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Constrained model predictive control for the quadruple-tank process

Zohra Zidane *

Team of Applied Physics and New Technologies, Department of Physic, Polydisciplinary Faculty, University of Sultan Moulay Slimane, B.P: 592, 23000 Beni-Mellal, Morocco

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ABSTRACT

Model Predictive Control (MPC) is an advanced method of controllers, explicitly uses of model to obtain control signal. MPC is popular in industry and academia because it is capable to deals with non-minimum phase, unstable, dead time and multivariable processes, and solves the problem of constraints. MPC with integral action method is used in this study for the quadruple tank system by taking the lower two tanks into account. The objective of this work is to design and study the MPC method for controlling the level of tanks in a quadruple tank process depending on type of constrained problems. However, to solve the problem of constraints is not easy way. The methods based on the quadratic programming function and 'if-else' technique are presented to solve the problem of the process constraints in MPC. A comparative study is performed with the quadratic programming function and 'if-else' technique. The performance of the proposed method is tested for reference tracking and disturbance rejection behavior. Simulation results are presented and discussed to show the performance of the controller.

1. Introduction

Developing effective control methods for the control of industrial processes in many areas of engineering is difficult because of the long and tedious identification of models [1].

The Model Predictive Control, MPC, has been the most used technique in more than 30 years and has become an important tool in many industrial process applications [2].

The predictive control applications that have been used successfully are [3]:

- Clinical anesthesia.
- Robotic.
- Chemical engineers.
- Cement industry.
- Electric servo motor, etc.

Predictive control is an effective strategy for solving constraints and dynamics of nonlinear systems, when the analytical computation of the control signal is difficult [4, 5]. This methodology is very used in industries, where the dynamics of the system are slow enough to allow its implementation [6].

The powerful of MPC controllers are its ability to manage constraints, non-minimal phase processes, changes in system parameters and its great applicability to the Multi Input Multi Output (MIMO) processes [7], [8].

Model Predictive Control, MPC, also known as receding horizon control, uses the range of control methods, making the use of the process model to predict the output and the control signal obtained by minimizing the quadratic cost function [9]. The effectiveness of the controller depends on the quality of the system dynamics captured by the input-output model used for controller design [10].

Constraints of different types are ubiquitous in the control of industrial processes; how to deal with them in the design of the control system is an important issue. Ignore the constraints or impose them on the control signal in a heuristically can lead to a deterioration of the performances, or even instability, in particular for the predictive control of the unstable systems.

* Corresponding author. Tel.: +212-(0)-523-4246 85/16 ; Fax: +212-(0)-523-4245 97.
E-mail addresses: zohra.zidane@usms.ma; zidane.zohra@gmail.com *(Z. Zidane)*
ORCID: 0000-0002-5603-8011 *(Z. Zidane)*
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Taking constraints into consideration in the design phase inherently leads to solving the constrained optimization problem. The quadratic programming (QP) techniques can be applied to the resolution of various types of constrained predictive control problems [11, 12].

The MPC algorithms usually assume that all signals have an unlimited range, although real processes have constraints (input constraint, input rate constraint, constrained output, etc). For this reason, it is necessary to use MPC controllers to cope with constrained inputs. In this work, constrained MPC is applied to the state space model of the four tank system after linearization. The MPC under constraints is given in more detail and constraint optimization techniques discussed here are based on 'quadprog' function and 'if-else' loop. Quadratic programming (QP) methods are widely used in constraint predictive control applications; see for example the comments given in [13] and [14].

In the previous paper given in [15], a comparative study of unconstrained and constrained control system behavior is developed and the method based on the Quadratic Programming (QP) technique are used to solve the constrained optimization problem. The aim of this work is to design and study the predictive controller for controlling the level of tanks in a quadruple tank process depending on type of constrained problems. However, there is no easy way to solve the problem of constraints. This study presents a comparison between the both named quadratic programming function and 'if-else' technique. The purpose of the tests will be to check if one of the two proposed methods is able to solve the problems of constraints with less calculation and lead MPC to best performance in term of good tracking and perturbation rejection capacity.

