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SEVENTH GRADE STUDENTS' USE OF MULTIPLE REPRESENTATIONS IN PATTERN RELATED ALGEBRA TASKS*

YEDİNCİ SINIF ÖĞRENCİLERİNİN ÖRÜNTÜLERLE İLGİLİ CEBİRSEL İŞLEMLERDE ÇOKLU TEMSİL KULLANIMLARI

Oylum AKKUŞ**, Erdinç ÇAKIROĞLU***

ABSTRACT: Using multiple representations in mathematical contexts can enhance mathematical learning. Firstly how students use and conceptualize multiple representations in mathematics should be investigated. Because of this, it was aimed to examine how students use multiple representations in algebra word problems and what are the reasons behind their usage of certain representational modes. For this purpose, 21 seventh grade students (11 female and 10 male) were chosen as interview participants. They were posed three algebra questions during interviews, and they were responsible to explain their representational preferences for each interview question. As a result, it can be argued that the participants used different representations according to the question. Their ways of using different representations varied in terms of the nature of the problems and their perception of the representations. Furthermore, their representational preferences can be varied with respect to the question type, the teacher, or emotional factors.

Keywords: mathematics education, multiple representations, algebra

ÖZET: Çoklu temsilleri matematik konularında kullanmak matematik öğrenmeyi zenginleştirir. Bundan önce öğrencilerin çoklu temsil becerilerini nasıl işe koştuklarını ve nasıl kavramsallaştırdıklarını araştırmak gereklidir. Bu nedenle, bu çalışmada yedinci sınıf öğrencilerinin cebir problemlerini çözerken çoklu temsilleri nasıl kullandıkları ve neden bazı temsil biçimlerini kullanmayı tercih ettikleri araştırılmıştır. Görüşmeler için 21 (11 kız, 10 erkek) yedinci sınıf öğrencisi seçilmiş, bu öğrencilere üç tane açık uçlu cebir problemi yöneltilmiştir. Öğrencilerden bu problemleri çözmeleri ve kullandıkları temsil biçimlerini açıklamaları istenmiştir. Araştırmanın bulgularına göre, öğrencilerin her bir soru için farklı temsilleri kullandıkları söylenebilir. Bu temsil biçimlerini tercih etme nedenleri, problemin yapısına ve onların temsil biçimlerini algılamalarına göre değişmektedir. Bunun yanı sıra, temsil biçimleri tercihleri de soru tipine, öğretmene ve duygusal etmenlere göre de farklılık göstermekte ve bu farklılıklar öğrencilere göre çeşitlilik içermektedir.

Anahtar Sözcükler: matematik eğitimi, çoklu temsiller, cebir

1. INTRODUCTION

The use of multiple representations in teaching mathematical concepts is highly recommended in reform oriented attempts to mathematics curriculum (Battista, 1999; Boaler, 1998; NCTM, 2000; MEB, 2005a; MEB, 2005b). Representations are defined as physical embodiments of the ideas, concepts, and procedures through which the mathematical ideas can be manipulated by the learners (Lesh, Post, & Behr, 1987). The concept of 'multiple representations' involves the utilization of more than one representational mode and representing the same concept in different modes (Even, 1998). Learning through multiple representations of mathematical concepts results in being flexible in passing through the representations. Being able to select the most suitable one among various representational modes, and realizing the advantages and disadvantages of representations are the crucial issues for conceptual understanding in mathematics (Even, 1998).

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** Dr., Hacettepe Üniversitesi, e-posta: oyluma@hacettepe.edu.tr

*** Yrd. Doç. Dr, Ortadoğu Teknik Üniversitesi, e-posta: erdinc@metu.edu.tr

Janvier (1987) defined understanding in mathematics as a cumulative process based upon the capacity of getting familiar with a rich set of representations. If a students' concept of a given mathematical idea evolves, the related underlying translations among different representational modes become more complex (Lesh & Kelly, 1997). For mathematics learners, to conceptualize a mathematical idea within a given representational system requires making a translation between representational modes (Lesh, 1979). From this point of view, a good problem solver should be able to be "sufficiently flexible" in using variety of representational systems (Lesh, 1979; Lesh & Kelly, 1997). In this case, understanding students' usage of multiple representations in mathematics has gaining importance to determine their conceptualization level in that mathematical topic. Thus, an immediate need occurs to reveal students' use of multiple representations for mathematical topics.

