Hacettepe Üniversitesi Eğitim Fakültesi Dergisi 18 : 85 - 92 [2000]

STUDENTS' ACHIEVEMENT AND COMMON MISTAKES IN SOLVING WORD PROBLEMS RELATED TO NUMBERS*

Yasar ERSOY** Halil ARDAHAN***

ÖZET: Bu makale 1996-97 öğretim yılında Türkiye'nin Konya ilinden rastgele seçilmiş on beş yaşındaki 191 öğrenci ile İngiltere'nin Leeds bölgesinden seçilmiş 109 öğrencinin yönlü sayılarla özellikle negatif sayılarla ilgili bir grup işlemsel (manipulative) ve sözel problemlerdeki başarıları ve ortak hataları üzerine bir karşılaştırmalı çalışma sunulmaktadır. Türk ve İngiliz öğrencilerinin gruplandırılmış verilerinin ortalaması sırasıy a 55.40 ve 66.91, standart sapmaları 21.26 ve 20.37 bulunmuştur. Aynı grupların ortancaları sırasıyla 52 ve 67 ve ortalama standart hataları 1.54 ve 1.95 dir. Böylece, Leeds deki 15 yaşındakiler, farklı içerikli sözel problemler üzerinde daha başarılı ve ortalama puanları göreceli olarak daha yüksektir. Son olarak, her iki grubun matematik öğretimcileri ve öğretmenleri hakkında bir fikir vermesi için öğrencilerin ortak yanılgıları ve hataları listelendi.

ANAHTAR SÖZCÜKLER: *Matematik Eğitimi, Sözel Problemler, Ortak Hatalar, Yanılgılar*

ABSTRACT: The present paper presents a comparative study on the achievement and common mistakes of fifteen-year- old 191 students chosen randomly from the province of Konya, Turkey and 109 students from Leeds, the UK in 1996-97 school year about solving a set of operational (manipulative) problems and of word problems related to the integer numbers, in particular negative numbers. The means of grouped data of students from Turkey and the UK are 55.40 and 66.91 respectively with their standard deviations 21.26 and 20.37. The medians of the same groups are 52 and 67, and standard error of means are 1.54 and 1.95 respectively. Thus fifteen-year-olds in Leeds achieved relatively high mean scores on the word problems with different contexts. Finally, students' common mistakes and errors were listed to give an idea to both mathematics educators and teachers.

KEY WORDS: *Mathematics Education, Word Problems, Common Mistakes, Misconceptions*

1. INTRODUCTION

People in the education community are excited about the development of students' mathematical power, student reasoning, common errors in various mathematical topics, comparisons of students' performance, achievement etc. Therefore, today's needs demand multiple methods of assessment, integrally connected to instruction that diagnose, inform, and empower teachers help their students. In this connection, some researches (eg. Hart, 1984; Glendon et al, 1990; Corte and Verschafll, 1991) focus on the analysis of students' errors in the performance tests, and try to construct models that fit the conceptual operations leading to certain misconceptions. The present paper aims to a comparative study on the achievement and common mistakes of fifteen-year-old (10th grade) 191 students chosen randomly from the province of Konya, Turkey and 109 students from Leeds, the UK in 1996-97 school year. This is a relatively new comparative study on the students' achievement related to integer numbers, which covers one- and multi-step operation (manipulative) problems and few word problems with different contexts. Such studies show us not only how much mathematical misunderstanding occurs as a result of teaching, but also how limited that understanding is even if it is "correct". More specifically, we have two fold objectives in our researches. The first one is to analyse what kind of mental representation (correct or incorrect) is constructed by the students on the 8th and 10th grade of a comprehensive school in the UK and of various junior and senior high schools in Turkey (Ardahan and Ersoy, 1998).The second one is to analyze the relationships between mental representation, strategy and correct answer within each word problem to design a set of instructional materials which will be used the in-service education and training of teachers and/or in classrooms. The

* A Summary of this paper was accepted to presenting the First Conference on European Research on Mathematics Education (CERME-1 *Aug 27-31, 1998; Osnabrück, Germany*

Prof. Dr., Dept. Science/Maths Education, METU, 06531 Ankara, Turkey * Dept. of Maths Education, Selcuk Uni. 42099 Konya, Turkey

descriptive statistical techniques were used for the analysis of data to see the general profile of students' achievement and performance, and to compare the scores of both group of students.

