



Research Article or Review Article

## An Investigation of Gross Error Angle in a Connecting Traverse with a $\pm 200^{gon}$ Difference

Hüseyin İnce<sup>1a</sup>, Nuri Erdem<sup>2</sup>, Fazlı Engin Tombuş<sup>1b</sup>

<sup>1a,b</sup>High School of Mapping and Land Survey, Hitit University, Çorum, Turkey,

<sup>2</sup>Engineering Faculty, Department of Geomatic Engineering, Osmaniye Korkut Ata University, Osmaniye, Turkey

### Abstract

The Global Navigation Satellite System (GNSS) cannot always be used efficiently in city surveys because of the effect of obstructions such as high-rise building and high trees in the city discussed in. In this situation the measurements of angles and lengths have to be carried out with the conventional (classical) traversing methods. Since geodetic surveys can be tedious and time consuming, it is inevitable that systematic errors, unsystematic errors and gross errors occur despite the care that is taken. An angle measured in a traverse with an error margin can be used in the calculation of traverse.

If there is an error in the measurement of a traverse angle of  $\pm 200^{gon}$  and this is then used in the manual traverse calculation, then the identification of this gross error is not possible in the calculation. In this case, the gross error is noticeable when the  $\alpha_n$  control bearing is checked. However, the Brönnimann formula can be used in manual calculations to determine the point where gross error occurred. After a wide literature survey it was found that there is no such study with concerning special case of gross error is seen in the literature. In this study a simple formula which can replace the Brönnimann formula is presented for the case where the error is  $\pm 200^{gon}$ . A new formula is created to calculate easier and simpler than classical formula for this special case. A flow chart of the program for scientific calculators and computers are given together with numerical examples.

Received  
21 July 2019

Accepted  
09 December 2019

### Keywords

Connecting traverse,  
blunder angle (gross error angle),  
single calculation method 1

## 1. Introduction

The Global Navigation Satellite System (GNSS) cannot always be used efficiently in city surveys because of the effect of obstructions such as urban canyon<sup>1</sup> and high trees discussed in [1, 2]. In this situation the measurements of angles and lengths have to be carried out with the conventional (classical) traversing methods. Since geodetic surveys can be tedious and time consuming, it is inevitable that systematic errors, unsystematic errors and gross errors occur despite the care that is taken. In this study, a method is proposed for the special case of gross error that can occur in angle surveys. This is when the error is to  $\pm 200^{gon}$  of the normal value of a traverse angle. After a wide literature survey it was found that there is no such study with concerning special case of gross error is seen in the literature [3-7].

A single calculation method has been developed by many researchers [8-13] to investigate the gross error angle and it is possible to use the formula developed by [14].

If there is an error in the measurement of a traverse angle of  $\pm 200^{gon}$  and this is then used in the manual traverse calculation, then the identification of this gross error is not possible in the calculation. In this case, the gross error is noticeable when the  $\alpha_n$  control bearing is checked (Fig. 1). However, the Brönnimann formula can be used in manual calculations to determine the point where gross error occurred. In this study a simple formula which can replace the Brönnimann formula (Brönnimann, Study On The Gross Error Angles Of Traverse Calculations (in German: Auffindung eines groben Winkelfehlers in einem Polygonzug), submitted to the *Zfv.*, 1888) is presented for the case where the error is  $\pm 200^{gon}$ . A new formula is created to calculate easier and simpler than classical formula for this special case.

---

<sup>1</sup> An urban canyon is community of very high buildings in a city

Currently the calculations of traverse coordinates, the search for gross angular error and gross error length are carried out by computer using CAD programs. If software used in traverse calculation has been coded without taking into consideration this special case give in this study, then the program application will fail this happens in Turkey with software such as Netcad [15]. This study will present the programming that is required to prevent the failure of certain programs when facing a gross error of  $\pm 200^{gon}$ . The flow chart of the program has been developed, which will even run on scientific calculators and computers.

This paper outlines the background to the study, then the evaluation of the gross error and gross error angle with the flow chart of the program are presented. Finally, sample numerical examples are given.

## 2. Investigation of Gross Error and Gross Angle Error Angle on a Traverse

### 2.1. Investigation of the Gross Angle in Checking of the Control Bearing

Where,  $\beta_0, \beta_1, \dots, \beta_n$  are the measured angles and  $\alpha_0, \alpha_1, \dots, \alpha_n$  are the bearings. The angular misclosure error ( $f_\beta$ ) in Figure1 is calculated as shown in equation (1).

$$f_\beta = \alpha_n - \alpha'_n \quad (1)$$

$\alpha'_n$  is calculated from equation (2).

$$\alpha'_n = \alpha_0 + [\beta] - k * 200^{gon} \quad (2)$$

The  $k$  value is determined to be a positive integer to calculate  $\alpha_n$  approximately from  $\alpha_0 + [\beta]$  in equation (2) and it is not a constant. The tolerable limit ( $F_\beta$ ) of the angular misclosure determined by equation (1) is calculated according to the Regulations of Map Making used in Turkey. If  $f_\beta < F_\beta$ ,  $d_\beta$  is computed the correction factor is as follows:

$$d_{\beta} = \frac{f_{\beta}}{n} \quad (3)$$

Thus,  $f_{\beta}$  is distributed over the angles using  $d_{\beta}$  as shown in equation (4).

$$\begin{aligned} (B1) &= \alpha_0 + \beta_0 + d_{\beta} \pm 200^{gon} \\ (12) &= (B1) + \beta_1 + d_{\beta} \pm 200^{gon} \\ (3C) &= (23) + \beta_3 + d_{\beta} \pm 200^{gon} \end{aligned} \quad (4)$$