1.1. Related Literature

In this part, some studies including using multiple representations in algebra context were presented. Yerushalmy (1997) reported that, multiple representations of functions would not be just tools for understanding mathematics for students; they are ends also, since they might be used for creating new mathematics. Although the students were familiar with the conventional representations of a function like f( ), they preferred to use their own representation when they were asked to write a representation for function, and therefore, their usage of representations affected their generalizations also. It was also noted that students used the symbolic representations in modeling accurately and appropriately. Tabular representations including two-column, three-column, and multi-column tables were used as an "organizational framework" for the students. They began with tables to solve the problems, they generalize the information situated in that table; and finally, they represented in an equation of graphical modes. They viewed tables as only a bridge representation to build graphs.

Swafford and Langrall (2000) studied on representations of students for describing algebraic problem solving situations. The researchers interviewed 10 sixth graders who had not taught formal algebra before. The interviewees were required to solve the problems; afterwards, they were asked to describe the situation verbally and symbolically. The results indicated that the participants solved the questions correctly, described the algebraic situations verbally and symbolically, and made accurate generalizations about functional relationships. It was reported that the most challenging procedure for students was to represent a situation symbolically and to write an equation. Moreover, Hines (2002) examined students' interpretation of linear functions in dynamic physical models which were mechanical tools for visualizing functions, and the way they used the tables, equations, and graphs to represent functions. This study was a case study with one eight-grade male student. The subject was required to explain the given problem in written words, to draw pictures, to create tables, equations, and graphs on the given task. The researcher mentioned that using different representations might help to build a deep knowledge of linear functions. This research showed that when students are given the opportunities to create and interpret representations, they can develop a better understanding for mathematics concepts by linking representations.

When the relevant research literature is examined, it is noticed that the researchers emphasized not only using multiple representations in certain topics, but also students' representation preferences. For instance, Keller and Hirsch (1998) investigated whether or not students have representation preferences, how they are contextually related, and the extent to which representation preference was influenced by the occurrence of number of representation. The sample of this study comprised of 39 students from graphics calculator section and 40 from regular calculus section. After administrating the representation preference survey to the both groups as a pre-test, 13 weeks of treatment was conducted. The post-test which had almost the same features with pre-test was administered followed by the treatment. Their study indicated that students have preferences for various representations, and they can be affected by the contextualized and purely mathematical problems. Most students had an orientation towards the equational mode of representation in purely mathematical settings; in contrast, they preferred tabular form of representations in contextualized settings.

A study carried out by Özgün-Koca (1998) was about examining students' representation preferences when they were asked to solve a mathematics problem. It was focused on students' thoughts, beliefs, and attitudes towards representations in computer and non-computer settings. The subjects were 16 students from remedial mathematics class. According to the results, students generally agreed that mathematics problems can be solved using lots of representational modes, however they found easier to focus on one mode and deal with it. The reasons for their representational preferences were presented as in Table 1.

Table 1. Reasons for students` preferences for representations

Internal Effects	Personal preferences
	Previous experience
	Previous knowledge
	Beliefs about mathematics
	Rote learning
External Effects	Presentation of problem
	Problem itself
	Sequential mathematics curriculum
	Dominance of algebraic representation in teaching
	Technology and graphing utilities

As presented in the above table, there can be many factors affecting students` representational preferences.

1.2. Purpose of the Study

The purpose of this study is to explore how students use multiple representations when they solve problems in pattern related algebra tasks and to evoke the reasons of using certain kind of representational modes for algebra context.

1.3. Research Question

How do seventh grade students' use multiple representations in pattern related algebra tasks?

2. METHOD

2.1. Research Design and Method of Data Collection

This is a case study using interviewing as a method of data collection. The case is defined here student's use of multiple representations in pattern related algebra. The standardized open-ended semi-structured interviews (Maxwell, 1996) were conducted to obtain data on how students used different representational modes when they were solving an algebra problem and to obtain deeper understanding about the possible reasons of their representation preferences.

