THE EFFECTS OF MULTIPLE LINKED REPRESENTATIONS ON STUDENTS' LEARNING OF LINEAR RELATIONSHIPS*

BİLGİSAYAR ORTAMINDAKİ ÇOĞUL BAĞLANTILI GÖSTERİMLERİN ÖĞRENCİLERİN DOĞRUSAL İLİŞKİLERİ ÖĞRENMELERİ ÜZERİNDEKİ ETKİLERİ

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ABSTRACT: The focus of this study was on comparing three groups of Algebra I 9th-year students: one group using linked representation software, the second group using similar software but with semi-linked representations, and the control group in order to examine the effects on students' understanding of linear relationships. Data collection methods included mathematics pre- and posttests, follow-up interviews, computer-based clinical interviews at the end of the treatment, classroom and lab observations, document analysis, and survey. The conclusion of this study was that semi-linked representations could be as effective as linked representations and that there was a role for each in different situations, at different levels, and with different mathematical concepts.

Keywords: Mathematics Education, Computer-Assisted Education, Multiple Representations, Linear Relationships

ÖZET: Bu çalışma, 9.sınıf cebir öğrencilerinin doğrusal ilişkiler konusunu öğrenmelerinde bilgisayar kullanımının etkilerini incelemiştir. Bu amaç ile bağlantılı ve yarı bağlantılı gösterim yazılımı kullanan iki deney ve bir kontrol grubu karşılaştırılmıştır. Veri toplama yöntemleri matematik ön ve son testleri, uygulama sonrası mülakatları ve bilgisayar ortamındaki klinik mülakatları, sınıf ve laboratuvar gözlemleri, döküman analizi ve matematiksel gösterimleri tercih anketinden oluşmuştur. Bu çalışmanın sonuçları, yarı bağlantılı gösterimlerin bağlantılı gösterimler kadar etkili olabileceğini ve her ikisinin de farklı durumlarda, değişik sınıf seviyelerinde ve matematik konularında kullanımı olduğunu göstermiştir.

Anahtar sözcükler: Matematik Eğitimi, Bilgisayar Destekli Eğitim, Çoğul Gösterimler, Doğrusal İlişkiler

1. INTRODUCTION

With the new millennium, the utilization of technology and the use of multiple representations in mathematics instruction have become one of the significant topics in mathematics education (NCTM, 2000). Here, multiple representations are defined as external mathematical embodiments of ideas and concepts to provide the same information in more than one form. One example of this type of environment is educational software with linked multiple representations. Linked multiple representations are a group of representations in which, upon altering a given representation, every other representation is automatically updated to reflect the same change (Rich, 1995/1996). We define semi-linked representations as those for which the corresponding update of changes within the representations are available only upon request but are not automatic.

Although there are a number of theories emphasizing multiple representations in the history of mathematics education, with Dienes' "multiple embodiment principle" this issue gained a significant prominence. The multi-embodiment principle suggests that conceptual learning of students is enhanced when students are exposed to a concept through a variety of embodiments (Dienes, 1960).

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Constructivism suggests that students construct their own knowledge by themselves actively in *their experiential world* (Von Glaserfeld, 1996). Through communication and interaction with other people, learners test how likely (consistent) their constructs are with others' (Confrey, 1990; Goldin, 1990). Because of differences in experience, we cannot expect that everyone will understand a concept in the same way from one representation or that one representation will be equally meaningful for everyone.

Integrating theoretical components from a number of mathematics educators, the theory of understanding relative to multiple representations is as follows:

- Students should be able to identify the given mathematical idea across different representations;
- Students should be able to manipulate the idea within a variety of representations;
- Students should be able to translate the idea from one representation to another;
- Students should be able to construct connections between internal representations;
- Students should be able to decide the appropriate representation to use in a mathematics problem;
- Students should be able to identify the strengths, weaknesses, differences, and similarities of various representations of a concept. (Dufour-Janvier, Bednarz, and Belanger, 1987; Hiebert and Carpenter, 1992; Lesh, Post, and Behr, 1987; Schwarz and Dreyfus, 1993).

