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STUDENTS' CONCEPTION OF FRACTIONS: A STUDY OF 5TH GRADE STUDENTS

ÖĞRENCİLERİN KESİRLERİ KAVRAMASI: 5.SINIF ÖĞRENCİLERİ ÜZERİNE BİR ÇALIŞMA

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ABSTRACT: This paper investigates students' conception of fractions in solving word-problems. An essay type test including 10 word-problems was administered to 5th grade students in a private elementary school (N=122). An analysis of the difficulties met by the students was given through examples; evidence was given about the crucial role of the meanings of a part and a quantity, and the units in the operations.

Keywords: fractions, problem solving, difficulties

ÖZET: Bu çalışma öğrencilerin kesirler ile ilgili sözel problemleri çözerken gösterdikleri kavramsal anlamayı incelemektedir. 10 sözel problem içeren sorudan oluşan sınav bir özel ilköğretim okulunda okuyan 122 5. sınıf öğrencisine uygulanmıştır. Öğrencilerin sahip oldukları zorluklar örnekler ile verildi; sonuçlar parça, bütün ve işlemlerdeki birimin önemini göstermiştir.

Anahtar Sözcükler: kesirler, problem çözme, zorluklar

1. INTRODUCTION

Fractions form a basis for various concepts in elementary school mathematics such as decimals, rational numbers, ratio, proportion, and percentage. It is also the first abstract concept in mathematics for the young learners (Booker, 1996). There is a complex relationship among the meanings and the representations of fractions since different meanings can be attached to the fractions in different contexts (Leinhardt & Smith, 1984). The different meanings or models for fractions are part-whole, quotient, ratio, operator and measure. The part-whole model is a basis for initial fraction idea. This model includes continuous representations and discrete representations (Orton & Frobisher, 1996).

Although students start to study fractions from the early grades, it was reported that many teachers needed to review the previous fraction concepts in each grade level (Kamii, 1994; Robitaille, 1992). Previous studies reported that students have difficulties in understanding the fundamental concepts in fractions in each grade level (Aksu, 1997; Booker, 1998; Hart, 1993; Haser & Ubuz, 2001; Haser & Ubuz, 2002; Leinhardt & Smith, 1984; Newstead & Murray, 1998; Orton & Frobisher, 1996). Dominance of limited contexts in initial exposure to fractions (e.g. area, half, unit fractions), interference of whole number schemes (e.g. considering a fraction as two distinct whole numbers), and ineffectiveness of current teaching methods are the main sources of the students' difficulties (Baturo & Cooper, 1999; Mack, 1995; Moss & Case, 1999; Newstead & Olivier, 1999).

Students' errors in dealing with discrete representations of part-whole model are a well-identified result of the lack of related classroom exercises. More emphasize on the role of the whole and problems involving fractions as the part of a collection of discrete objects was documented in long-term projects about teaching fractions (Newstead & Murray, 1998; Newstead & Olivier, 1999).

It is claimed that even though students can

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easily carry out the algorithms with fractions, they do not understand the meanings of such algorithms (Aksu, 1997; Mick & Snicrope, 1989; Wearne-Hiebert & Hiebert, 1983). Students who seem to perform the operations correctly can easily get confused when operations are presented in word problems (Aksu, 1997).

Students' initial approaches to solve a word-problem is searching for a single idea, equation or computation rather than trying to combine various basic procedures in a new path. The reason is the restricted type of problems that the students solve in their mathematics classes. Hence, when students are presented with problems involving multiple concepts and steps, they face several difficulties (Lesh & Zawojewski, 1992).

Hecht (1998) investigated the individual differences in students' fractions skills in relation to their procedural knowledge defined as students' awareness of the processing steps required to solve a problem, and conceptual knowledge that involves connecting meanings to mathematical symbols. The results indicated that procedural and conceptual knowledge explain the variability in students' word-problem solving strategies involving fractions.

