,0), (2,0)$.

By [2, 3], we have the following lemma.

3.1. Lemma. *Let p be a prime and X a connected cubic symmetric graph of order $14p^2$, where $p < 13$. Then X is isomorphic to one of the 1-regular graphs $F56A$, $F126$, $F350$, $F686A$, $F686C$ and $F1694$, or to the 2-regular graphs $F56B$ or $F686B$, or to the 3-regular graph $F56C$.* □

3.2. Lemma. *Let $p \geq 13$ be a prime and X a connected cubic symmetric graph of order $14p^2$. Then X is isomorphic to one of the 1-regular graphs $EF14p^2$ or $Cay(G, S)$, where $G = < a, b \mid a^2 = b^{7p^2} = 1,\ aba = b^{-1} > \cong D_{14p^2}$ and $S = \{a, ba, b^{t+1}a\}$, such that $t^2 + t + 1 = 0$ (mod $7p^2$) and $3 | (p-1)$.*

Proof. By Tutte [20], X is at most 5-regular and hence $|A| = 2^s \cdot 3 \cdot 7 \cdot p^2$ for some s, where $1 \leq s \leq 5$. Let $Q = O_p(A)$ be the maximal normal p-subgroup of A. We show that $|Q| = p^2$ as follows.

Let N be a minimal normal subgroup of A. Thus $N \cong L \times \cdots \times L = L^k$, where L is a simple group. If N is unsolvable then by [4], $L \cong PSL(2,7)$ or $PSL(2,13)$ of orders $2^3 \cdot 3 \cdot 7$ and $2^2 \cdot 3 \cdot 7 \cdot 13$, respectively. Since $3^2 \nmid |A|$, we have $k = 1$ and so $N \cong PSL(2,7)$ or $PSL(2,13)$. Thus N has more than two orbits and then by Proposition 2.5, N is semiregular. Therefore, $|N| \mid 14p^2$, and this is impossible. Hence N is solvable and so elementary abelian.

Suppose first that $Q = 1$. Thus N is an elementary abelian q-group, for $q = 2, 3$ or 7 and so N has more than two orbits on X. By Proposition 2.5, N is semiregular and hence $|N| \mid 14p^2$. It follows that $|N| = 2$ or 7. If $|N| = 2$, by Proposition 2.5 X_N is a cubic symmetric graph of odd order $7p^2$, a contradiction.

Suppose that $|N| = 7$. By Proposition 2.5, X_N is a cubic A/N-symmetric graph of order $2p^2$. Let T/N be a minimal normal subgroup of A/N. By a similar argument as above, T/N is elementary abelian and hence $|T/N| = 2$ or p. If $|T/N| = 2$, then $|T| = 14$ and X_T is a cubic symmetric graph of odd order p^2, a contradiction. So, $|T/N| = p$ and also $|T| = 7p$. Since $p \geq 13$, the Sylow p-subgroup of T is characteristic and so normal in A, a contrary to the our assumption that $Q = 1$.

We now suppose that $|Q| = p$. Let P be a Sylow p-subgroup of A and $C = C_A(Q)$ the centralizer of Q in A. Clearly, $Q < P$ and also $P \leq C$ because P is abelian. Thus $p^2 \mid |C|$. If $p^2 \mid |C'|$ (C' is the derived subgroup of C) then $Q \leq C'$ and hence $p \mid |C' \bigcap Q|$, forcing that $p \mid |C' \bigcap Z(C|$ because $Q \leq Z(C)$. This contradicts Proposition 2.3. Consequently, $p^2 \nmid |C'|$ and so C' has more than two orbits on X. By Proposition 2.5, C' is semiregular on X and hence $|C'| \mid 14p^2$.

Let K/C' be a Sylow p-subgroup of C/C'. Since C/C' is abelian, K/C' is characteristic and hence normal in A/C', implying that $K \lhd A$. Note that $p^2 \mid |K|$ and $|K| \mid 14p^2$. If $|K| = 14p^2$ then K has a normal subgroup of order $7p^2$, say H. Since $p \geq 13$, the Sylow p-subgroup of H is characteristic and consequently normal in K and also normal in A. Also, if $|K| < 14p^2$, K has a characteristic Sylow p-subgroup of order p^2 which is normal in A. However, this is contrary to our assumption $|Q| = p$. Therefore, $|Q| = p^2$.

