

**ONE DOF THUMB MECHANISM AND ITS DIMENSIONAL OPTIMIZATION**

Samet Yavuz<sup>1\*</sup>

<sup>1</sup>*Mechanical Engineering Department, Çankaya University, Ankara, Turkey*

*\*Corresponding author: syavuz@cankaya.edu.tr*

**Abstract** – This paper involves the geometrical representation, kinematic solution and dimensional optimization of a one degree of freedom parallel thumb mechanism. The aim of the designing of this mechanism grasping objects by mimicking of a human thumb. Link length optimization of the mechanism are made around an average human thumb sizes by using Firefly Algorithm which is a nature inspired optimization algorithm.

**Keywords** – thumb mechanism, numerical synthesis, kinematic synthesis, parallel mechanisms, Firefly Algorithm

I. INTRODUCTION

Producing prosthesis for human body parts is not a new subject for human history. We can see basic prosthesis for humans in ancient civilizations. Today, with advancing technology we can produce more complex and useful prosthesis. One can be seen that multi degree of freedom thumb and finger mechanisms are commonly used in prosthesis and humanoid robots. But in this paper a one degree of freedom thumb mechanism offered. In literature there are few research are submitted as one degree of freedom finger mechanism [1, 2, 3, 4, 5].

There are two main ways to synthesize mechanisms, analytical and numeric methods. For this case analytical methods remain incapable because the presented mechanism has highly non-linear kinematic equations. At this point optimization algorithms get involved to obtain link parameters. Firefly Algorithm is used in this paper [6]. Selvi and Yavuz were used Firefly Algorithm several times to synthesize mechanisms [7, 8, 9].

First, geometry of the mechanism is submitted, its mobility is calculated and mimicking of a thumb with the mechanism is presented. Second, inverse kinematic solution of the mechanism is given to construct objective function and constraints for optimization algorithm. Then, optimization procedure is performed by using Firefly Algorithm and link parameters are obtained. Finally, obtained link parameters are tested to see whether the mechanism follow the desired motion.

II. GEOMETRY OF MECHANISM

The thumb mechanism consists of a five bar which ground link is prismatic and a four bar which is linked directly to five bar (Fig. 1). Motion of the mechanism is given by the prismatic link which is attached to the ground. One should be noticed that we have a constant angle between links  $a_5$  and  $a_3$  and a revolute joint at point B which is rotating with respect to link  $a_2$ .

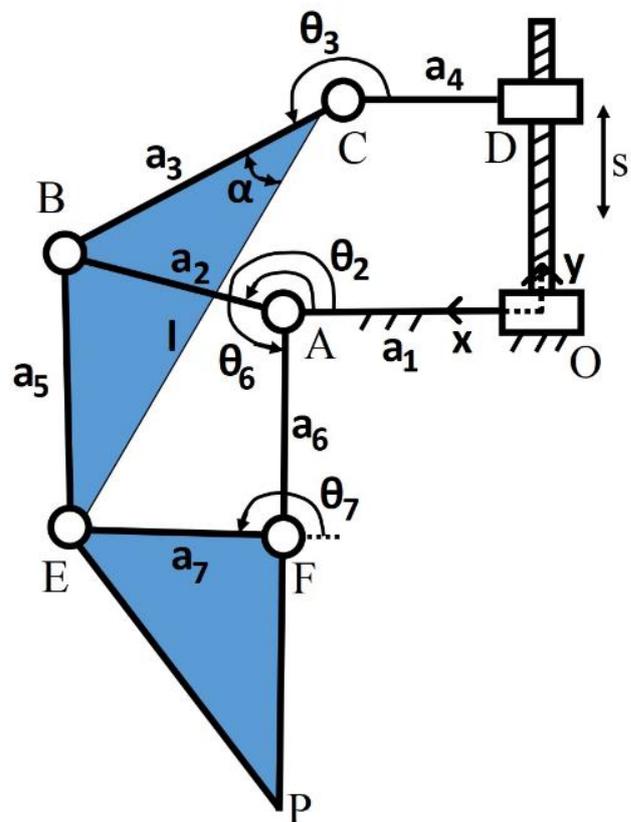


Fig. 1 Geometry of thumb mechanism

Thumb positions and joint angles with respect to x axis are given in the Fig. 2 below. Mechanism has 6 links and 7 joints. Mobility of the mechanism can be calculated as follow.

$$m = 3(n - 1) - 2J_p - J_h \quad (1)$$

Where n,  $J_p$  and  $J_h$  are link, primary joint and higher order joint numbers respectively. According to equation 1 mobility of the mechanism is calculated as 1.