The question is, now, how understanding across multiple representations can be improved with educational technology. Kaput (1992) advocates the use of linked representations as follows:

> All aspects of a complex idea cannot be adequately represented within a single notation system, and hence require multiple systems for their full expression, meaning that multiple, linked representations will grow in importance as an application of the new, dynamic, interactive media (p.530).

According to Piaget's theory, cognitive development is driven by a series of equilibrium-disequilibrium states. If everything is in equilibrium, we do not need to change anything in our cognitive structures. Linked representational software gives students immediate feedback on the consequences of their actions with machine accuracy, but it may not engender the disequilibrium necessary for learning. Semi-linked software, by not showing the corresponding changes in other representations, by giving time to reflect or asking questions about what kind of changes will result from a change in any representation, forces students to resolve the dissonance in their cognitive structures. If their organization of knowledge is well established, they can deal with the question. However, if not, then they will need accommodations in their cognitive structures. Thus, semi linked representational environment puts students in a more active role as learners.

1.1. Review of Literature

The studies in the literature indicate that the use of multiple representations with or without technology helps students to construct mathematical concepts in more empowering ways. Studies without the use of technology emphasized the use of various representations in-class instruction (Brenner et al., 1995; Harel, 1989; Moseley & Brenner, 1997; Poppe, 1993/1994; Knuth, 2000a and 2000b). Technology oriented studies, on the other hand, utilized computer (Dyer, 1994/1995; Jiang & McClintock, 2000; Noble et al., 2001; O'Keefe, 1992/1993; Porzio, 1994/1995) or calculator (Porzio, 1997; Ruthven, 1990; Tolias, 1993) technology in order to investigate the effects of technology on learning with multiple representations. Many of them concluded that students gained more understanding not only in mathematical concepts but also in the use of multiple representations in mathematics.

The studies utilizing the linked multiple representational software are divided into two groups: comparative studies and case studies. The former ones (Rich, 1995/1996; Rosenheck, 1991/1992) compared groups of students by using different technologies in treatments. Due to the crucial differences in the environments (e.g., computer versus non-computer or calculators versus computers), it is difficult to draw clear conclusions. In fact, results of these studies showed no significant differences between groups. On the other hand, the case studies indicated more encouraging results because of the use of linked multiple representation software (Borba, 1993; Borba & Confrey, 1993; Lin, 1993; Rizzuti, 1991/1992; Yerushalmy & Gafni, 1992). The linkage among representations in the computer-based environment was powerful tool which provide a visual environment for students to develop and test their mathematical conjectures.

Studies reviewed above investigated various effects of multiple linked representation software. However, the present study focused directly on the effects of the linking property of the software on students' learning, by using two different groups of students in a classroom environment with the same computer software, except for the linking property. The goal was to see how this feature of the software affected their learning and understanding of the relationships between the representations and the mathematics content itself. Therefore, the focus of this study was comparing three groups of students: one group using linked representation software, the second group using similar software but with semi-linked representations, and the control group. Briefly, the research question of this study was:

- What are the effects of using linked representation software compared to using semi-linked representation software on students' understanding linear relationships?

2. METHOD

Subjects of this study were ninth-grade Algebra I students. The main reason for selecting this class was that Algebra I is the first time students learn formally about subjects like linear equations that are easy to address with multiple representations. The goal of this study was to examine the effects of technological features on students' learning when they are first encountering a topic. Linear relationships, one of the mathematical concepts included in Algebra I, was well-suited for teaching via multi-representational software because it has real world applications and a variety of representations such as tables, graphs, or equations. The class was divided randomly into three groups of students—two experimental groups ($n_1 = n_2 = 10$) and a control group ($n_3 = 5$). Two experimental groups used the same software but with different linking properties—linked and semi-linked.

