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Broadband Matching Network Design via Explicit Solutions of Two-Variable Scattering Equations

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Abstract **–** In many applications, circuits containing lumped elements are preferred because they are small in size. Also the losses in the designed circuits should be kept as low as possible. Unfortunately, especially at microwave frequencies, it is not possible to avoid the losses caused by the connections between the lumped elements. However, the use of these connections as circuit elements will improve the performance of the circuit. Therefore, it is inevitable to use circuits containing mixed (lumped and distributed) elements at microwave frequencies. Mixed lumped and distributed element two-port networks are described by means of two-variable scattering equations. So it is necessary to obtain the solutions of these equations to design this kind of networks. In literate, the solutions of these equations for some classes of low-order ladder networks are given under some restrictions. But in this paper, a broadband matching network is designed by using explicit solutions of the equations without any restrictions. Then the obtained results and the results obtained in the literature have been compared.

Keywords – Scattering equations, two-port networks, broadband networks, matching networks, ladder networks.

I. INTRODUCTION

Mixed-element networks (lumped and distributed element) are important for microwave engineers [1]. If the connections between lumped elements are considered as transmission lines, and they are utilized to form network functions, they do not destroy the performance of the network, and they are used to obtain the desired response from the network.

Since these networks have two different kinds of elements, their network functions are defined in terms of two variables; for lumped elements $p=\sigma+j\omega$ classical frequency variable and for distributed elements $\lambda=\tanh(p\tau)$ Richards variable (here τ is the delay of distributed elements). Notice that there is a hyperbolic dependence between two variables, so transcendental functions can be used to define these kinds of networks. But if p and λ are assumed to be independent, mixed-element networks can be defined by using two-variable functions [2-5]. In the literature, although there are many works about these kinds of networks, there is no any analytical general method to solve these functions. But there is a semi-analytic approach [6-13]. In this approach, two-variable scattering equations are utilized and it can be applied for only a limited range of network topologies; LC ladder networks separated by unit element (UE).

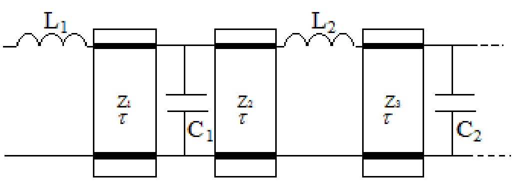

Fig. 1 Mixed-element low-pass structure (LPLU) [9]

In the literature, the solutions of scattering equations under some restrictions are given for low-order low-pass LC ladder networks separated by unit elements (Fig. 1) by means of the mentioned semi-analytic approach. But in this work, by means of the solved equations without any restrictions, a broadband matching network is designed, and the obtained results are compared with the results obtained in the literature.

II. DEFINITION OF MIXED-ELEMENT TWO-PORT NETWORKS

By means of two-variable polynomials g, h, f, scattering parameters of a two-port with mixed lumped and distributed elements can be written as [6-15]

$$S(p,\lambda)=\begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)}\begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix} \quad (1)$$

here $|\mu|=1$ is a constant.

These two-variable polynomials have the following features:

- $g(p,\lambda)$, $h(p,\lambda)$ and $f(p,\lambda)$ are two-variable polynomials with real coefficients.
- $g(p,\lambda)$ is a scattering Hurwitz polynomial.
- $f(p,\lambda)$ is formed by using the transmission zeros of the two-port network.
- $g(p,\lambda)$, $h(p,\lambda)$ and $f(p,\lambda)$ have the following relation;
  $g(p,\lambda)g(-p,-\lambda) = h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda).$

The polynomial $g(p,\lambda)$ is a $n_p + n_\lambda$ order scattering Hurwitz polynomial with real coefficients [14] and can be defined as

$$g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P} \tag{2a}$$

here

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p 0} & \cdots & \cdots & g_{n_p n_\lambda} \end{bmatrix}, \tag{2b}$$

$$\begin{aligned} \mathbf{P}^T &= \left[1\ p\ p^2 \ldots p^{n_p}\right] \\ \boldsymbol{\lambda}^T &= \left[1\ \lambda\ \lambda^2 \ldots \lambda^{n_\lambda}\right]. \end{aligned} \tag{2c}$$

In a similar manner, the polynomial $h(p,\lambda)$ is a $n_p + n_\lambda$ order polynomial with real coefficients [14] and can be defined as

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p 0} & \cdots & \cdots & h_{n_p n_\lambda} \end{bmatrix}. \tag{3}$$

The polynomial $f(p,\lambda)$ can be formed by using the transmission zeros of the two-port network [14] and can be defined as

$$f(p,\lambda) = f_L(p) f_D(\lambda) \tag{4}$$

here the polynomials $f_L(p)$ and $f_D(\lambda)$ are formed by using the transmission zeros of lumped and distributed sections, respectively.