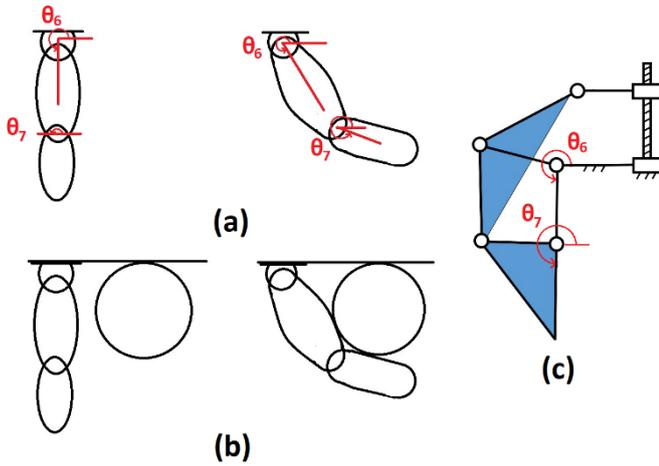


Fig 2. Thumb positions and joint angles with respect to x axis  
(a) representative thumb angles, (b) grasping of an object, (c) angles of thumb mechanism

### III. INVERSE KINEMATIC SOLUTION OF MECHANISM

In this section, kinematic analysis of the thumb mechanism is done for both to obtaining kinematic equations, objective function and constraints for optimization algorithm. Firstly, kinematic equations of five bar mechanism is solved then kinematic equations of four bar is solved.

Equations for first mechanism;

$$a_4 + a_3 \cos(\theta_3) = a_1 + a_2 \cos(\theta_2) \quad (2)$$

$$s + a_3 \sin(\theta_3) = a_2 \sin(\theta_2) \quad (3)$$

Eliminating  $\theta_2$  gives,

$$A_1 + B_1 \cos(\theta_3) + C_1 \sin(\theta_3) = 0 \quad (4)$$

Where,

$$A_1 = a_3^2 + (a_1 - a_4)^2 + s^2$$

$$B_1 = 2a_3(-a_1 + a_4) \quad (5)$$

$$C_1 = 2a_3s$$

Solving equation 4 for  $\theta_3$  gives;

$$\theta_3 = \text{ArcTan}\left[-\frac{A_1 B_1 + \sqrt{C_1^2(-A_1^2 + B_1^2 + C_1^2)}}{B_1^2 + C_1^2}, -\frac{A_1 C_1 - B_1 \sqrt{C_1^2(-A_1^2 + B_1^2 + C_1^2)}}{B_1^2 C_1 + C_1^3}\right] \quad (6)$$

From equation 2 and 3,  $\theta_2$  becomes,

$$\theta_2 = \text{ArcTan}\left[\frac{a_3 \cos(\theta_3) + a_4 - a_1}{a_2}, \frac{s + a_3 \sin(\theta_3)}{a_2}\right] \quad (7)$$

Equations for second mechanism;

$$a_4 + l \cos(\theta_l) = a_1 + a_6 \cos(\theta_6) + a_7 \cos(\theta_7) \quad (8)$$

$$s + l \sin(\theta_l) = a_6 \sin(\theta_6) + a_7 \sin(\theta_7) \quad (9)$$

Eliminating  $\theta_6$  gives,

$$A_2 + B_2 \cos(\theta_7) + C_2 \sin(\theta_7) = 0 \quad (10)$$

Where,

$$A_2 = a_1^2 + a_4^2 - a_6^2 + a_7^2 + l^2 + s^2 - 2a_1 a_4 - 2l(a_1 - a_4) \cos(\theta_l) + 2ls \sin(\theta_l)$$

$$B_2 = 2a_7(a_1 - a_4 - l \cos(\theta_l)) \quad (11)$$

$$C_2 = -2a_7(s + l \sin(\theta_l))$$

Solving equation 10 for  $\theta_7$  gives;

$$\theta_7 = \text{ArcTan}\left[-\frac{A_2 B_2 + \sqrt{C_2^2(-A_2^2 + B_2^2 + C_2^2)}}{B_2^2 + C_2^2}, -\frac{A_2 C_2 - B_2 \sqrt{C_2^2(-A_2^2 + B_2^2 + C_2^2)}}{B_2^2 C_2 + C_2^3}\right] \quad (12)$$

From equation 8 and 9,  $\theta_6$  becomes,

$$\theta_6 = \text{ArcTan}\left[\frac{l \cos(\theta_l) - a_7 \cos(\theta_7) + a_4 - a_1}{a_6}, \frac{s + l \sin(\theta_l) - a_7 \sin(\theta_7)}{a_6}\right] \quad (13)$$

Where,

$$\theta_l = \alpha + \theta_3 \quad (14)$$

### IV. OPTIMIZATION OF LINK PARAMETERS

To obtain dimensional parameters for thumb mechanism Firefly Algorithm is used. For an optimization algorithm constraints and objective function should be determined. Constraints and objective function for Firefly Algorithm are defined as follows.