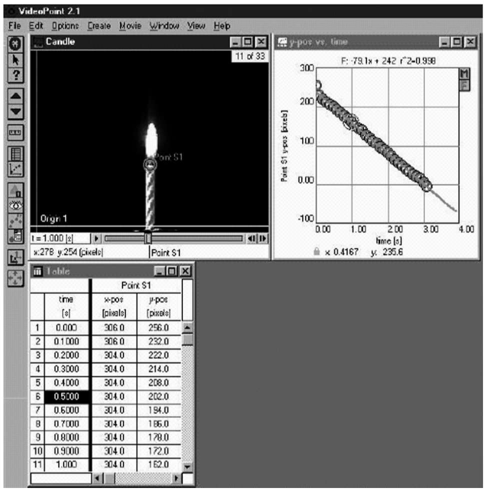
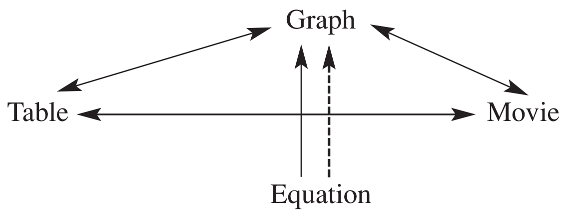


Figure 1: Computer screen from VideoPoint 2.1.2. (2000) (Candle movie was obtained from Measurement in Motion (1996) sofware.)

VideoPoint is a software package that allows one to collect position and time data from QuickTime movies of a candle burning out, two cars driving in the same direction with different constant speeds or two fish swimming towards each other (Figure 1). These data can be combined to form calculations such as distances between points and can be presented using different representations such as tables, graphs, and equations. As can be seen in Figure 1, this environment provides access to all four types of representation. For instance, while the movie presents the real-life phenomena, the mathematical model is presented in the equation form above the graph. Moreover, in order to answer a question about the height of a candle at a particular time, table values could be more efficient to use, whereas height changes over time can be clearly seen with the graph.

Although VideoPoint was designed as linked representational software, the linkage for the table representation was not two-way. So, the software developer made changes, at the request of the investigator, to create the fully linked and semi-linked versions of the VideoPoint for this study. The differences between the linkages in the linked and the semi-linked representations are summarized in Figure 2. As one can observe, the graph, table, and movie representations are linked two-way in the linked version. This means that when the user clicks on a point in those representations, the corresponding data points in all other two representations are highlighted. This can be observed in Figure 1 between the graph and the movie. Moreover, when the linked version user clicks to see the algebraic form (the equation of best fit) of the phenomena, the line of best fit is graphed in the graph window and its equation appears above the graph automatically. On the other hand, the user of the semi-linked version is not able to see any updates when s/he clicks on one representation. The only linkage that is available in the semi-linked version is between the graph and equation form. When the user estimates the coefficients in the algebraic form, s/he has an option to see the graph of the predicted equation.



___ Linked Version
- - - Semi-Linked Version
A→ B A change in A automatically effects a change in B
A→B A change in A effects a change in B upon request

Figure 2: The Linkages among Linked and Semi-Linked Representations in VideoPoint

Four computer lab sessions took place during the data collection period of 11 weeks. Since this particular school schedules its classes for 78 minutes, one group was taken out of the classroom one day per week for a 35-minute computer lab at the first part of the class session; then the second part of this class session, the other group. At this time, the control group stayed in the class for the whole period and they did extra activities or questions.

Data collection methods included mathematics pre-and posttests, follow-up interviews with all students after the mathematics posttest, computer-based clinical interviews at the end of the treatment with 5 students from each experimental group, and classroom and computer lab observations. A survey was conducted at the end of the study in order to see students' opinions about mathematics, representations in general, and the computer environment. A panel of experts helped the researcher assure both the content and face validity of the instruments. Instruments were continuously updated according to feedback from students both during the pilot and throughout the actual study. Although there are a lot of techniques available for increasing the credibility of a study; triangulation, member check, and peer debriefing were used to provide trustworthiness of this study. Table 1 summarizes the research design with data collection and data analysis methods.