Only a research study carried out by Aksu (1997) examined the 6^{th} grade Turkish students' performance on fractions in three different contexts: (a) understanding the meaning of fractions, (b) computation with fractions and (c) solving word problem involving fractions. Her findings indicated that students' performance was highest in the computational tasks and lowest in the problem-solving tasks. In detail, addition problems were the simplest but the multiplication problems were the most difficult for the students. She also investigated the relationship between the students' performance on fractions and their previous mathematics achievement and the gender. No significant differences according to gender but significant differences according to previous mathematics performance were detected. But, what is needed more is to find out how students construct the knowledge. The study done by Aksu (1997) provided only statistical interpretations of the 6^{th} grade students' performances in dealing with fractions. The aim of this study, however, is to provide a detailed qualitative analysis of students' conceptions of fractions, particularly part-whole relation.

2. METHOD AND PROCEDURE

Fractions are introduced in the 2^{nd} grade of elementary school in order to solve the daily life problems where natural numbers are not sufficient (Milli Eğitim Bakanlığı, 1998). Only a limited number of fractions, such as 1/2, 1/3 and 1/6 are introduced. Besides that, solving and writing problems by using these fractions are given. But, fractions are taught as a separate subject in the 4^{th} grade in which different types of fractions and equivalent fractions are explored. Following the 4^{th} grade, transition between the different types of fractions, reducing and expanding fractions, and ordering of fractions are taught. The time allowed for teaching fractions is eight hours in the 2^{nd}, 3^{rd}, and the 5^{th} grade, and 12 hours in the 4^{th} grade. These time spans do not include the time spent on operations.

Operations with fractions are introduced in the 3^{rd} grade. Until the 5^{th} grade, students add and subtract fractions with equal denominator and multiply a fraction with a natural number. Addition and subtraction of fractions with unequal denominators and multiplication of fractions are explored in the 5^{th} grade. Each operation on fractions takes 4 hours to be introduced. This means that the time spent on fractions in the 5^{th} grade is 25 hours as a total.

The sample of the study was 122 fifth grade students (age 10) in a private elementary school in Ankara. Sixty-six of these students were female and 56 were male. These students were all the 5^{th} grade students in the school. The study was carried out at the beginning of January, 2000.

The instrument of the study was developed according to the objectives of the Turkish National Elementary Mathematics Education Curriculum. It was an essay type test including 10 word problems about the part-whole concept in fractions, where part is usually given as a quantity. The test consisted of two types of problems, which are routine or non-routine. Routine type of problems were the ones that students were familiar with (problems 4, 5, 6, 7, 8, 9, and 10). In these problems, students were required to find: (i) the quantity of a whole when the quantity of a part was given, and (ii) the quantity of a fraction when the whole was given as a quantity, or (iii) both. In non-routine type of problems, students were asked to complete or find the missing information to solve the problem (problems 1, 2, and 3). The test was prepared in Turkish and administered to the students upon the completion of the fraction unit. The time allowed for the test was 50 minutes.

In scoring each problem, 1 point was assigned for the correct answer and 0 for the incorrect answer. The Kuder-Richardson reliability coefficient of the test was calculated as 0.72 by using SPSS 7.0 statistical program.

3. RESULTS AND THEIR INTERPRETATION

Table 1 below indicates the problems together with the percentages of correct and incorrect answers and, most common correct answers and classified errors. While classifying the errors, computational errors not related to the fractions and partially correct answers were not taken into consideration. For example, in problem 2, the correct problem statement without a solution was not taken as an error. While scoring the problem, however, it was taken as an incorrect answer. Among 100 students who wrote a correct problem statement, only 38 solved it.

Considering the percentages in Table 1, students' success in solving word-problems including more than one operation is rather low. If the success rates of the students on problems with one operation (problems 6 and 7) were compared, it was seen that the type of the fraction to be multiplicated affects students' answers. Over 96 students who wrote the correct operation with either improper or mixed fraction, only 41 were able to compute it correctly in problem 7.

The results also showed that students used different ways for solving the problems. Nature of the correct and incorrect answers gives us the idea about how they perceive fractions, particularly the part-whole concept, and how they attempt to solve problems by using their conceptions on fractions. In solving the problems, students tried various operations with the given fractions and quantities to reach the solution. The incorrect solutions mainly resulted from not understanding the problems together with the misconceptions in part-whole construct and in operations with fractions.

