APPROXIMATION BY GENUINE $q$-BERNSTEIN-DURRMEYER POLYNOMIALS IN COMPACT DISKS

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Abstract

In this paper, the order of simultaneous approximation and Voronovskaja type theorems with quantitative estimate for complex genuine $q$-Bernstein-Durrmeyer polynomials ($0 < q < 1$) attached to analytic functions on compact disks are obtained. Our results show that extension of the complex genuine $q$-Bernstein-Durrmeyer polynomials from real intervals to compact disks in the complex plane extends approximation properties (with quantitative estimates).

Keywords: Complex genuine $q$-Bernstein–Durrmeyer operators, Voronovskaja’s theorem, Approximation in compact disks.

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1. Introduction

Recent studies demonstrate that the theory of $q$-calculus plays an important role in analytic number theory and theoretical physics. For example, various applications of this theory have appeared in the study of hypergeometric series [3], in approximation theory [24], while other important applications have been related with quantum theory [20]. In this paper, with the help of techniques from the $q$-calculus, we study the approximation properties of a general family of genuine $q$-Bernstein–Durrmeyer operators, which are well-known positive linear operators in approximation theory.

Genuine Bernstein–Durrmeyer operators were first considered by W. Chen [4] and T. N. T. Goodman and A. Sharma [16] around 1987. In recent years, genuine Bernstein–Durrmeyer operators have been investigated intensively by a number of authors. Among the many articles written on the $U_n$, we mention here only the ones by H. Gonska et al. [15], by P. Parvanov and B. Popov [23], by T. Sauer [25], by S. Waldron [26], and the

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