SUBORDINATION AND SUPERORDINATION
FOR HIGHER-ORDER DERIVATIVES OF
p-VALENT ANALYTIC FUNCTIONS

R. M. El-Ashwah∗† and M. K. Aouf‡

Received 17:10:2009 : Accepted 12:02:2011

Abstract
In this paper some subordination and superordination results for
higher-order derivatives of certain $p$-valent analytic functions in the
open unit disc are derived. Relevant connections of the results, which
are obtained in this paper, with various known results are also consid-
ered.

Keywords: Analytic functions, Differential subordinations, Superor-
dination, Subordi-
nants, Dominants, Higher-order derivatives.

2000 AMS Classification: 30 C 45.

1. Introduction
Let $A(p)$ denote the class of analytic functions of the form:

\begin{equation}
 f(z) = z^p + \sum_{k=1}^{\infty} a_{k+p}z^{k+p} \quad (p \in \mathbb{N} = \{1, 2, 3, \ldots\}),
\end{equation}

which are $p$-valent in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$ and let $A(1) = A$. Upon
differentiating both sides of (1.1) $m$-times with respect to $z$, we obtain (see [6])

\begin{equation}
 f^{(m)}(z) = \delta(p, m)z^{p-m} + \sum_{k=1}^{\infty} \delta(k, m)a_{k+p}z^{k+p-m},
\end{equation}

$(p \in \mathbb{N}; m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; p > m)$,

∗Department of Mathematics, Faculty of Science (Damietta Branch), Mansoura University,
New Damietta 34517, Egypt. E-mail: r_elashwah@yahoo.com
†Corresponding Author.
‡Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516,
Egypt. E-mail: mkaouf127@yahoo.com