The paper is organized as follows. The one classical formulation of constrained MPC controller algorithm is presented. A benchmark quadruple tank process is considered. The efficiency and the superiority of MPC under constraints are illustrated by an example of simulation. Some concluding remarks and future prospects complete the paper.

2. MPC Algorithm

The general strategy of the MPC is shown in Figure 1 [16, 17].

1) Definition of a numerical model of the system to calculate the predicted future outputs $\hat{y}$. These depend upon the known values up to instance k, taking the current $y(k)$ into consideration and calculate the future control signals $u(k+i), i = 1 \dots p-1$.
2) The sequence of future control signals is computed to minimize the objective function.

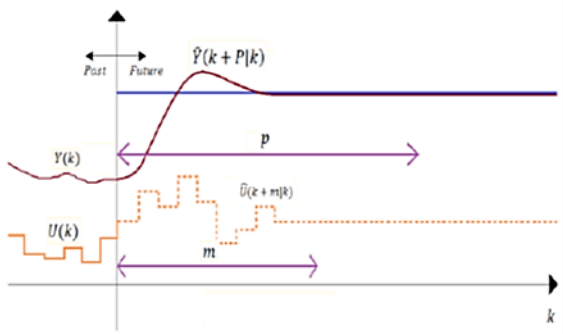

Figure 1. The receding horizon strategy of MPC [17]

3) Only the current control signal $u(k)$ is applied to the system. Next time, measure $y(k+1)$, repeat step 1 and all sequences are updated. And the control signal $u(k+1)$ is then calculated using the concept of moving horizon.

In MPC, the model used to analyze the behavior of the system [17] is linear or nonlinear. The future moves of the variables studied are obtained by minimizing the quadratic cost function.

2.1 Performance Criterion

The quadratic cost function to be minimized is:

$$J_k = \left(y_{k+1/L} - w_{k+1/L}\right)^T Q \left(y_{k+1/L} - w_{k+1/L}\right) + u_{k/L}{}^T P u_{k/L} + \Delta u_{k/L}{}^T R \Delta u_{k/L} \quad (1)$$

Where **w** is the reference signal, **y** is the output signal, **Δu** is the input changes, and **u** is the input signal. Q, R and P are the weight matrices. **L** prediction horizon.

The control law minimizing the cost is, [18]:

$$u_{k/L}{}^* = arg\ min\ J_k(u_{k/L}) \quad (2)$$

The purpose of the quadratic cost function is to reduce the difference $\left(y_{k+1/L} - w_{k+1/L}\right)$ and at the same time reduce the control $u_{k/L}$ [19].

3. Constraints Implementation

A constraint is a limitation; in practice all processes have constraints. In MPC one normally defines these constraints to minimize inequalities [20],

$$C\Delta u_{k/L} \le b \quad (3)$$

Where $C \in R^{m \times n}$ is a matrix and $b \in R^m$ is a vector.

The **input constraints** are given in linear inequality form as:

$$\begin{bmatrix} S \\ -S \end{bmatrix} \Delta u_{k/L} \le \begin{bmatrix} u_{k/L}^{max} - cu_{k-1} \\ -u_{k/L}^{min} + cu_{k-1} \end{bmatrix} \quad (4)$$

The **input rate constraint** written in linear inequality form as:

$$\begin{bmatrix} I \\ -I \end{bmatrix} \Delta u_{k/L} \leq \begin{bmatrix} \Delta u_{k/L}^{max} \\ -\Delta u_{k/L}^{min} \end{bmatrix} \quad (5)$$