Mathematica software is used to run optimization algorithm. A module is constructed to evaluate kinematic equations, constraints and objective function. Written Mathematica module is given below.

```
*List of wanted  $\theta_6$  and  $\theta_7$  angles
EvaluateVector[vector ] :=
Module[{},
a1 = 25 + vector[[1]];
a2 = 30 + vector[[2]];
a3 = 40 + vector[[3]];
a4 = 25 + vector[[4]];
a5 = 40 + vector[[5]];
a6 = 30 + vector[[6]];
a7 = 25 + vector[[7]];
l = 50 + vector[[8]];
(*Link lengths are optimized around approximated thumb sizes. *)
For[i = 20, i < 40, i ++,
error1 = 0;
s = i;
A1 = a3^2 - a2^2 + (a1 - a4)^2 + s^2;
B1 = 2a3(-a1 + a4); C1 = 2a3s;
```

```

θ3
= ArcTan [ - (A1B1 + √(C1²(-A1² + B1² + C1²))) / (B1² + C1²) ,
            - (A1C1² - B1√(C1²(-A1² + B1² + C1²))) / (B1²C1 + C1³) ];
θ2 = ArcTan [ (a3Cos[θ3] + a4 - a1) / a2 , (s + a3Sin[θ3]) / a2 ];
α=π/6;
θl=α+θ3;
A2=a1²-2 a1 a4+a4²-a6²+a7²+l²+s²-2 a1 l Cos[θl]+2 a4 l
Cos[θl]+2 l s Sin[θl];
B2=(2 a1 a7-2 a4 a7-2 a7 l Cos[θl]);
C2=(-2 a7 s -2 a7 l Sin[θl]);
θ7[i]
= ArcTan [ - (A2B2 + √(C2²(-A2² + B2² + C2²))) / (B2² + C2²) ,
            - (A2C2² - B2√(C2²(-A2² + B2² + C2²))) / (B2²C2 + C2³) ];
θ6[i] = ArcTan [ (a4 + lCos[θl] - a1 - a7Cos[θ7[i]]) / a6 ,
                (s + lSin[θl] - a7Sin[θ7[i]]) / a6 ];
(*Constraints are defined as; *)

If[
C1²(-A1² + B1² + C1²)
< 0 || B1² + C1² == 0 || B1²C1 + C1³ =
= 0 | |
a3Cos[θ3] + a4 - a1
a2
= 0 || C2²(-A2² + B2² + C2²)
< 0 || B2² + C2² == 0 | |
B2²C2 + C2³ == 0 || (a4 + lCos[θl] - a1 - a7Cos[θ7[i]])
a6
== 0 |
|Abs[Im[θ3]] > 0 || Abs[Im[θ2]] > 0 || Abs[Im[θ7[i]]]
> 0 || Abs[Im[θ6[i]]] > 0,
error1 = 1; Break[]
];

(*And objective function becomes; *)

sum6 = Sum [ √((θ6e[i] - θ6[i])²), {i, 20, 39} ];
sum7 = Sum [ √((θ7e[i] - θ7[i])²), {i, 20, 39} ];
error2 = sum6 + sum7;
1
1 + error1 + error2
]
    
```

Used Firefly Algorithm in this paper is given below as fellow.

```

Objective function f(x_i),
x_i = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, l)^T, where
f(x_i) = 1/(1 +
Sum [(Error_1+Error_2), (i, 1, n)])
    
```

```

Generate initial population of fireflies x_i (i =
1, 2, ..., n_f).
Light intensity I at x_i is determined by f(x_i)
Define light absorption coefficient γ
for (m_i; 1, MaxGen)
    for i = 1: n_f
        for j = 1: n_f
            if (I_i < I_j),
                r_{i,j} = √(Sum [(x_{k,i} - x_{k,j})², (k, 1, n_c)]);
                Do [ x_{k,i} = x_{k,i} + (β_0 / (1 + γ r_{i,j}²)) (x_{k,j} - x_{k,i}) +
                    α (Random[] - 0.5), {k, 1, n_c} ], (move firefly i towards to
                    j)
                else Do [ x_{k,i} = x_{k,i} + α (Random[] -
                    0.5), {k, 1, n_c} ], (move firefly random)
            end if
            Evaluate new solutions of f(x_i) and update light
            intensity
        end for j
    end for i
    Rank the fireflies and find the current global best
g*
end for
    
```