2. BACKGROUND INFORMATION

Pupils and students' misconceptions, common error and mistake in various topics in school mathematics is one of the current research area. With regard to the acquisition of new knowledge it is assumed that the learner's attention is focused by his or her existing schemes, and these schemes are also regarded as means of achieving understanding.

2.1. Conceptual Understanding and Common Mistakes

Over the last ten years, conceptual understanding and identification of misconception have been used in science and mathematics education, both as a teaching strategy and as a research tool to investigate students' performance and cognitive achievement with respect to many topics. In mathematics education community, many researches study students' common errors and mistakes in various mathe-matical topics based on different theoretical and methodological approaches. Some re-searches mainly focus on the analysis of pupils' errors only and construct models that fit the conceptual operations leading to certain misconceptions (Hart, 1984). Some other researches (for example, Onslow, 1990) also examine the role of instruction in overcoming the misconceptions. Although many questions wait to be answered some general recommendations for practice can be abstracted from researches. Glendon, Clements and Del Campo (1990), for example, stress that a child's mental representation is a function of many variables, including syntax and semantics of the problem, and the extent of the child's preference for analytic as distinct from visual thinking, or vice versa (Lean & Clements, 1981). Clement (1982) studied on algebra word problem solutions and pointed out that teaching a student a standard method is no guarantee that the student's own intuitive method will not "take over" in a later problem-solving situation. Corte and Verschafell (1991), on the other hand, have studied on the problem structure and some factors influencing on children's solution strategies. Teubal and Nesher (1991) studied on the order of mention versus order of events as determining factors in additive word problems. The extent to which students can succeed in overcoming their errors, depends on the teacher's efforts. He/she should devise challenging problem situations at an appropriate level of difficulty, and to create a classroom environment conductive to students' conflict discussions, which will allow him/her to delve deeply into the student underlying misconceptions leading to errors.

2.2. Development of Instrument

The Number Test (NuT), the instrument for this research, was designed by the authors, and applied in Turkey for the development of instrument a year ago (Ardahan and Ersoy, 1997).

The test involves thirteen problems on integer numbers, in particular positive and negative numbers. These problems generally classified into two groups: (a) one or multi-steps arithmetic operation (manipulative) problems, and (b) real-word problems chosen from the subjects of National Curriculum such as money, pop lists, temperature, geographic directions and mathematics modelling.

To give an idea a sample of questions is given in Table1a and Table 1b.

Table-1a. Examples of One- and Multi-Step Operations Problems in the NuT

QNr	Questions	
2	What is (-9)- (-2)?	*Explain how you do this.*
4	What is 5-12 + 8- 2?	*Explain how you do this.*
5	What is 2 x 3 + 2 x (-3) - (-2) (-3)?	*Explain how you do this.*

Table-1b. Examples of Word Problems in the NuT

QNr	Questions
6	Ahmet will get 10 TL (Turkish lira) from Mehmet. Also, Mehmet owes 5 TL to Ahmet. How much will Mehmet pay to Ahmet? *Explain how you do this.*
7	The pop song " My love" has gone up 6 places from 9th position. What position is it now? *Explain how you do this.*
9	The afternoon temperature was 8° Celsius. In the evening it fell by 6° Celsius. What was the temperature in the evening? *Explain how you do this.*
10	From 6.00am to 11.00am the temperature changed from -6° Celsius to -2° Celsius. By how much did the temperature change and was it a rise or a fall ? *Explain how you do this.*
13	A driver set off from Ankara (Leeds) and went 10 km toward the north. But then he realized that he should have gone in the opposite direction. So, he turned back and went 150 km to the south and arrived at his destination. How far was the driver from Ankara (Leeds)? *Explain how you do this.*

During the development phase of the instrument, the statement of each problem was controlled by a group of teachers, and the consistency of its content with the National Curriculum was checked before administrating test in the class (Ardahan and Ersoy, 1997). The original version of the NuT was in Turkish, and it was then translated to English and adopted by changing the name of children, unit of currency and of town or city when it was applied in the UK. The allocated time for the NuT was 30-mintes; and the calculators were not allowed in the administration of the test.