2.2. Participants

The interview process involved the purposeful sampling of 21 students from two different elementary schools. Purposeful sampling, as used in qualitative research methods, selects information rich cases for in-depth studies (Maxwell, 1996; Patton, 1990). 21 seventh grade students (11 female and 10 male) were selected as participants. The mathematics classes of those students were observed for three months before this study. They were accounted as good informants for interviews (Bogdan & Biklen, 1992) since they were willing to participate in class discussions and attempted to use mathematical language in daily mathematical activities.

2.3. Interviews

For face validity of the interview protocol, the interview questions were given to three experts (2 mathematics educators and 1 seventh grade mathematics teacher) to be examined. Considering their suggestions, the protocol was improved. After this, the think aloud interview protocol was piloted with two seventh grade students who were chosen randomly from the low and high achiever groups. After explaining the aim of this pilot study to the pupils, they were given the questions separately. In the light of their answers the interview questions were modified.

Qualitative data for this study were obtained by this instrument in which two types of questions took place. The aim of asking the first type of questions is to obtain information about their use of multiple representations in algebra situations. The students were questioned on why they chose one type of representation over others in the second part.

After selecting the interview participants, each participant was interviewed individually, going through the interview questions in order, asking further questions when it was essential and appropriate to clarify in more detail some of the responses of the interviewees. These interviews were conducted in the teacher's room in each school at times that suited to students' schedules and also all the interviews were audio taped with the permission of the student. During the interviews, there were some rules that the researcher must obey and situations that the researcher should provide for the participating students. First of all, the researcher informed the interview participants about the purpose and the content of the interview, and then she asked each of the participant's permission to record all the interview session by audio recorder. After the questions, probes and follow-up questions were directed to deepen the interview responses.

Each interview lasted approximately 120 minutes. The analyses of the interview data were conducted with respect to the interview questions which can be classified as knowledge and opinion type of questions (Patton, 1990; Zazkis & Hazzan, 1998). The researcher as an interviewer was represented by "I". Letters for the names of the participants were assigned for purposes of protecting their identities.

3. RESULTS AND DISCUSSIONS

One aim of this study was to explore the nature of factual information the participants had about the question conceptually-oriented algebraic tasks. During the interviews, the participants were given three tasks related to growing patterns and they were asked questions that could be answered through using representations such as tables, figures, real objects, symbols, or graphs. These tasks were asked in order and the results were given accordingly.

In the first task the participants were given pattern of toothpicks (see Figure 1) by which they were expected to find the number of toothpicks in the *n*th figure.

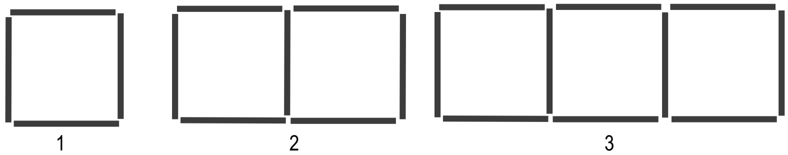

Figure 1. A sequence of boxes created by toothpicks for the first question

Participants B and U tried to use this figure to find the answer. They added new squares to this figure without attempting to use any other representational mode. Participants A and N had difficulties in understanding what was meant by "*n*th" figure; they tended to replace *n* with a specific number, as 4 or 7. Participant N had obvious problems in using letters to represent numbers. Both participant A and N insisted on saying that the *n*th figure means the fourth or seventh figure. They did not recognize that *n* represents an algebraic way of looking at the pattern to make a generalization.

Participant C, on the other hand, formed a table in order to see the pattern but she could not notice a relationship between this table and the necessary algorithm to find the number of toothpicks in the *n*th figure. In fact, she was unsuccessful to make a translation between the tabular and symbolic representations. Participants P and T chose symbolic mode of representation. Participant P, for example, first wrote $y=2x$, and tried to put the number of the toothpick as x and the figure number as y to verify this equation. He, then, used trial-and-error method to reach the solution; he put 2 for x, and found y as 4. However he found by counting that in the second pattern, there are 7 toothpicks. So, after trying few equations, he decided that he did not formulate the correct equation and gave up.