The **output constraint** in linear inequality form, defined as,

$$\begin{bmatrix} G_L^\Delta \\ -G_L^\Delta \end{bmatrix} \Delta u_{k/L} \leq \begin{bmatrix} y_{max} - F_L^\Delta \\ -y_{min} + F_L^\Delta \end{bmatrix} \quad (6)$$

Input, rate and output constraints from Equations (4), (5) and (6) can be given as $C\Delta u_{k/L} \leq b$ respectively.

$$\begin{bmatrix} S \\ -S \\ I \\ -I \\ G_L^\Delta \\ -G_L^\Delta \end{bmatrix} \Delta u_{k/L} \leq \begin{bmatrix} u_{k/L}^{max} - cu_{k-1} \\ -u_{k/L}^{min} + cu_{k-1} \\ \Delta u_{k/L}^{max} \\ -\Delta u_{k/L}^{min} \\ y_{max} - F_L^\Delta \\ -y_{min} + F_L^\Delta \end{bmatrix} \quad (7)$$

Where

$$C = \begin{bmatrix} S \\ -S \\ I \\ -I \\ G_L^\Delta \\ -G_L^\Delta \end{bmatrix} \text{ and } b = \begin{bmatrix} u_{k/L}^{max} - cu_{k-1} \\ -u_{k/L}^{min} + cu_{k-1} \\ \Delta u_{k/L}^{max} \\ -\Delta u_{k/L}^{min} \\ y_{max} - F_L^\Delta \\ -y_{min} + F_L^\Delta \end{bmatrix}$$

4. MPC with Integral Action

Model predictive control has many different algorithms, depending on the numerical model of the process used for the objective function. These formulations have some problems with offset. To solve this, using the integral action it is an effective approach [21], [22].

Consider discrete-time state space model with disturbance.

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + \zeta \\ y_k &= Cx_k + \epsilon \end{aligned} \quad (8)$$

Where A, B and C are system matrices, $x_k \in R^n$ is a state vector, $u_k \in R^r$ is input vector, $y_k \in R^m$ is output vector. ζ is process disturbance vector and ϵ is measurement noise vector.

In order to solve the problem of optimal control of the MPC, Firstly, a model without unknown perturbations is considered.

From the Equation (8), the state space model becomes,

$$\begin{aligned} \Delta x_{k+1} &= A\Delta x_k + B\Delta u_k \\ y_k &= y_{k-1} + C\Delta x_k \end{aligned} \quad (9)$$

Where,
$\Delta x_{k+1} = x_{k+1} - x_k$
$\Delta x_k = x_k - x_{k-1}$
$\Delta u_k = u_k - u_{k-1}$

The augmented form of the model shown below is obtained using the model of Equation (15),

$$\begin{aligned} \underbrace{\begin{bmatrix} \Delta x_{k+1} \\ y_k \end{bmatrix}}_{\tilde{x}_{k+1}} &= \underbrace{\begin{bmatrix} A & 0 \\ C & I \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} \Delta x_k \\ y_{k-1} \end{bmatrix}}_{\tilde{x}_k} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\tilde{B}} \Delta u_k \\ y_k &= \underbrace{\begin{bmatrix} C & I \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} \Delta x_k \\ y_{k-1} \end{bmatrix}}_{\tilde{x}_k} \end{aligned} \quad (10)$$

A strictly proper state model written as,

$$\begin{aligned} \tilde{x}_{k+1} &= \tilde{A}\tilde{x}_k + \tilde{B}\Delta u_k \\ y_k &= \tilde{C}\tilde{x}_k \end{aligned} \quad (11)$$

$\tilde{A}, \tilde{B}, \tilde{C}$: Augmented model matrices

The predicted value of the output along the horizon will be;

$$y_{k+1/L} = F_L + G_L u_{k/L} \quad (12)$$

Where

$$F_L = O_L \widetilde{A}_L \quad (13)$$

$$G_L = [O_L \widetilde{B} \quad H_L^d] \quad (14)$$

Where O_L is extended observability matrix for the $(\widetilde{C}, \widetilde{A})$, and H_L^d is a Toeplitz matrix for $(\widetilde{C}, \widetilde{A}, \widetilde{B})$ matrices [19].