3. METHOD AND PROCEDURE

The procedure and the results of the analysis of collected data, ie. the rate of success of students from both countries were explained below.

3.1. Procedure of Administration and Scoring of Number Test

The devised NuT was applied to fifteen-year-old pupils, 191 of whom chosen randomly from the province Konya in Turkey and 109 of whom from Leeds in the UK in the second semester of 1996-97 academic year. Data were collected and used for statistical analysis to compare scores of both groups of students. The frequency distribution of scores and the percentage of success of students were calculated; and the lists of common mistakes concerned with each group of students were tabulated in the following section. Such comparisons, of course, give an idea to understand the current situations in other countries,and help us find out the common aspects of the same phenomena. However, we keep in mind that each European country has its own characteristic school culture and mathematics teaching/learning. More clearly, each item of the test was marked by a set of criteria, ie. Problem structure, one or multi-steps operations and model approach. Furthermore, each item of the NuT was then marked by considering certain criteria, eg. the complexity of problem structure (ie. One or multi-step operation problem or word problem with different contexts) to evaluate of students' performance and achievement. In scoring the student's achievement, each of the item was given a value of 3, 4, 5,...,14 or 0 over 100, according to whether the solution was correct with the appropriate and full explanation, correct but inadequate explanation, and incorrect or unattempted, respectively.

3.2. Analysis of Data and Results

The gathered data were analysed, in particular those linked to the concepts of integer numbers, in particular negative number, operations and real-word problems. The descriptive statistical techniques were used for the analysis of data to see the general profile of students' achievement in solving operational (manipulative) and word problems with integer numbers. Thus the mean, median, standard deviation, and errors of means etc. were calculated to compare and eva-

luate the achievement of two groups of students from Turkey and the UK.[1]

The means of grouped data of students from Konya, Turkey and Leeds, the UK are 55.40 and 66.91 respectively with their standard deviations 21.26 and 20.37. The medians of the same groups of students are 52 and 67, and standard error of means are 1.54 and 1.95 respectively. It is found that the lower and upper quartiles of scores of the same students from Konya are 37 and 67 while those of students from Leeds are 56 and 78 respectively. Thus fifteen-year-olds in Leeds achieved relatively high mean scores on the word problems in the given NuT. However, the achievement of a group of students from Anatolian High Schools in Konya is comparable with those of students in Leeds. Similarly, the thirteen-year-olds in Leeds achieved relatively high mean scores on the word problems in the same NT (Ardahan and Ersoy, 1998).[2]

4. COMMON MISTAKES OF STUDENTS AND THEIR REASONING

The common mistakes of the fifteen-year-old Turkish and English students, and how they did the operations and solved word problems on integer numbers were displayed in Table 2a and Table 2b.

Table-2a. Common Errors and Mistakes of a Group of Students in One-and Multi-stage Operation Problems