In algorithm which was given above,  $n_f$  denotes the population of fireflies and is selected 25 and  $n_c$  is the number of coordinates and signs the number of variables which are expected to be optimized.  $\gamma$  is called approach speed or absorption coefficient and it states multifariousness with escalating distance from interacted firefly and is selected as 0.2.  $\beta_0$  is the attractiveness, it indicates the capability of a firefly to draw in other fireflies and is selected as 0.8.  $\alpha$  is defined as randomness and it remarks the how much fireflies move randomly. Light intensity of a firefly is measured by  $I$  and it directly impresses the movement of fireflies. Here  $f(x_i)$  is the objective function and  $x_i$  is the solution for parameters which are wanted to be optimized at each iteration. Finally  $r_{i,j}$  is the monotonically decreasing function of the distance between fireflies.

### V. RESULTS AND DISCUSSIONS

Obtained link lengths are given in Table 1. below. Also one can be seen below graphics that the differences between desired and obtained joint angles with obtained link lengths (Fig. 3 and 4). Different configurations of the mechanism can be shown Fig. 5 below.

Table 1. Obtained link lengths

a <sub>1</sub>	24.5507 mm	a <sub>2</sub>	28.3635 mm
a <sub>3</sub>	38.2505 mm	a <sub>4</sub>	30.2384 mm
a <sub>5</sub>	38.2062 mm	a <sub>6</sub>	26.282 mm
a <sub>7</sub>	36.9597 mm	l	47.0929 mm

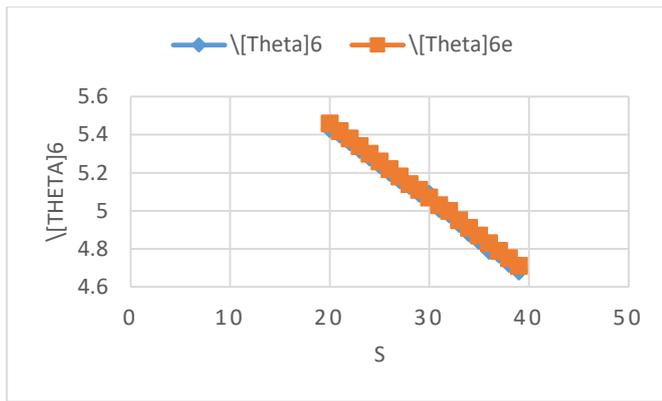


Fig. 3 Differences between desired and obtained angle  $\theta_6$

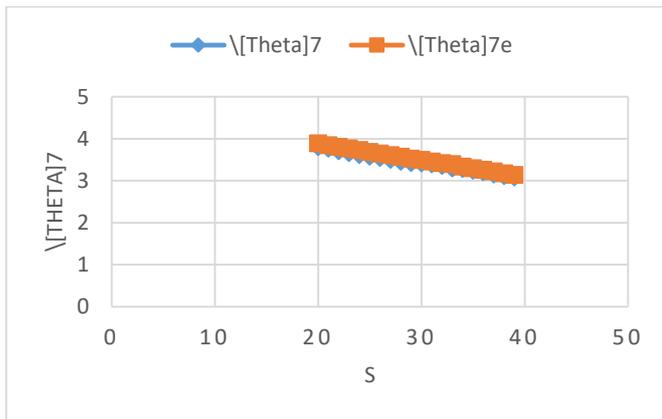


Fig. 4 Differences between desired and obtained angle  $\theta_7$

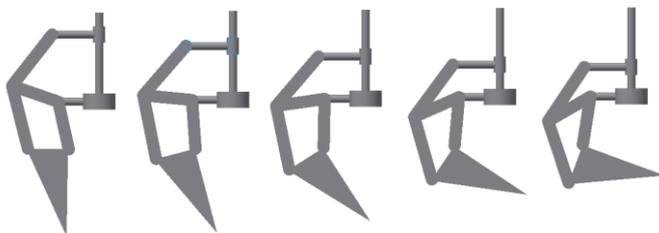


Fig. 5 Different configurations of the mechanism.

One can be clearly seen that from above figures, obtained link length parameters closely follow the desired paths.

### VI. CONCLUSION

In this study, a one degree of freedom parallel thumb mechanism is represented. Mechanism is presented geometrically and it's input and output parameters are stated clearly. Inverse kinematic equations of the mechanism are solved and input-output relationship are established mathematically and also constraints are obtained from these equations to use them in the optimization algorithm later. Link length optimization of the mechanism is made by using Firefly Algorithm and it is seen that obtained link parameters are provide the desired motion closely.

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