The cost index Equation (1) with the model prediction Equation (12) can be given as quadratic objective function on standard for as,

$$J_k = \Delta u_{k/L}^T H \Delta u_{k/L} + 2{f_k}^T \Delta u_{k/L} + J_0 \quad (15)$$

Where

$$H = G_L^T Q G_L + R \quad (16)$$

$$f_k = G_L^T Q (F_L - w_{k+1/L}) \quad (17)$$

$$J_0 = (F_L - w_{k+1/L})^T Q (F_L - w_{k+1/L}) \quad (18)$$

The optimal control deviation vector written as,

$$\Delta u_{k/L}^* = -H^{-1} f_k \quad (19)$$

Note that, the first element of $\Delta u_{k/L}^*$ is $\Delta u_{k/L}$ and the current control $u_{k/L}$ is calculated as $u_k = \Delta u_k + u_{k-1}$ [19].

5. Process Description

Quadruple tank process contains four interconnected water tanks and two pumps as given in Figure 2 [23], [24]. The process inputs are u_1 and u_2 and the outputs are $y_1 = k_c l_1$ and $y_2 = k_c l_2$.

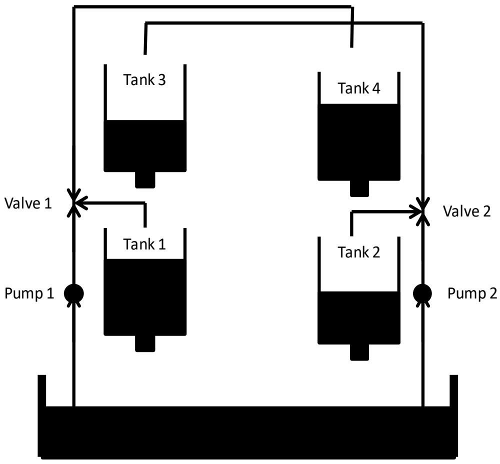

Figure 2. Schematic of the quadruple tank process [23]

The mathematical model of four tank systems is as follows [25], [26]:

$$\frac{dl_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gl_1} + \frac{a_3}{A_1}\sqrt{2gl_3} + \frac{\gamma_1 k_1}{A_1}u_1 \tag{20}$$

$$\frac{dl_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gl_2} + \frac{a_4}{A_2}\sqrt{2gl_4} + \frac{\gamma_2 k_2}{A_2}u_2 \tag{21}$$

$$\frac{dl_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gl_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \tag{22}$$

$$\frac{dl_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gl_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \tag{23}$$

Where;
A_i: Surface of cross section of the tank i;
a_i: Surface of cross section of the exit hole i;
l_i: Level of water in the tank i;
u_i: Voltage of the pump i;
γ_i: Constant of valve i;
k_i: Constant of pump i;
g: Acceleration due tothe gravity;
k_c: Pump gain.

The linearized state space model is [25], [26]:

$$\frac{dl}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_1} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_2} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} l + \begin{bmatrix} \frac{\gamma_1k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \tag{24}$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} l \tag{25}$$

The discrete four-tank plant model is as:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned} \tag{26}$$

Where;

$$A = \begin{bmatrix} 0.9984 & 0 & 0.0026 & 0 \\ 0 & 0.9989 & 0 & 0.0018 \\ 0 & 0 & 0.9974 & 0 \\ 0 & 0 & 0 & 0.9982 \end{bmatrix};$$

$$B = \begin{bmatrix} 0.0048 & 0 \\ 0 & 0.0035 \\ 0 & 0.0077 \\ 0.0056 & 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

6. Simulation Analysis

In this paper, the MPC method under imposed constraints is implemented in the state space model after linearization of the quadruple tank in the non-minimum phase region. A comparative study is performed with the quadratic programming function and 'if-else' technique.