Clearly, $Q \cong Z_{p^2}$ or $Z_p \times Z_p$. Then by Proposition 2.5, X is a regular Q-covering of the symmetric graph X_Q of order 14. By [3] the only cubic symmetric graph of order 14 is the Heawood graph $F14$. Suppose that $Q \cong Z_{p^2}$. Since $p \geq 13$, 7 is coprime to p^2 and hence by Proposition 2.6, X is isomorphic to a 1-regular graph $\mathrm{Cay}(G, S)$, where $G =< a, b \mid a^2 = b^{7p^2} = 1,\ aba = b^{-1} >\cong D_{14p^2}$, $S = \{a, ba, b^{t+1}a\}$, $t^2 + t + 1 = 0 \pmod{7p^2}$, $p \geq 13$ and $3|(p-1)$.

Now, suppose that $Q \cong Z_p \times Z_p$. Then by [18, Table 2], X is isomorphic to $EF14p^2$, where $p \geq 13$. Hence the result follows. □

3.3. Lemma. *Let p be a prime. Then, $S126$ is the only cubic semisymmetric graph of order $14p^2$.*

Proof. Let X be a cubic semisymmetric graph of order $14p^2$. If $p < 11$, then by [4] there is only one cubic semisymmetric graph $S126$ of order $14p^2$, in which $p = 3$. Hence we can assume that $p \geq 11$. Set $A := \mathrm{Aut}(X)$. By Proposition 2.2, $|A_v| = 2^r \cdot 3$, where $0 \leq r \leq 7$ and hence $|A| = 2^r \cdot 3 \cdot 7 \cdot p^2$. Let $Q = O_p(A)$ be the maximal normal p-subgroup of A. We show that $|Q| = p^2$ as follows.

Let N be a minimal normal subgroup of A. Thus $N \cong L^k$, where L is a simple group. Let N be unsolvable. By [5], L is isomorphic to $PSL(2, 7)$ or $PSL(2, 13)$ of orders $2^3 \cdot 3 \cdot 7$ and $2^2 \cdot 3 \cdot 7 \cdot 13$, respectively. Note that $3^2 \nmid |A|$, forcing $k = 1$. Also, 3 does not divide $|A/N|$, and hence by Proposition 2.1 N is semisymmetric on X. Consequently, $7p^2 \mid |N|$, a contradiction because $p \geq 11$. Therefore, N is solvable and so elementary abelian. It follows that N acts intransitively on the bipartition sets of X, and by Proposition 2.1 it is semiregular on each partition. Hence $|N| \mid 7p^2$.

Suppose first that $Q = 1$. This implies that $N \cong Z_7$. Consequently, by Proposition 2.1, X_N is a cubic A/N-semisymmetric graph of order $2p^2$. Let T/N be a minimal normal subgroup of A/N. If T/N is unsolvable then by a similar argument as above, T/N is isomorphic to one of the two simple groups in the previous paragraph, implying that $7^2 \mid |T|$ and this is impossible. Hence, T/N is solvable and so elementary abelian. If T/N acts transitively on one partition of X_N, by Proposition 2.4 $|T/N| = p^2$ and hence $|T| = 7p^2$. Since $p \geq 11$, the Sylow p-subgroup of T is characteristic and consequently normal in A. It contradicts our assumption $Q = 1$. Therefore, T/N acts intransitively on the bipartition sets of X_N and by Proposition 2.1, it is semiregular on each partition, which force $|T/N| \mid p^2$. Hence $|T/N| = p$ and so $|T| = 7p$. Again, A has a normal p-subgroup, a contradiction.

We now suppose that $|Q| = p$. Let $C = C_A(Q)$ be the centralizer of Q in A and C' the derived subgroup of C. By the same argument as in the previous lemma, $p^2 \nmid |C'|$ and so C' acts intransitively on the bipartition sets of X. Then by Proposition 2.1, it is semiregular and hence $|C'| \mid 7p^2$.

Let K/C' be a Sylow p-subgroup of C/C'. Since C/C' is abelian, K/C' is characteristic and hence normal in A/C', implying that $K \lhd A$. Note that $p^2 \mid |K|$ and $|K| \mid 7p^2$. Then, K has a characteristic Sylow p-subgroup of order p^2 which is normal in A, contrary to our assumption $|Q| = p$.

Therefore, $|Q| = p^2$. Clearly, $Q \cong Z_{p^2}$ or $Z_p \times Z_p$. By Proposition 2.1, the semisymmetric graph X is a regular Q-covering of a A/Q-semisymmetric graph X_Q of order 14 which is the Heawood graph F_{14} under a projection, say $\wp$. Since $Q \lhd A$, the group A is projected along $\wp$ and consequently, $\wp$ is a semisymmetric Q-covering projection and also, X is a semisymmetric Q-covering of the Heawood graph. But by Proposition 2.6, there is no semisymmetric Z_{p^2}-covering of the Heawood graph and also by [16, Theorem 7.1], there is no semisymmetric $Z_p \times Z_p$-covering projection of the Heawood graph, a contradiction. Hence the result follows. □

Now, the proof of Theorem 1.1 follows by Lemmas 3.1, 3.2 and 3.3.