The participants who gave acceptable answers to the first question were all used symbolic representation. Nine of the respondents preferred to write an equation that represents the given pattern. They represented the figure number and the number of toothpicks by letters and they directly wrote an equation that generalizes the given pattern without attempting to use any other representational mode. Participant D, for example, mentioned that he just focused on the figure and thought the numbers of the toothpicks in his mind and then he formed an equation related to these numbers.

A translation from tabular mode of representation to symbolic mode was established by the respondent H and M. They approached the problem first by forming a table and tried to solve the problem in this form of representation, but they could not. Therefore, they constructed an equation by using the numbers located in the columns and rows of the table. Participant M used table in a different way. She said that table helped her to find the formula. She further explained;

M: In tables, everything you need is there, it is a suitable vehicle to find out the relationship between numbers. I make use of table to find equation.

Only one student preferred to describe the problem verbally. He wrote what he understood from the given diagram in words and then attempted to write an equation.

In summary, for the first question, the representational modes that were used were symbolic, tabular, diagrammatic, arithmetic, and verbal. It can be revealed that, students' use of representational modes can be altered with respect to the question. For this question, some students tried to reach a conclusion by just using the given figure, few preferred to construct a table related to the figure, and some of them wrote the necessary equation. Students used the tables as a mean to reach the symbolic mode of representation, not an end for this question.

The second question was based on a figure which represented a growing pattern of squares (see Figure 2). The participants were asked to find the number of small squares in the next figure, 7th and *n*th figure.

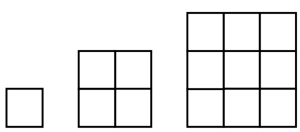

Figure 2. Figure for the second question.

Among 21 participants 19 reached a correct answer for this question. Surprisingly half of the students preferred to establish a translation from the tabular representation to the symbolic mode. They first constructed a table that included number of squares in each figure. Then to find the number of squares in 7th figure participants put the number 7 in the table and found the regarding number for the squares as 49. Figure 3 is a sample student work in response to the second question.

Figure	Coefficient
1	1
2	4
3	9
4	16
5	25
6	36
7	49

Figure = x

Square = y

y = x^2

y = r^2

Figure 3. A sample of student work in response to the second question

When they were probed the reasons of preferring this method, they explained that by constructing a table they could summarize the essential numbers. After drawing a table, they remarked that an equation should be written to find the answer for the *n*th figure. So they wrote the necessary equation and carried out the procedures to find the number of the squares in the *n*th figure. Students Z and V, for instance, conveyed the ideas about their preferences like following;

Z: First I think in my mind, and then I realize that I should organize the numbers, table is the best way to do it. And also it helps to find the relationship. In table you can see clearly. Till the third figure, I could not see the relationship, but in the fourth figure I figured it out and wrote the equation.

V: I tried to construct an equation here, but before the equation I should make a table then go with the equation. In table I searched the relationship between numbers, and then I use this relationship in the equation.

Participants H and K constructed a table only to obtain a clear pattern of numbers and then they wrote an equation. As a third step, they put 7 into an equation then find 49, and carried out the same procedures for *n*. They put *n* and find n^2 as a correct response. They did not use the tabular representation to find the number of squares for the seventh figure; instead they used the equation they found. Other translations among representational modes were also utilized by the participants. For instance F preferred to explain the pattern verbally;

F: From the figures I understood that it is a growing figure and it includes squares of the numbers, like 1 is the square of 1, 4 is the square of 2, and 9 is the square of 3, so 49 should be the answer, because it is the square of 7. For the nth figure the results is the square of n.

This showed that, participant F used her reasoning on the given figure, and made a translation from this figure to verbal mode of representation; afterwards she translated this mode into an equation for the *n*th figure.

Participant U preferred to go on with the given figures only, and found an equation that represents the pattern. He did not draw a table or a graph; he directly wrote the equation by considering the relationship between numbers. He noticed that the figure gave a clue that there are numbers whose squares were taken, so he constructed the equation.