The constant weight matrices Q and R are chosen in terms to obtain the better performances respectively,

$$Q = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}, \text{ and } R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The initial levels in tanks 1 and 2 are 9.5 and 10.5 [cm].

The prediction horizon L of 16 is used.

The control action amplitude constraints $0 \le u \le 5$ is applied.

The reference is chosen as a square wave.

Simulation results are developed with Matlab® codes written to simulate a quadruple tank process.

6.1 Constrained MPC with 'quadprog' Function

The main advantages of MPC method are its capability to handle constraints [1]. Here the constraints are provided on both the input voltages to the pumps at the amplitude constraints $0 \le u \le 5$. The process outputs and control input signal, under constrained MPC method with 'quadprog' function, are shown in Figure 3.

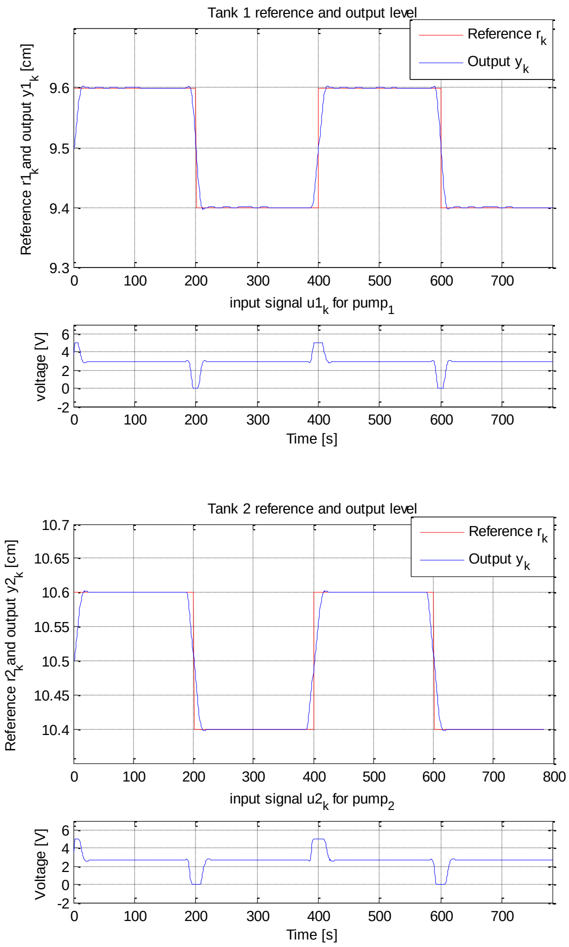

Figure 3. Quadruple tank process responses and control input signal using 'quadprog' function

