ON SEMISYMMETRIC CUBIC GRAPHS OF ORDER 10p³

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Abstract

Connected cubic graphs of order 10p³ which admit an automorphism group acting semisymmetrically are investigated. We prove that every connected cubic edge-transitive graph of order 10p³ is vertex-transitive, where p is a prime.

Keywords: Automorphism group, Regular cover, Semisymmetric graph.

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1. Introduction

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For the group-theoretic concepts and notation not defined here we refer the reader to [4, 8, 14]. Given a graph X, we let \( V(X) \), \( E(X) \), \( A(X) \) and \( \text{Aut}(X) \) be the vertex set, the edge set, the arc set and the automorphism group of \( X \), respectively.

If a subgroup \( G \) of \( \text{Aut}(X) \) acts transitively on \( V(X) \), \( E(X) \) and \( A(X) \), then \( X \) is said to be \( G \)-vertex-transitive, \( G \)-edge-transitive and \( G \)-arc-transitive, respectively. It is easily seen that a graph \( X \) which is \( G \)-edge- but not \( G \)-vertex-transitive is necessarily bipartite, with the two parts of the bipartition coinciding with the orbits of \( G \). In particular, if \( X \) is a regular, then these two parts have equal cardinalities, and such a graph is then referred to as being \( G \)-semisymmetric. In the case where \( G = \text{Aut}(X) \) the symbol \( G \) may be omitted from the definitions above, so that \( X \) is called semisymmetric if it is regular and \( \text{Aut}(X) \)-edge-transitive but not \( \text{Aut}(X) \)-vertex-transitive.

An \( s \)-arc in a graph \( X \) is an ordered \((s + 1)\)-tuple \((v_0, v_1, \ldots, v_s)\) of vertices of \( X \) such that \( v_{i-1} \) is adjacent to \( v_i \) for \( 1 \leq i \leq s \), and \( v_{i-1} \neq v_{i+1} \) for \( 1 \leq i < s \). A graph \( X \) is said to be \( s \)-arc-transitive if \( \text{Aut}(X) \) is transitive on the set of \( s \)-arcs of \( X \). In particular, \( 0 \)-arc-transitive means vertex-transitive, and \( 1 \)-arc-transitive means arc-transitive or symmetric.

The study of semisymmetric graphs was initiated by Folkman [7].

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