The reasons of wide range of error types can, however, be summarized in terms of seven categories:

I. Confusing the parts of a whole with the quantities (problems 1, 2, 9, and 10).

II. Performing operations between different kinds of units (problems 3 and 7).

III. Taking the amount of a part as the amount of a whole (problems 6, and 8).

IV. Interpreting fractions in conceptually incorrect ways (problem 5).

V. Taking the parts of more than one whole as a set of whole (problem 4).

VI. Using incorrect procedures for computations between fractions (problems 5, 7, 8, 9, and 10).

VII. Choosing incorrect operations (problem 10).

Attempts of addition or subtraction between the given quantity and the given fraction show that students do not understand the relationship between a part and a quantity. Some errors in problems 1, 6, 9, and 10 are rooted in a lack of understanding of the fact that fractions refer to the parts of a whole, not the quantity of a part. Although most of the students can find the part required, they do not think that the quantity of the part is required to do an operation with the given quantity. Thus, they subtract the fraction that shows the part from the quantity. But, quantity is the numerical measurement or count, where part is the proportion of some quantity to another. This shows that they do not consider quantity and part as different things.

Operations between wrong quantities point out that students are not aware of the resultant unit of an operation. When students were required to write a problem for a given operation in problem 3, they wrote a problem without considering the units of the known and the unknown in the problem. The reason seems to stem from the lack of the connection between the units of the known and the unknown. Similarly, in problem 7, although the length of the road was asked, students tried to solve the problem by converting the time from hour to minute and then did some computations with the given length of the road.

Considering the amount of a part given in the problem as the amount of a whole is very com-

Table 1: The Percentages of Correct and Incorrect Answers, and Frequencies of Most Common Ones.

Problems	**correct(%)/ incorrect(%)**	**Most common correct answers (frequency)**	**Classified errors (frequency)**
1. If we know that a train went 3/7 of the road, can we find the length of the rest of the road? Write the reason if we cannot.	66% / 21%	"No, we cannot. There is no length given to us." **(80)**	"Yes, we can. It is 4/7." **(26)**
2. "Deniz read 3/5 of her 240-page storybook." Write a problem by using the information above and solve the problem that you have written.	40% / 54%	"How many pages are there left? (240 · 3/5 = 144, 240 - 144 = 96)" **(26)** "How many pages did she read? (240 · 3/5 = 144)" **(12)**	"How many pages are there left? 240 - 3/5 = … or 5/5 – 3/5 = 2/5" **(5 + 5)** "How many books can she read in 3 days?" **(3)**
3. Write a problem that requires the following operation 44 **.** 1/4.	68% / 13%	"1/4 of 44 apples are given, how many apples are given?" **(31)** "If 1/4 of 44 books are given, then how many books are left?" **(15)** "What is the result of 44 · 1/4?" **(14)**	"1/4 of a number is 44. What is this number?" **(7)** "If Ali reads his book 1/4 hours a day, how many books does he read in 44 days?" **(2)**
4. A group of students bought 3 pieces of cardboard. Their teacher divides each cardboard into 8 equal parts. Each student gets 3/8 pieces of cardboard. How many students are there in this group?	36% / 50%	By using a rectangular region. **(29)** "A cardboard is divided into 8 parts and there are 3 cardboards. Each student gets 3/8 parts, so the answer is 8." **(8)**	"3 · 8 = 24, 24 · 3/8 = 9" **(17)** "3 · 8 = 24" **(8)**
5. Kerem bought a book by the half of his money and an eraser by 2/7 of his money. Write the rest of his money as a fraction.	35% / 41%	"2/7 +1/2 = 11/14, 14/14 - 11/14 = 3/14" **(42)**	"7 / 2 = 3, 3/7 + 2/7 = 5/7, 7/7 – 5/7 = 2/7" **(4)** "7/7 – 3.5/7 = 3.5/7, 3.5/7 – 2/7 = 1.5/7" **(1)**
6. If 1/6 of a herd is 15 sheep, how many sheep are there in the herd?	84% / 13%	"15 · 6 = 90" **(106)**	"15 · 1/6 = 5/2" **(5)**
7. If a car's speed is 90 km. per hour, find how far can it go in 3 1/5 hours.	38% / 51%	"90 · 16/5 = 1440/5 = 288"**(30)** "90 · 3 1/5 = 270 90/5 = 288" **(11)**	"60 / 5 =12, 90 · 3 = 270, 270 + 12 = 282" **(5)**
8. If 3/4 of a bag of walnuts is 69, what is 4/23 of it?	31% / 54%	"69 / 3 = 23, 23 · 4 = 92, 92 · 4/23 = 16" **(39)**	"69 · 4/ 23 = 12" **(7)** "69 : 23/4 = 12" **(6)**
9. The sum of 4/9 and 2/5 of a number is 380. What is this number?	26% / 52%	"4/9 + 2/5 = 38/45, 380 / 38 = 10, 10 · 45 = 450" **(32)**	"380 - 38/45 =" **(6)**
10. Pınar gives 2/9 of her 90-color pencils to her sister and 1/5 of them to her friend. Later, her mother gives 10 more color pencils to her. How many color pencils does Pınar have?	47% / 50%	"(90 · 2/9) + (90 · 1/5) = 38, 90 - 38 + 10 = 62" **(32)** "2/9 + 1/5 = 19/45, 90 · 19/45 = 38, 90 - 38 + 10 = 62" **(28)**	"90 · 2/9 = 20, 90 - 20 = 70, 70 · 1/5 = 14, [90 - (20 + 14)] + 10 = 66" **(9)** "90 - 19/45 =" **(8)** " 90 - 2/9 =" **(6)**