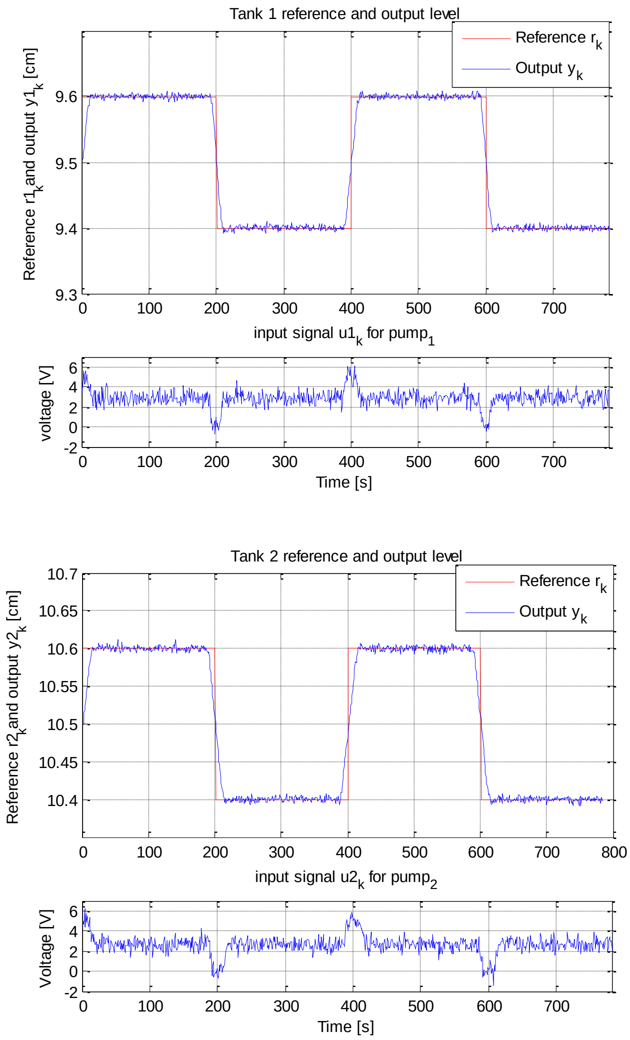

Figure 4. Quadruple tank process responses and control input signal using 'quadprog' function with a random disturbance

From the responses it is clearly shown that the output variables are able to track the set points given with no over/undershoots in both the tanks 1 and 2. When the load disturbance is applied as given in Figure 4, it can be seen that the proposed method allows guaranteeing the disturbance rejection and the tracking performance is achieved successfully. The running time of the simulation is 9.97 seconds due to complex calculation.

6.2 Constrained MPC with "if-else" Technique

The constraints are handled using "if-else" technique such as:

$u_{min} = 0$
$u_{max} = 5$

if $\quad u < u_{min}$
$\quad u = u_{min}$;
elseif $u > u_{max}$
$\quad u = u_{max}$
end

The results are plotted and are given in Figure 5. From the responses it is clearly shown that the output variables are able to track the set points given with a small undershoots and overshoots in both responses of the system. When a random disturbance is applied to the system as seen in Figure 6, it can be observed that the disturbance is rejected, and the tracking performance is achieved successfully. The running time of the simulation is 2.13 seconds.

6.3 Comparison Results

In the presented Figures, it can be seen the comparative results between MPC under imposed constraints using the function 'quadprog' and the technique "if-else". The output levels for tanks 1 and 2, and the control input signal for pumps 1 and 2, under constrained MPC method with the function 'quadprog' and with the technique 'if-else' without and with the random disturbance, are shown, respectively, in Figures 3, 4, 5 and 6. Best performance is characterized by good set point tracking, robustness, lower or no over/undershoots. Based on this, it can be observed that the constrained MPC method with the function 'quadprog' produces the good response in terms of tracking and overshoot, and the effect of disturbance is well rejected. But the running time of simulation with the technique "if-else" is reduced compared to the case with the function 'quadprog'. As a result, the calculation time can be shortened and for the process with rapid dynamic, the 'if-else' technique is most interesting.

7. Conclusion

In this paper, a model predictive controller under constraints was designed using a linearized state-space model of the quadruple tank process in the non-minimum phase region. From the simulation results, it is clear that, the constrained MPC controller with 'quadprog' function has a good set point tracking and the effect of disturbance is well rejected. The running time of simulation is reduced with 'if-else' technique compared to 'quadprog' function. For the process with rapid dynamic, the 'if-else' technique is most interesting, this is particularly important in real times application.

Table 1. Comparison of Constrained MPC using 'quadprog' function and Constrained MPC using 'if-else' technique.