Q	Students from Konya, Turkey	Students from Leeds, the UK
2	• "It is subtracted smaller number than bigger one and given the sign of bigger one" • "Minus subtracted by minus is minus. So, -9-2= -11" • "If we subtract two negative numbers it gives negative result". • "I took away -2 from 9 and found +7" • "The difference of two negative numbers is similar to addition. So, we find -11".	• "A minus plus a minus is a plus so add inside the brackets". • "If you subtract two minus number you get a positive one. So, -9 turns 9 and -2 to 2". • "A minus - a minus = a minus. So, -9 take away another -2 makes -11". • "Ans. -7. By taking -2 away from -9". • "There is two minuses so it makes a plus. So, it works out at 9-2=7".
4	• "I did the subtractions first and then added the results and got -13" • "First subtraction gives -7 and second one gives 6. So, we subtract smaller number than bigger one to get -1 giving the sign of bigger one" • "In the set of iNTegers we write (-) + (-) = (-) and get -1" • "We add the positive numbers and negative numbers separately and than subtract the smaller number than absolutely bigger one and give the sign of bigger one"	• "A negative and a positive = a negative and a + and a - is a - . So, 5-12+8-2= -17" • "5 - (12 + 8) -2 = 2-2 = 18 and 5-18 = -13". • "Ans. 13. I added 12 and 8 then took away 5 and 2".
5	• "We calculate according to the sequence of operations" • "I omit the brackets first and then I get the answer by the order of operations" • "These rules belong to mathematicians. The result is 42" • "By heart"	• "2x3 = 6, 6 + 2 = 8, 8 x (-3)= -24, -24 + 2 = -22, -22 x -3 = 66". • "2 x 3 = 6, -3 x 2 = -5 + -6 = -11 x 2 = - 22 + 6 = -16". • "Ans. -5. I did 2 x 3 then 2x (-3) and add together then took away (-2) and (-3) ". • "There are many differeNT results. Such as -46, -40,-32,-8,-6,-3, 0, 1, 5, 6, 10, 19, 22, 24, 66, 88"

1 In order the reader can understand and situate the results, the Turkey's education system and teaching of mathematics is described in Appendix A very briefly.

2 We continue to work on the detail of the analysis and interpretations of the results and find out the common mistakes in order to get more insights on the misconception related to integer numbers and solving simple word problems with different contexts.

Table-2b. Common Errors and Mistakes of a Group of Students in Solving Word Problems with Different Context

Qu	Students from Konya, Turkey	Students from Leeds, the UK
6	• "We take the debt away from the due of Ahmet and get 5 TL" • "Subtracting 5 TL of Mehmet's debt from Ahmet's due we get 5 TL" • "The debt means subtraction and the due means addition" • "Take 5 TL away from 10 TL to get the answer"	• "Tom will pay £5 to Peter. Because Peter gave Tom £10 and all Tom had to do is half the £10 and give £5 back". • "I said Tom owes £5 pounds so that how much he would give him". • "I had 10 and took away 5 for Peter".
9	• "Since the temperature falls - 3°, we subtract it from 310 and get -28 °" • "There is 28° difference between 31° and -3°" • "When the temperature falls, we subtract" • "We subtract as much as it falls from 310 "	• "Ans. -9. Take 12°C then count back to 0 then count back to -3". • "Ans. -9. I minused 12 from -3 and when misusing a negative from a positive the answer is negative" • "Ans.+9. By taking 3 away from 13" • "Ans. 16. I added 3° to 12° and 1° far zero"
10	• "When the temperature falls 6° from 8°, it means 6° " • "Falling means subtraction" • "I added 6° by 8° and got 14°"	• "I added together 8 and 6" • "6° -8° = -2°C"
13	• "The driver has gone 150 km west" • "The driver confused the directions, it is a pity" • "He is 140 km far away from the initial point" • "He has gone 170 km as a total distance."	• "10 - 150 = -140 so it is 140 miles south" • "I added 10 + 150 =160". " 160 miles. I guessed by adding" • "150 - 20 = 130 miles."

We can see the similar reasoning and mistakes in solving the other manipulative problems such as Q1, Q3, and the word problems such as Q7, Q8, Q11,Q12. Therefore we will not list the similar mistakes.

4.3. Discussions on Students' Achievements and Common Mistakes

This paper presents a comparative study on the achievements and common mistakes and errors of fifteen-year- old students. It is difficult to judge from this research how the 10th grade students (15 year-old) might make a set of various mistakes in the given test. Partial answers to this question can be provided because the number of questions in the test was not enough to classify the misconceptions and bring out general trends. However, fifteen-year- old students from Leeds, the UK achieved relatively high mean scores on the word problems in the given test on integer number. We think that students' reasoning and computations displayed in Tables 2a and 2b are self explanatory to a certain extent, but the underlying misconceptions are hidden and should be lightened. We have therefore planed to interview a group of students from both countries and few teachers so that we would be able to reveal the background and underlying misconceptions leading to mistakes and errors. However, such list of common mistakes and errors can be used to identify weakness in students' understanding of various topics in mathematics, and help teachers design new teaching/learning materials.