References

[1] Alaeiyan, M. and Ghasemi, M. *Cubic edge-transitive graphs of order* $8p^2$, Bull. Austral. Math. Soc. **77**,315–323, 2008.

[2] Conder, M. *Trivalent (cubic) symmetric graphs on up to 2048 vertices*, 2006. `http://www.math.auckland.ac.nz/∼conder/∼conder/symmcubic2048list.txt`.

[3] Conder, M. and Nedela, R. *A refined classification of cubic symmetric graphs*, Jornal of Algebra **322**, 722–740, 2009.

[4] Conder, M and Malnič, A., Marušič, D. and Potočnik, P. *A census of semisymmetric cubic graphs on up to 768 vertices*, J. Algebr. Comb. **23**, 255–294, 2006.

[5] Conway, J. H., Curtis, R. T., Norton, S. P., Parker, R. A. and Wilson, R. A. *An ATLAS of Finite Groups* (Oxford University Press, Oxford, 1985).

[6] Djoković, Ž. D. and Miller, G. L. *Regular groups of automorphisms of cubic graphs*, J. Combin. Theory Ser. B **29**, 195–230, 1980.

[7] Du, S. F. and Xu, M. Y. *A classification of semisymmetric graphs of order* $2pq$, Com. in Algebra **28** (6), 2685–2715, 2000.

[8] Feng, Y. Q. and Kwak, J. H. *On regular cubic graphs of order a small number times a prime or a prime square*, J. Aust. Math. Soc. **76**, 345–356, 2004.

[9] Feng, Y. Q. and Kwak, J. H. *Classifying cubic symmetric graphs of order* $10p$ *or* $10p^2$, Sci. China Ser. A. Math. **49**, 300–319, 2006.

[10] Feng, Y. Q. and Kwak, J. H. *Cubic symmetric graphs of order twice an odd prime-power*, J. Aust. Math. Soc. **81**, 153–164, 2006.

[11] Feng, Y. Q. and Kwak, J. H. *Cubic symmetric graphs of order a small number times a prime or a prime square*, J. Combin. Theory Ser. B **97**, 627–646, 2007.

[12] Y. Q. Feng, J. H. Kwak and Xu, M. Y. *Cubic s-regular graphs of order* $2p^3$, J. Graph Theory **52**, 341–352, 2006.

[13] Folkman, J. *Regular line-symmetric graphs*, J. Combin. Theory **3**, 215–232, 1967.

[14] Gross, J. L. and Tucker, T. W. *Topological Graph Theory* (Wiley-Interscience, New York, 1987).

[15] Lu, Z., Wang, C. Q. and Xu, M. Y. *On semisymmetric cubic graphs of order* $6p^2$, Science in China Ser. A Mathematics **47** (1), 1–17, 2004.

[16] Malnič, A., Marušič, D. and Potočnik, P. *Elementary abelian covers of graphs*, J. Alg. Combin. **20**, 71–97, 2004.

[17] Malnič, A., Marušič, D. and Wang, C. Q. *Cubic edge-transitive graphs of order* $2p^3$, Discrete Math. **274**, 187–198, 2004.

[18] Oh, J. M. *A classification of cubic s-regular graphs of order* $14p$, Discrete Math. **309**, 2721–2726, 2009.

[19] Rose, J. S. *A Course On Group Theory* (Cambridg University Press, Cambridge, 1978).

[20] Tutte, W. T. *A family of cubical graphs*, Proc. Cambridge Philos. Soc. **43**, 459–474, 1947.

[21] Tutte, W. T. *Connectivity in graphs* (Toronto University Press, Toronto, 1966).

[22] Wang, C. Q. and Chen, T. S. *Semisymmetric cubic graphs as regular covers of* $K_{3,3}$, Acta Mathematica Sinica **24**, 405–416, 2008.

[23] Wang, C. Q. and Hao, Y. *Edge-transitive regular* Z_n*-covers of the Heawood graph*, Discrete Mathematics **310**, 1752–1758, 2010.

[24] Wielandant, H. *Finite Permutation Groups* (Acadamic Press. New York, 1964).

[25] Zhou, J. X. and Feng, Y. Q. *Cubic vertex-transitive graphs of order* $2pq$, J. Graph Theory, DOI 10.1002/jgt.20481.