3. FINDINGS

Instead of studying each question separately, questions in all written tests used in this study were clustered into categories, and those categories were compared across the three groups—linked, semi-linked, and

Table 1: Data Sources

Research Question	Data Collection Methods	Description of the Data Collectiuon Methods	Criteria or Indicators for Analysis
What are the effects on students' understanding of linear relationships using linked representation software compared to using semi-linked representation software?	Clinical and Follow-up Interviews	Five students from each experimental group were interviewed while using the computer software. Follow-up interviews after the pre-and posttests provided informatiyon about their reasoning in answering the questions.	Codes, patterns and themes were searched throughout the data.
	Mathematical Pre-and Posttest	Students' paper and pencil performance were analyzed.	Descriptive analysis. Nonparametric tests for group differences and for achievement differences between pre-and posttest.
	Teacher Interviews	To see the teacher's views about students' growth mathematically and their preferences	Codes, patterns and themes were searched throughout the data.
	Observations	Everday classroom and computer lab sessions observations	
	Survery	Students' attitudes towards mathematics mathematical representations in both technology and non-technology enviroments were studied with Likert scale and open-ended questions.	Descriptive analysis Nonparametric tests for group differences. Qualitative analysis for open-ended questions.

control. The categories were: Word Problems, Interpreting/Constructing and Reading Graphs, Solving and Constructing Equations, Reading and Constructing Tables, and Misconceptions (Height/Slope, Point/Interval, Graph as Picture). These categories were compared using a nonparametric test—Kruskal-Wallis (a test for several independent samples)—to identify differences between the linked, semi-linked, and control groups. The results of this test showed that there were no differences in achievement between the groups in any category of problems on either the pretest or posttest at either the .05 or .1 confidence level. A nonparametric test—the Wilcoxon Test (a test for dependent samples)—was used to identify the improvement or decline from pretest to posttest within groups in each category (see Table 2). Some of the improvements were significant at the .05 level, such as experimental groups in the categories of interpreting graphs and constructing equations, the semi-linked group for the height/slope misconception category, and the linked group for the graph as picture category. Other improvements were significant at the .1 level, such as the linked group for the height/slope category.

Table 2: Improvement Significance Scores

		Improvement Significance Scores		
		Control	Linked	Semi-Linked
Word Problems (Verbal)		0.083*	0.058*	0.014**
Graphs	Interpreting/Constructing Graphs	0.059*	0.026**	0.007**
	Reading Graphs	0.785	0.038**	0.729
Equations	Solving Equations	0.157	0.317	0.180
	Constructing Equations	0.066*	0.042**	0.042**
Tables	Reading Tablesa	0.317	0.02**	0.102
	Constructing Tables	1	1	0.317
Height/Slope Misconception		0.684	0.061*	0.024**
Point/Interval Misconception		0.102	0.317	0.317
Graph as Picture Misconception		0.317	0.014**	0.157

*.1 significant **.05 significant

In order to study the mathematical learning within the computerized environment, clinical interviews were conducted. It was found that in the linked software environment, when a question was asked, students either used the linkage directly to answer the question or they assimilated this new information and drew upon their previous knowledge to answer the question. When they used the linkage, their explanation for their answer was based more on the software; especially the movie. However, their answers were more based on the mathematical aspects of the question, when they did not use the linkage. When students provided an inappropriate answer to a question and they saw that they were wrong according to the linkage or computer feedback, disequilibrium occurred. Then they needed to go back and interpret this new information with their existing knowledge. If they could not interpret the new information, they needed to accommodate their preexisting knowledge in order to reach equilibrium. Sometimes students did not have the enough background to interpret this new information with their existing knowledge. Some students did not use the linkage at all, when they trusted their knowledge and answers.