There are several issues concerning the teaching and learning of school mathematics that required immediate attention. Firstly, it appeared from the data that the students did not have sound understanding of negative number concepts. Hence, the majority of the students did not perform well on both the operation (manipulative) problems as well as word problems.

With this information the teacher is more equipped to plan for remedial actions. However, this method provides the teacher with only a general picture of his or her students' ability. The teacher would need to probe further in order to determine specifically the difficulties faced by students and the means of rectifying the problems. On the other hand, sometimes it can happen that the students find the solution without any understanding of the word problem or without using a strategy in a perfect way. There is an obvious pedagogical implication of this conclusion, ie., teachers have to be aware that a correct answer does not necessarily imply a real comprehension of the problem. In the same cases, the contrary can also happen, ie. some students can give us an incorrect answer and to have construct a correct mental representation and choose a correct strategy; the fail can be found on the calculation.

It is obvious that the correct solution of the word problem is more dependent on the way the strategy is used than on the construction of the mental representation. From a pedagogical point of view, it means that the teachers have to pay attentions to the construction of the mental representation of a word problem as they normally pay to the construction of the strategies. Indeed, to solve a word problem the student has to choose the important data, to organise a correct mental representation and, further, he/she needs to find a strategy and to employ it without fails. We think that the use of correct teaching/learning material is very useful to help the students, particularly in the primary and the junior high school, to be aware of the importance of this construction.

5. CONCLUDING REMARKS

The main objective of the present research was to identify student's strategies in solving word problems with integer number and find out the common mistakes. These are the results from our first analyses of the data collected from various schools in a province of Turkey but from one school in the UK. Although many questions wait to be answered some general recommendations for practice can be abstracted from the present research. The results of this research have provided clues about the successes and failures of a group of pupils from Turkey and Leeds, the UK in the NuT. In this test, for example 15 year-olds students from Leeds, the UK achieved relatively high mean scores on the word problems with the NuT.

In concluding this study, five thoughts come to mind:

- To be more effective in mathematics education, National Curricula must be reorganised and sequenced differently.
- Pupils need more mathematics skills to represent such word problems with mathematics expressions. This procedure of modelling a problem situation with mathematical sentence is very important skills through all mathematics.
- The teaching of negative numbers in conventional way does not help the solution of real-world problems they faced. This subject should be thought students by using the real-life situation and/or concrete objects such as thermometer, maps, league fixtures, pop lists and money.
- Some teaching/learning materials such as booklets and work-sheets should be designed and developed to use in the classroom.
- Interactive and student centred teaching methods should be preferred instead of exposition method in mathematics education.

More detailed investigations should indeed yield further insights into the relevant factors related to achievement in mathematics, and findings help us enhance our understanding of teaching and learning in mathematics. In this respect, our study brings new research problems how to overcome the obstacles and common mistakes. It is important to notice here that whi-

le learning mathematics new rules are constantly being formed due to the fact that new ways are found to extend conceptual networks previously developed. Furthermore, the question that also has to be answered is where a student's prior knowledge comes from. Was it established in former teaching, in everyday life experience, or in some other way? The meaning of knowledge is not the results of a genesis but derives directly from reality: the mere confrontation suffices to provide adequate meaning