Participant A was the only student who preferred graphical and symbolic representations for the first and second parts of the question separately. He did not make a translation between two modes of representations; rather he used graphical representation of $y=x^2$ for the first part and symbolic representation for the second. Plotting this graph was very challenging for seventh grade students. Although respondent A said that he preferred the graphical representation, he did not draw the graph correctly. He just put the numbers in the axes of graph, like constructing a table. He could not plot the graph of $y=x^2$. An equation as symbolic representation was used by respondent A for the second part of the question.

Eight of the participants used a symbolic representation for this question. E, S, and T preferred to construct an equation by referring the number of figures as *ş*, and the number of squares as *k*, and then wrote the equation of $k=2ş$. They put 7 and *n* into this equation to find the number of squares. E explained his idea like this;

E: When someone says "n" I understand that the figures go infinity, there is no end. If I use an equation to represent the situation, this is the most suitable way since in an equation you can put any number, so it is infinity. So if there is an equation, why do I waste time with using other kind of representations?

The participant E saw no need for other kind of representations if she has an equation. Participants B and C reached the wrong answer since their tabular representation was remained along with the second part of the question. They insisted on representing the mathematical situation in tabular form for seventh and *n*th figure. However, they normally could not find the answer for the *n*th figure in the table; therefore, they decided to replace *n* by a number 8, and found the result as 64. C said that:

C: I considered n as 8. So the answer is 64.

I: Is this answer true for the nth figure?

C: Actually, no. But in a table how can I find the number of squares for n? I should translate n to a number.

I: Why did you choose 8?

C: Because 7 was followed by 8.

As a result, the participants showed examples of using different modes of representations for an algebra problem. Although, it can not be argued that symbolic mode is the most convenient representational mode to solve this question, some of the participants' efforts to construct a graph or to write sentences about the figures did not seem practical for this question. From the given figure, the relationship between the number of the figures and the small squares can be seen for the seventh figure and $y=x^2$ can be constructed as an essential equation. Not being familiar with the second degree equations might be a reason for difficulties in writing an equation for most of the participants. In this question, the same situation in the first question can be observed. Some of the participants treated *n* as an unknown, not a variable. They tried to replace *n* by certain numbers. These students can be classified as the students who think arithmetically, not algebraically to solve the problems. For them using the tabular mode of representation seemed to be the only way to construct an equation, they should be persuaded that all the mathematical situations cannot be represented by numbers.

The third question asked the relationship between the numbers of toothpicks in a growing pattern (see Figure 4). They were asked to find out the number of toothpicks in figure (n+2).

This question had common features with the second question. Both of them included a figure, and asked questions about the consecutive figures. The responses for this interview question did not vary. Among the participants, only one student followed wrong procedures and concluded with an incorrect answer. She started with a verbal type of representation indicating the relationship between the number of

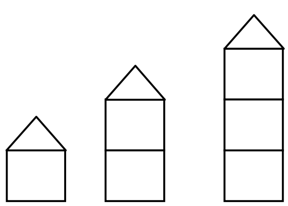

Figure 4. Figure for the third question.

toothpicks and the number of figures. However, she could not pass to the other type of representations. She thought that the only way to solve the question was to represent the mathematical situation verbally, so she replaced (n+2) by a number. She first tried 4 for n, and found 6 for (n+2). Then she decided that 6 was the correct answer. The researcher tried to make her aware of something wrong however, she did not recognize her failure.

All the other students got the correct answers for the question three. Among them only D used a symbolic representation for expressing the relationship, and for finding the number of toothpicks in the (n+2). figure. He constructed an equation representing the number of figures by x, and the number of toothpicks by y, then wrote y=3x+3 as required relationship, and then he put (n+2) in place of x, carried out the necessary algorithms, and found y as (3n+9) which was the correct answer.

Setting up a translation between the verbal and symbolic representational modes was preferred favorably. Respondents B, C, E, L, and P described the mathematical context in this question in words. C mentioned that;

C: There is a relationship between the given things. The number of the figures is less than the toothpicks. And when I multiply the figure numbers by 3 and add another 3 to what I have, I found the number of toothpicks.

