FUZZY SETS OVER THE POSET \( \mathbb{I} \)

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Abstract

The author studies fuzzy sets over the poset \( \mathbb{I} = [0, 1] \) with the usual order. These form a canonical example of fuzzy sets over a poset discussed in (Tiryaki, İ. U. and Brown, L. M. Plain textures and fuzzy sets via posets, preprint). Characterizations of these so called “soft fuzzy sets” are obtained, and soft fuzzy sets are shown to have a richer mathematical theory than classical \( \mathbb{I} \)-fuzzy sets. In particular soft fuzzy points behave like the points of crisp set theory with respect to join, and moreover there exists a Lowen type functor from \( \textbf{Top} \) to the construct \( \textbf{SF-Top} \) that preserves both separation and compactness.

Keywords: Texture, Unit interval texture, Hutton algebra, Fuzzy subset, Soft fuzzy subset, Point, Copoint, Construct, \( SF \)-topology, Ditopology, Separation, Compactness, Generalized Lowen functor, Rotund soft fuzzy set, Lowen rotund functor, Preservation of topological properties.

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1. Introduction

In [17] the author and L. M. Brown used the characterization of plain textures in terms of posets given in [8] to present several new results in the theory of plain ditopological texture spaces. As part of this investigation they considered fuzzy sets over a poset and mentioned a canonical example of such fuzzy sets that coincides with the notion of “soft fuzzy set” introduced by the author in his PhD thesis [15] from a different view-point. This paper presents an updated account of the theory of soft fuzzy sets based on the discussion in [17] and placed within a more suitable categorical framework than that given in [15]. As mentioned in [17], fuzzy sets over a poset have properties that make them potentially useful in applications. Naturally, soft fuzzy sets share these properties and it is anticipated that they will find useful applications in various areas.

If \((N, \leq)\) is a partially ordered set (poset, for short) we denote by \( \mathcal{L}_N \) the set of lower sets of \( N \) as in [17]. Hence \((N, \mathcal{L}_N)\) is a plain texture, and all plain textures can

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