If unit elements are connected cascade, then the polynomial $f_D(\lambda)$ can be defined as

$$f_D(\lambda) = (1-\lambda^2)^{n_\lambda/2} \tag{5}$$

here n_λ is the number of unit elements [14].

If there are transmission zeros at only DC, the polynomial $f_L(p)$ can be defined as

$$f_L(p) = p^k \tag{6}$$

here k is the number of transmission zeros at DC [14].

As a result, the following form is a practical one for the polynomial $f(p,\lambda)$

$$f(p,\lambda) = p^k (1-\lambda^2)^{n_\lambda/2}. \tag{7}$$

If $\lambda = 0$ is used in $g(p,\lambda)$ and $h(p,\lambda)$, single-variable polynomials describing lumped element section are obtained. The coefficients of these polynomials can be found in the first column of (2b) and (3). In a similar manner, if $p = 0$ is used in $g(p,\lambda)$ and $h(p,\lambda)$, single-variable polynomials describing unit element section are obtained. The coefficients of these polynomials can be found in the first row of (2b) and (3).

Since two-port network is lossless, then the following equation is satisfied

$$S(p,\lambda) S^T(-p,-\lambda) = I \tag{8}$$

here I is the unity matrix [14]. If (1) is substituted in this equation, the following equation is reached

$$\begin{aligned} G(p,\lambda) &= g(p,\lambda) g(-p,-\lambda) \\ &= h(p,\lambda) h(-p,-\lambda) + f(p,\lambda) f(-p,-\lambda). \end{aligned} \tag{9}$$

In the design of this kind on networks, to be able to write the coefficients in matrices (2b) and (3), (9) must be solved. In the literature, if $h_{00} = 0$, there are solutions for low-order low-pass structures [9]. In [16], without any restriction, the solutions are given for low-order low-pass structures. In the example below, a broadband matching network is designed by using the explicit solutions given in [16] and the results are compared with the results obtained in the literature.

III. EXAMPLE

In this work, the example given in [9] is solved to be able to compare the results. A broadband low-pass matching network with mixed elements (two lumped and two init elements) is designed by using the explicit solutions without any restriction.

In the design, the first row and column coefficients ($h_{00}, h_{01}, h_{02}, h_{10}, h_{20}$) of the matrix Λ_h and the delay (τ) are selected as the optimization parameters. The constant μ_2 is assumed to be –1. The sign of the other constant μ_1 is going to be the sign of h_{20} and it is defined after optimization. If h_{20} is negative, then $\mu_1 = -1$, and if h_{20} is positive, $\mu_1 = +1$. The unknown coefficients of the matrices Λ_h and Λ_g are computed by using the explicit solutions without any restriction.

In broadband matching problems, it is desired to transfer power from a complex generator to a complex load. So the selected optimization parameters ($h_{00}, h_{01}, h_{02}, h_{10}, h_{20}$ and τ) are optimized until obtaining maximum power transfer within the passband. After completing the non-linear optimization process, the following coefficient matrices are reached:

$$\Lambda_h = \begin{bmatrix} -0.1076 & -3.4667 & -2.7720 \\ 0.3723 & -5.6308 & -11.6498 \\ 1.1199 & -8.2039 & 0 \end{bmatrix},$$

$$\Lambda_g = \begin{bmatrix} 1.0058 & 4.3988 & 2.9468 \\ 1.6225 & 8.9814 & 11.6498 \\ 1.1199 & 8.2039 & 0 \end{bmatrix}.$$

The designed matching network with normalized element values is given in Fig. 2. The simulation of the network is realized by using “Microwave Office (Applied Wave Research Inc.)” [17].

