ON THE VALUES OF SOME GENERALIZED LACUNARY POWER SERIES WITH ALGEBRAIC COEFFICIENTS FOR LIOUVille NUMBER ARGUMENTS†

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Abstract
In this work, it is shown that under certain conditions, the values of some generalized lacunary power series with algebraic coefficients from a certain algebraic number field \( K \) of degree \( m \) for Liouville number arguments belong to either the algebraic number field \( K \) or \( \bigcup_{i=1}^{m} U_i \) in Mahler’s classification of the complex numbers.

Keywords: Generalized lacunary power series, \( U \)-numbers in Mahler’s classification.

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1. Introduction
A power series \( F(z) = \sum_{h=0}^{\infty} c_h z^h \) \((c_h \in \mathbb{C}, h = 0, 1, 2, \ldots)\) with a positive radius of convergence, satisfying the following conditions
\[
\begin{align*}
    c_h &= 0, \quad r_n < h < s_n \quad (n = 1, 2, 3, \ldots), \\
    c_h &\neq 0, \quad h = r_n \quad (n = 1, 2, 3, \ldots), \\
    c_h &\neq 0, \quad h = s_n \quad (n = 0, 1, 2, \ldots),
\end{align*}
\]
where \( \{s_n\}_{n=0}^{\infty} \) and \( \{r_n\}_{n=1}^{\infty} \) are two infinite sequences of non-negative rational integers with
\[
0 = s_0 \leq r_1 < s_1 \leq r_2 < s_2 \leq r_3 < s_3 \leq \ldots, \quad \lim_{n \to \infty} \frac{s_n}{r_n} = \infty,
\]
is called a generalized lacunary power series.

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