(*) The percentages of the omitted answers are not included in the table.

mon when students cannot understand the problem clearly. Students related quantity of a part either to the wrong part or to the whole given and then performed operations according to the relation they have built. In problem 8, 69 was given as the quantity of 3/4 but taken as the quantity of the whole or the quantity of 4/23. Similarly, 15 which was the quantity of 1/6 was considered as the whole in problem 6.

Incorrect conception of the definition of fractions is an evidence of students' misconceptions of basic fraction knowledge. In problem 5, 1/2 was misinterpreted in various ways such as 3.5/7 or 3/7. This was because students sought to convert 1/2 to a fraction with denominator 7 to be able to add 1/2 with 2/7. Even the answer given, 3.5/7, was unique, it showed an important misconception that the student was not aware of the fact that the numerator and the denominator of a fraction must be a natural number, not a decimal. Students who gave the answer as 3/7 were aware of the fact that the numerator and the denominator must be a natural number. They, however, did not think that 3/7 is not equal to 1/2.

The difficulty in doing operations with the fractions having different denominators was also seen in the correct answers given. Therefore, they showed the whole as 14/14 and found the half of the money as 7/14 and continued operations by using this fraction.

Attempts of taking the parts of more than one whole as a set of whole show that students have difficulty in dealing with more than one whole. Low success rate in problem 4 is a result of students' tendency to take the parts of more than one whole as a set of whole. Although there are three pieces of cardboards, each divided into eight equal parts, students found the total number of pieces as 24, and considered the set of 24 pieces as the whole. It seems that students have difficulty in dealing with wholes more than one.

Computational errors in students' solutions result in incorrect answers, although the idea for solution was correct. Students had difficulties in multiplication of an integer with a mixed fraction in problem 7, and expanding fractions in problems 9 and 10. Here one serious misconception, which also appeared in other problems involving multiplication, was equalizing the denominators of fractions. This is because the multiplication concept is not well developed or, methods of addition are generalized to multiplication.

Choosing incorrect operations is an evidence of not understanding the problem clearly. Students had difficulties in choosing the appropriate operations such as addition, subtraction or multiplication. Main difficulty appeared in choosing subtraction instead of addition.

4. CONCLUSION

The errors identified in students' answers seem to mainly originate from not understanding the meanings of a part and a quantity. To avoid this undesirable consequence it seems reasonable to propose a chance in teaching so as to deepen the difference between a part and a quantity contemplated in a situation related to a discrete set of object. To overcome this misconception, students should be confronted with questions related to a quantity and a part.

In the curriculum, more emphasize is given to the parts of a rectangular region or objects rather than the parts of a discrete set of objects. The examples given at the beginning of an instruction like "piece of a cake" are not suitable to explain the difference between a part and a quantity. As a result, it can be said that, the findings in this study tend to confirm the previous studies' suggestions about more emphasize on discrete representation of part-whole model.

In understanding fractions the role of constructing and interpreting units (the kind of the quantity such as day, book…etc.) is also crucial. Taking this issue into account should be another basic ground for designing activities oriented toward a meaningful learning.

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