When a question was proposed in the semi-linked environment, students relied mainly on their own existing knowledge with the help of the software. They assimilated new information and drew upon their existing knowledge to answer the question. Although the semi-linked environment did not provide such rich feedback as in the linked environment, a ready-made graph or table presented powerful visual information/feedback for students to use while answering the questions. Lack of linkage forced more mathematically-based explanations instead of movie-based explanations and empowered students to trust their answers and convince themselves and construct the linkages between representations by themselves. Some students needed the linkage in some situations in order to construct more empowering mathematical concepts.

Most of the students indicated that they found VideoPoint helpful in learning mathematics. Easy access to all representations at once was a common theme mentioned by students as a reason for finding VideoPoint helpful. Some students also mentioned how VideoPoint helped in constructing relationships among representations. They reported that they liked being able to see different kinds of representations all at once since it gave them a choice to work with one that they were more comfortable with or showed them there were various forms available. Several students also pointed out that VideoPoint was helpful in comparing different representations or checking their answers.

4. RESULTS and SUGGESTIONS

Having access to multiple mathematical representations provided by VideoPoint enabled students to choose the types of representation they were most comfortable with. Another advantage of technological multiple representations was that more focus was spent on the relationships among representations and the mathematical content instead of calculating or drawing. Drudgery-free access to multiple representations allowed students to concentrate on the mathematical ideas. Moreover, the software was able to offer an environment with resources and constraints for students to construct new schemes or change their existing ones through a series of equilibrium-disequilibrium states.

VideoPoint offered a link among those representations. Connections among representations were even highlighted by the linked version of VideoPoint. Linkage offers an interesting potential for the teaching and learning of mathematics. However, the results of this study illustrate that the mathematics education community can benefit from both versions, linked and semi-linked. Having two versions of the software would be more helpful than having either one of them alone. Being able to switch between the linked and semi-linked versions would be invaluable because the linked and semi-linked versions have their own different advantages. This emphasizes the importance of the teacher's role in the classroom. Technology is a very effective tool in the process of teaching and learning of mathematics. However, there are many important decisions to be made by the teacher, such as when to use linked or semi-linked versions and with whom as the results of the present study suggest.

The best example of a significant benefit of the linkage was helping students to overcome the graph as picture misconception. The experimental groups, semi-linked and linked, performed better than the control group in the category of graph as picture misconception. With linkage, students were able to see corresponding highlighted points in the movie and on the graphical representation. As expected, the improvement of the linked group in separating the graph from the real-life picture was significant. If a student has weak cognitive schemes about the connections among representations, the linked version could be very helpful. However, students who have already constructed the relationships among representations mentally might be better off with the semi-linked version. Some advantages of the semi-linked software were enabling students to be more active in their learning process, to make use of their existing knowledge and the information provided by the software, and to construct new mathematical ideas or connections among the representations with the help of new information. Mathematics teachers might prefer linked or semi-linked versions of software for different age groups or grade levels. The most beneficial usage could come from using a linked version to introduce a mathematical idea and help students construct their schemes. Once accomplished, the linkage could be removed and the semi-linked version could be turned on in order to make students use their newly constructed schemes.

One of the limitations of this study was time. A longer study in which students have more experience with the software could be more informative. Moreover, students encountered the software on only one topic—linear relationships. The study of more advanced topics such as quadratic relationships or exponential relationships with VideoPoint could reveal how the linked or semi-linked version might help students to understand more sophisticated ideas. Furthermore, more participants from different age groups or grade levels could bring different issues to bear. Limitation of the software was that the semi-linked version did not provide semi-linkages among the movie, the graph, and the table windows at all but only a semi-linkage between the graph and the algebraic form. It would be very interesting to develop another version of the software that provides linkages among all the representations not automatically but upon request. Based on conclusions drawn from this study, it is evident that more studies are needed to investigate various aspects of the electronic linkage provided by some software. Some aspects of the linked and semi-linked versions of the software have been explored by this study, while many other possible areas of research have been waiting to be explored.

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