Parameters	Constrained MPC using 'quadprog' function	Constrained MPC using "if-else" technique
$\sum(y(k) - r(k))^2$ in tank 1	$2.6336e^{-004}$	$3.4953e^{-004}$
$\sum(u(k))^2$ in tank 1	8.6946	8.7327
$\sum(y(k) - r(k))^2$ in tank 2	$3.6109e^{-004}$	$5.3257e^{-004}$
$\sum(u(k))^2$ in tank 2	7.5232	7.6158
Execution time	9.97 seconds	2.13 seconds

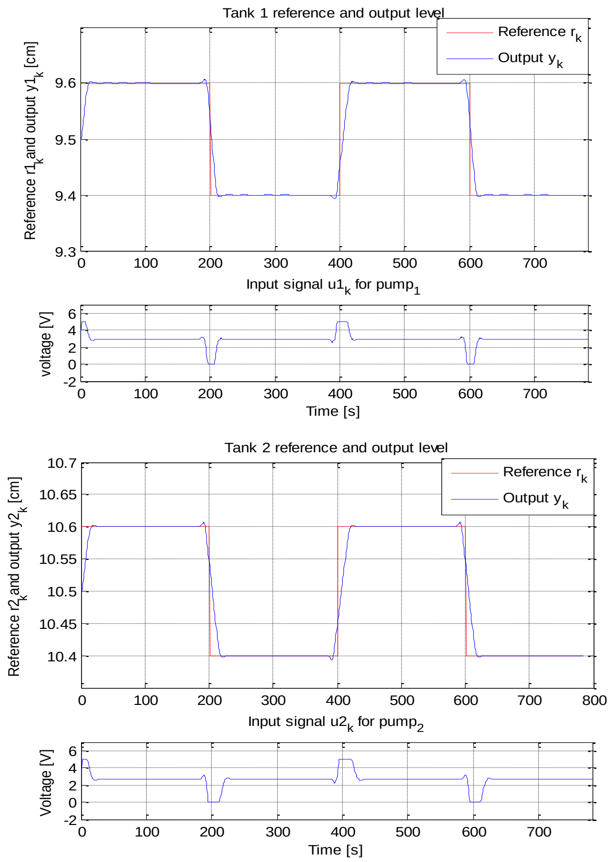

Figure 5. Quadruple tank process responses and control input signal using 'if-else' technique

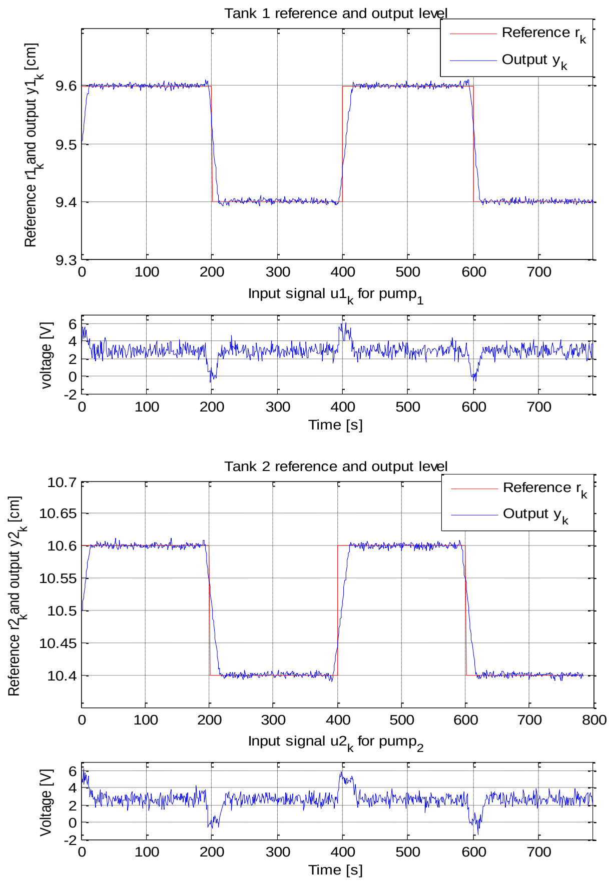

Figure 6. Quadruple tank process responses and control input signal using 'if-else' technique with a random disturbance

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