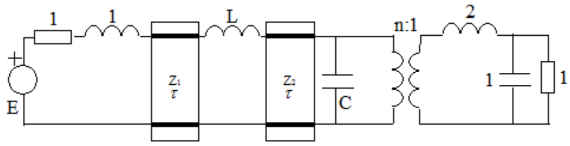

Fig. 2 Designed mixed-element matching network; proposed: $L = 1.7916$, $C = 1.392$, $Z_1 = 0.137$, $Z_2 = 0.701$, $\tau = 0.2$, $n = 0.8982$, Ref [9]: $L = 2.126$, $C = 0.751$, $Z_1 = 0.161$, $Z_2 = 0.341$, $\tau = 0.21$.

Transducer power gain graph of the designed network is depicted in Fig. 3. For comparison purposes, the gain graph obtained in [9] is also given in the same figure.

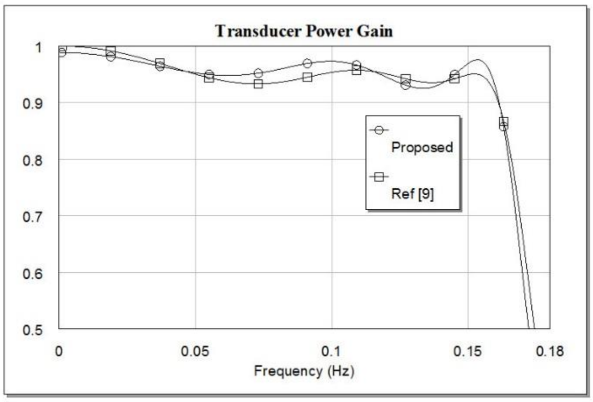

Fig. 3 The performance of the designed matching network with mixed-elements.

It can be seen in Fig. 3 that the gain value obtained in [9] at DC is unity. But then the gain drops to about 0.95 [18,19]. Since the ideal gain level for the interested load is very close to unity, the drop in the gain graph is not noticible. If the normalized capacitor value in the load is raised to 3, then the ideal gain level is 0.8 [18,19]. In this case, if the matching network is designed via the solutions given in [9], the gain at DC is stil unity, but then it drops to about 0.6 as can be seen in Fig. 4. If the explicit solutions without any restriction are used during the design, a flatter gain graph fluctiating about 0.8 is obtained (Fig. 4).

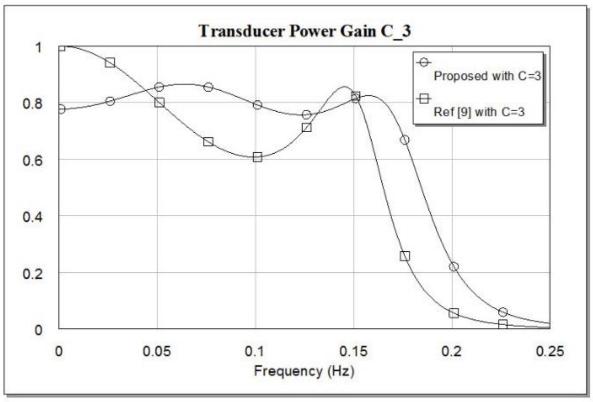

Fig. 4 The performance of the matching network with C=3.

IV. Results

In the given broadband matching network design example, it is seen that it is possible to design networks with mixed lumped and distributed elements. Since the distributed elements between lumped elements are being used as circuit elements, they are not destroying the performance of the circuit; on the contrary, they contribute to the performance of the circuit.

V. Discussion

The networks with mixed lumped and distributed elements are defined in terms of two variables: p for lumped elements and λ for distributed elements. In this work, these kinds of networks are defined in terms of scattering parameters with the polynomials g, h, f. Then as an example, a broadband matching network with mixed-elements is designed. The explicit expressions obtained from the solutions of the equations derived from the losslessness equation ($G(p,\lambda) = g(p,\lambda)g(-p,-\lambda)$) are used to compute the coefficients of the matrices Λ_h and Λ_g. Then an optimization process is started to make the transferred gain computed by using these matrices maximum. Obtained gain graph and the graph in the literature are compared. It is seen that there is no sudden drop in the obtained graph and the gain value at DC is more compatible with the values in the passband. So as a result, a flatter gain graph is obtained in the passband.

VI. Conclusion

The explicit solutions of the scattering equations without any restrictions can be used to design mixed lumped and distributed element networks.

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