After explaining the relationship verbally, they translated this verbal statement into an equation. Among all participants, 14 had very similar responses, which involved forming a table. Some of the students filled the table till the fourth term, some of them continued with the table till they found the regarding equation. After the equation was found, the students put (n+2) into the equation then found the correct answer. Two of the students explained this process as follows;

M: When you said a relationship, I firstly think a path from a table to an equation. I created a table, then I looked at this table, I tried to interpret the numbers in the table. I searched the common things among the numbers, and I catch a pattern. After this I translate this pattern to an equation, and finally I ended with an answer.

R: When I am searching a relationship in a table I try several relationships like "multiply 4, and add 3", or "divide 2". I try until one of them fits. Then I write "table relationship" in symbols, equation I mean.

In all the representation modes, a translation from a tabular to symbolic mode of representation was appeared to be most preferred one by the students. It was interesting to note that, students' representation preferences varied according to the given figure in the question. In the previous question, the given figure in which growing squares were placed was simple. It involved only one type of geometric figure which was a square. However, in this question a figure was perceived a little bit complicated by most of the students. It included a house combining squares and one triangle. Therefore the complex feature of the figure might be a reason for students to have a tendency for constructing a table to solve the question. In addition to the table, verbal explanations for the given algebraic situations were also used by the participants.

To summarize the frequency of correct and incorrect responses, it can be said that for the first question there were 14 correct, 6 incorrect and 1 no response. For the second and third questions 19 participants answered correctly, 2 participants could not provide a correct response. It can be said that, the number of correct responses were higher than the incorrect ones. This indicated that the participants performed well on the interview questions.

In Table 2, the representation preferences of the participants were displayed. In general, all representational modes and translations among these modes were used by them. The most popular representational mode seemed to be symbolic one. Tabular mode and arithmetical procedures were also appealing to the participants. The diagrammatic mode of representation had the least frequency. The translation from the tabular mode to the symbolic mode was commonly used among participants; however, making translations to solve the questions were less used compared to using only one mode of representation.

Table 2. The frequency of representation preferences of participants with respect to each question.

	Q1	Q2	Q3
Representations			
Arithmetic	3	–	–
Table	1	2	–
Symbolic	4	2	1
Verbal	–	–	1
Graph	–	1	–
Diagram	–	3	–
Translations			
Diagram to symbolic	6	1	–
Verbal to symbolic	1	1	5
Arithmetic to symbolic	2	–	–
Table to symbolic	3	12	14

3.1. Representation Preferences of the Participants

To understand the factors that influence students' representation preferences, they were asked to explain the reasons for their preferences during the interviews. There were three groups of participants based on their preferences; (a) participants who prefer equations for all question types, (b) participants who prefer tables for all question types, (c) participants whose preferences depend on the question type.

Participants A, D, F, Z, K, L, M, O, and P were included in the first group. They said that they always prefer equations for solving algebra questions. They explained their reasons, like;

It gives me a pleasure (participant A), it is easy (participant D), there is less risk of failure than other methods (participant F), it is practical to use (participant Z), I like to construct and solve an equation (participant K), It takes less time than the other methods (participant L), it is challenging (participant s O), it is certain, and more mathematical (participant M), it makes sense to me (participant O), it is mysterious with its all unknowns (participant L), it gives me enthusiasm (participant K), it arouses my curiosity (participant A), it doesn't bother me, it is easy (participant D).

When the above responses were looked over, it is amazing that the reason of students' tendencies for symbolic mode of representation was mostly emotional. They like to deal with equations, and felt pleasure from them. They were also found using an equation more mathematical and certain.

There were four respondents in the second group. Tabular representation was preferred by them for all types of questions. They preferred using a table for solving algebraic problems since it was a visual vehicle for them. All the numbers were given in a table, so it is easy to discover the relationship between them, and also in a table, information was more organized.