REFERENCES

1. Ardahan, H. and Ersoy, Y (1997)."Deficiencies in solving problems with directed numbers in secondary schools". *(Unpublished Report)*. The summary of the research was presented in the European Research Conference on Mathematics Education (**ERCME'97**), Aug 31- Sep 4, 1997; Podébrady
2. Ardahan, H. and Ersoy, Y. (1998). "A comparative study on pupils' achievement and common mistakes in solving word problems related to directed numbers". Paper was presented in the International Conference on the Teaching of Mathematics (**ICTM'98**) July 3-6, 1998; Samos, Greece
3. Clement, J.(1982). "Algebra word problem solutions". *Journal for Research in Maths Education* **13** (1), 16-30.
4. Corte de E. and Verschafell, L.(1987). " The effect of semantic structure on first graders' strategies for solving addition and subtraction problems", *Journal for Research in Maths Education* **18** (5), 363-381.
5. Corte de E. and Verschafell, L.(1991). "Some factors influencing the solution of addition and subtraction word problems", Language in Mathematical Education, Philadelphia: Open Univ. Press,118-130.
6. Glendon, A. L., Clements, M. A. and Del Campo, G. D. (1990). "Linguistics and pedagogical factors affecting children's understanding of arithmetic word problems: A comparative study". *Educational Studies in Mathematics* **21**, 165-191.
7. Hart, K.M. (1984). Ratio: Children's Strategies and Errors. A report of the strategies and errors in secondary school mathematics project. London: NFRE-Nelson.
8. Lean, G. A. & Clements, M. A. (1981). "Spatial ability, visual imaginary, and mathematical performance". *Educational Studies in Mathematics* **12**, 267-299.
9. Onslow, B. (1990). "Overcoming conceptual obstacles: The qualified use of a game". *School Science and Mathematics* **90** (7), 581-592.
10. Teubal, E. and Nesher, P. (1991). "Order of motion vs order of events as determining factors in addition and subtraction word problems", Language in Mathematical Education. Philadelphia: Open University Press.

APPENDIX A: **TURKEY'S EDUCATION SYSTEM**

In order to help the international audience understand the background of this study, it would be necessary to introduce briefly the Turkey's education system (TES). Some main features of Turkey and of the TES are summarised as follows.

Turkey in General: Turkey is located at the south-western extremity of Asia and at the South East of Europe.

Certainly some students succeed in school mathematics, ie. they learn how to do the mathematical techniques, get the right answers, use the right methods, and pass the examination. Most of the successful ones never questions their mathematical knowledge or their mathematics education. The situation is rather different for the majority of young people who do not succeed. They sometimes blame the teachers for never understanding them and the mathematics curriculum for all its irrelevant and mind numbing exercises.

To meet the challenges that confront children and young people now and in the future they need to develop the capability: (a) to learn how to learn, (b) to think logically, (c) to develop and use knowledge, (d) to analyse and solve problems, (e) to make relational decisions, and (f) to communicate with others in speech and writing. Thus both primary and secondary school education aim at preparing children to beco-

me useful and responsible citizens and to cope with the rapid changes of society.

A land of constant and exciting contrasts, Turkey is both a very old and a very new country. When the 600 years-old Ottoman Empire came to end, it was replaced by a new and dynamic republic in 1923. Under the leadership of M. Kemal Atatürk , the Turkish Republic adopted a secular system, in which religion and politics are kept strictly separate. Turkey has 63 million inhabitants, 36 of whom live in the countryside.

Turkey's economy is progressively moving away from dependence on labour-intensive manufacturing towards dependence on finance, commerce and other service industries. There are areas of knowledge and understanding and particular skills, which are plainly necessary for the individual in coping with life and work in this society. Developments in science and technology today mean that children need some knowledge and understanding of mathematics, and almost certainly the ability to cope with these new developments.

Education System: The formal educational opportunities in Turkey encompass kinder-gardens, primary schools, secondary schools (including technical and vocational schools), higher education institutions, ie. universities, higher vocational institutions. All children are required by law to be in full-time education between the ages of six and fourteen. Pre-school education begins for a few children in a kindergarten, at the age of four. Primary school, which begins at the age of six and lasts fourteen years, consists of two sequential stages. At about eleven, children progress to a three-year course in either the same school or other one. After the eight year two-staged schools most continue on their education in a three /four-year secondary school leading to the university entrance examination.

Mathematics Education and Issues: Educating people mathematically consists of much more than just teaching them some mathematics. It is much more difficult to do, and the problems and issues are much more challenging. It requires a fundamental awareness of values, which underlie mathematics, and the recognition of the complexity of educating children and young people about those values. It is not enough merely to teach them mathematics, we need also to educate them about mathematics, to educate them through mathematics, and to educate them with mathematics.

Note: This research was financially supported witer the grant, ODTÜ. AFP. 99.06.02.35.