Participants B, C, H, J, N, S, and V were in the third group. Their preferences changed accordingly with the type of question. They were open to use any representation mode if the question requires. Respondents J and V said that knowing and using many ways to solve questions are better than knowing only one way. Participant N and S claimed that, if they were given with a table they would certainly use this table to reach the result, but since there was no table, they had to construct it for reaching a solution. They did not prefer to construct a table, rather they continued with symbolic mode of representation. On the other hand participants B, C, and H's preferences depend on the given numbers in the question. If the numbers were small, as the participants indicated less than 20, they were willing to make a table including the numbers.

Furthermore, participants S and N stated that they could prefer tabular, graphical and symbolic mode of representations depending on the situation. If they were dealing with questions including unknowns like x, y, and z they preferred symbolic mode of representations since an equation means unknowns for them. If finding a relationship between the numbers was asked in a question, they said that their representation preference would be tabular, because the sequence of numbers can be easily seen so the relationship can be directly found. And the graphical mode of representation was preferred by them if a question asks about any increase or decrease in a given mathematical situation. They indicated that sometimes they used one representational mode to verify other mode of representation. When they were having any trouble in symbolic mode of representation, they preferred to construct a table of the equation to verify the symbolic representational mode.

4. CONCLUSIONS and RECOMMENDATIONS

As a conclusion, it can be argued that the participants used different representations to reach an answer to the given algebraic questions. Their ways of using different representations varied based on the nature of the problems and their perception of the representations. However, students' level of involvement in certain mode of representation influenced their way of using it for solving algebra questions. If they were familiar with a graphical mode of representation, they tried to use it, or if they enjoyed using tabular mode of representation, they used it for every question.

The reason why one representation is preferred over another for solving algebra problems involves the perception of students about the representations and what is mathematically sound, the nature of a given problem, the belief about the level of accuracy that a certain mode of representation can produce a solution for the problem, and also whether or not they enjoy to use it. The results of the study were in line with the results of Hines (2002) who investigated one student's experiences with linear functions. He argued that students could use variety of representational modes in dealing with algebraic concepts. The results of Herman's (2002) study were also supportive. She argued that, students' representational preferences can be varied according to the content of the problem, students' perceptions of the representations, and being familiar with the certain types of representations.

This study contributes an additional component to the reasons for representational preferences. In the theory of multiple representations, several factors influencing students` representation preferences were listed in the literature part (Keller and Hirsch ,1998; Özgün-Koca, 1998). These all stand out important factors determining the reasons of students` choices for certain representational modes. However according to numerous comments from interview subjects, one factor can be added to the above list; which can be called as "emotional factor". It was observed that if students like to practice one of representation modes, whether or not using this type of representation is suitable for this problem situation, they use it. In interviews, one of the students said that;

G: I know using a table is time consuming here and I love to use tables, I like to put numbers in it, so I am going to use it.

This emotional factor might be coming from the fact that; as students practiced certain type of representation and recognized the benefits of it, they seem hardly willing to use another type of representation over the one they are familiar with. Therefore, it was recognized that students` appreciation and enjoyment of the type of representation has an influence on choosing a representation mode for solving algebra problems.

As a classroom implication, it can be stated that teachers should emphasize applications of multiple representations in their classrooms since in interviews students agreed the idea that they like to be engaged in all kinds of representations for solving algebra problems. In a very simple way, establishing relationships between representational modes can be conducted in part of a daily lesson by making students to think about any situation that represents a mathematical object. It can be a daily-life situation, a table, or a poem. Afterwards, students can be provided opportunities to discuss the similarities and differences of variety of representations, as Herman (2002) suggested, so that students can recognize relationships between different modes of representations and appreciate the superiorities and disadvantages of some kind of representational modes over others. As it can be understood, discussion about representations should be an inevitable part of mathematics lessons.

Future research could combine data from students and their teachers, because teachers have also an impact on shaping students' representational preferences. What teaching strategies and representation types are used within algebra classrooms by teachers and how those representations are conceptualized by the students seems to be worthwhile to study. Some students during the interviews claimed that they prefer to use an equation as a representational tool to solve algebra problems because it seems to them more mathematical and this kind of representation was taught with more emphasis. Such study examining the reasons of that belief and the degree of teacher effects on that belief would be a deeper level of investigation after this study.

In this study the open-ended semi structured interviews were conducted. However, there is a need for a structured task based interviews to answer the question of how students create new representations, how they use the representational modes in problem solving, and how they demonstrate these representations in mathematical situations. In my opinion, it would be interesting to examine more closely students' behaviors when they are dealing with multiple representations in mathematics.

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EXTENDED ABSTRACT (Uzun İngilizce Özet)

Representations are defined as physical embodiments of the ideas, concepts, and procedures through which the mathematical ideas can be manipulated by the learners (Lesh, Post, & Behr, 1987). The concept of ‘multiple representations’ involves the utilization of more than one representational mode and representing the same concept in different modes (Even, 1998). Learning through multiple representations of mathematical concepts results in being flexible in passing through the representations, being able to select the most suitable one among various representational modes, and realizing the advantages and disadvantages of representations are the crucial issues for conceptual understanding in mathematics (Even, 1998). For mathematics learners, to conceptualize a mathematical idea within a given representational system requires making a translation between representational modes (Lesh, 1979). From this point of view, a good problem solver should be able to “sufficiently flexible” in using variety of representational systems (Lesh, 1979; Lesh & Kelly, 1997). In this case, understanding students’ usage of multiple representations in mathematics has gaining importance to determine their conceptualization level in that mathematical topic. Thus, an immediate need occurs to reveal students’ use of multiple representations for mathematical topics.

The purpose of this study was to explore how students use multiple representations when they solve problems related to algebra and to evoke the reasons of using certain kind of representational modes for algebra context. The research question can be stated as follows; “How do seventh grade students’ use multiple representations in pattern related algebra tasks?”

To achieve the purpose, the standardized open-ended semi-structured interviews (Maxwell, 1996) were conducted to obtain data on how students used different representational modes when they were solving algebra problems and to obtain a deeper understanding of the possible reasons of their representational preferences. The sample of this study consists of 21 students from two different elementary schools. After selecting the interview participants, each participant was interviewed individually, going through the interview questions in order, asking further questions when it was necessary and appropriate to clarify some of the responses of the interviewees. After the questions, probes and follow-up questions were directed to deepen the interview responses. Each interview lasted approximately 120 minutes.

According to the results, the participants used different representations to reach an answer to the given algebraic questions. Their ways of using different representations varied based on the nature of the problems and their perception of the representations. However, students’ level of involvement in certain mode of representation influenced their way of using it for solving algebra questions. If they were familiar with a graphical mode of representation, they tried to use it, or if they enjoyed using tabular mode of representation, they used it for every question. The reason why one representation is preferred over another for solving algebra problems involves the perception of students about the representations and what is mathematically sound, the nature of a given problem, the belief about the level of accuracy that a certain mode of representation can produce a solution for the problem, and also whether or not they enjoy to use it. These all stand out important factors determining the reasons of students` choices for certain representational modes. An additional factor can be added to the reasons for representational mode choices. This mode can be called as “emotional factor”. It was observed that if students like to practice one of representation modes, whether or not using this type of representation is suitable for the problem situation, they use it. This emotional factor might be coming from the fact that; as students practiced certain type of representation and recognized the benefits of it, they seem hardly willing to use another type of representation over the one they are familiar with. Therefore, it was recognized that students’ appreciation and enjoyment of the type of representation has an influence on choosing a representation mode for solving algebra problems.

As a classroom implication, it can be stated that teachers should emphasize applications of multiple representations in their classrooms since in interviews students agreed the idea that they like to be engaged in all kinds of representations for solving algebra problems. In a very simple way, establishing relationships between representational modes can be conducted in part of a daily lesson by making students to think about any situation that represents a mathematical object. It can be a daily-life situation, a table, or a poem. Future research could combine data from students and their teachers, because teachers have also impact on shaping students’ representation preferences. What teaching strategies and representation types are used within algebra classrooms by teachers and how those representations are conceptualized by the students seems to be worthwhile to study. Some students during the interviews claimed that they prefer to use equational mode of representation to solve algebra problems because it seems to them more mathematical was taught with more emphasis. Such study examining the reasons of that belief and the degree of teacher effects on that belief would be a deeper level of investigation after this study.