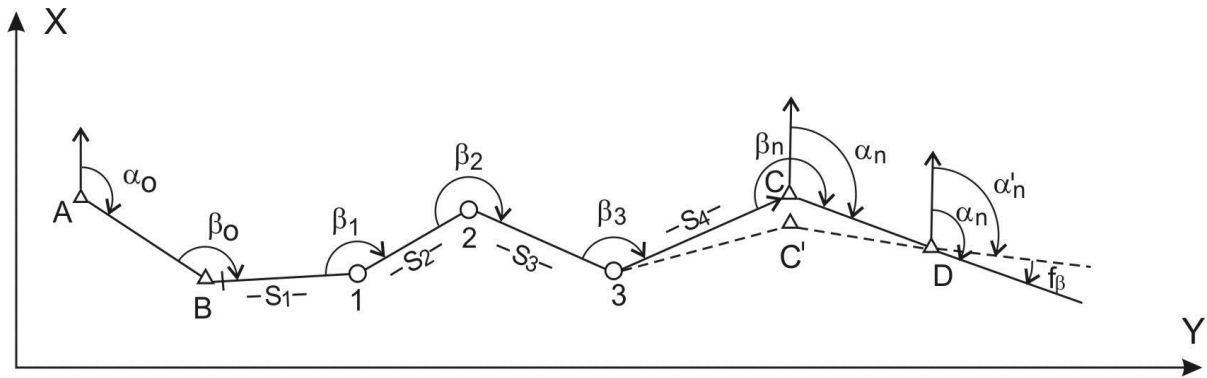
Then, as shown in Figure 1 using the formula given in equation (4), all the bearings (azimuths) are calculated for all points starting from where the traverse is initially connected. The bearing from  $C$  to  $D$  is calculated with the first basic geodetic rule using the arctangent function (equation (5)).

$$(CD) = \arctan \left( \frac{Y_D - Y_C}{X_D - X_C} \right) \quad (5)$$

$(CD)$  is also controlled by the relationship in equation (6).

$$(CD) = (3C) + \beta_n + d_{\beta} \pm 200^{gon} \quad (6)$$

If one of the angles in this traverse route is mistakenly taken as  $\pm 200^{gon}$ ,  $(CD)$  calculated using equation (6) will be  $\pm 200^{gon}$  different than when calculated using equation (5). An angular error of  $\pm 200^{gon}$  to an angle cannot be detected in normal traverse calculations.



**Figure 1.** Locations of a connecting traverse route without a gross angular error ( $\alpha_0$ : starting bearing,  $\alpha_n$ : control bearing,  $\beta$ : traverse angles at the points).

## 2.2. Investigation of Gross Error Angle Using Single Calculation Method

The investigation of the gross angular error will be explained in two stages, theoretically and practically.

### 2.2.1. Theoretical Investigation of The Point Where a Gross Error Angle Measurement is made

Taking the situation shown in Figure 2 where the error is applied to the angle at station 1. Thus,  $\beta'_1 = \beta_1 - 200^{gon}$  this error will cause points 2 and C to be located at 2' and C' respectively. Drawing a line from C to C', two similar triangles are formed, namely C'2'1 and C21. M is the midpoint on the line and its position coincides with the position of point 1. From these similar triangles, the following relationships exist:

$$C'2' = 2'C, 12' = 12, C'2' \parallel 2C \text{ and } \beta_2 = \beta'_2 \text{ then } C'1 = 1C$$

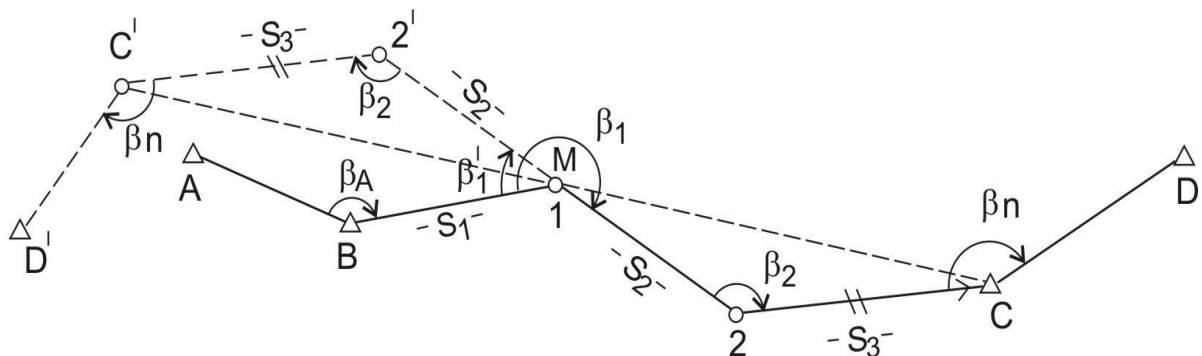


Figure 2- The effect of the error on the location of the following points when the angle on the connecting traverse route is taken as  $\beta_1' = \beta_1 - 200^{gon}$  at point 1.

Taking the situation shown in Figure 3 where the error is applied to the angle at station 3. Thus,  $\beta_3' = \beta_3 + 200^{gon}$  this error will cause points 4 and C to be located at 4' and C' respectively. Drawing a line from C to C', two similar triangles are formed, namely 34'C' and 34C triangles. M is the midpoint on the line and its position coincides with the position of point 3. From these similar triangles, the following relationships exist:

$$C'4' = 4C, 34' = 34, C'4' \parallel 4C \text{ and } \varepsilon = \varepsilon' = 400 - \beta_4 \text{ then } C'3 = 3C$$

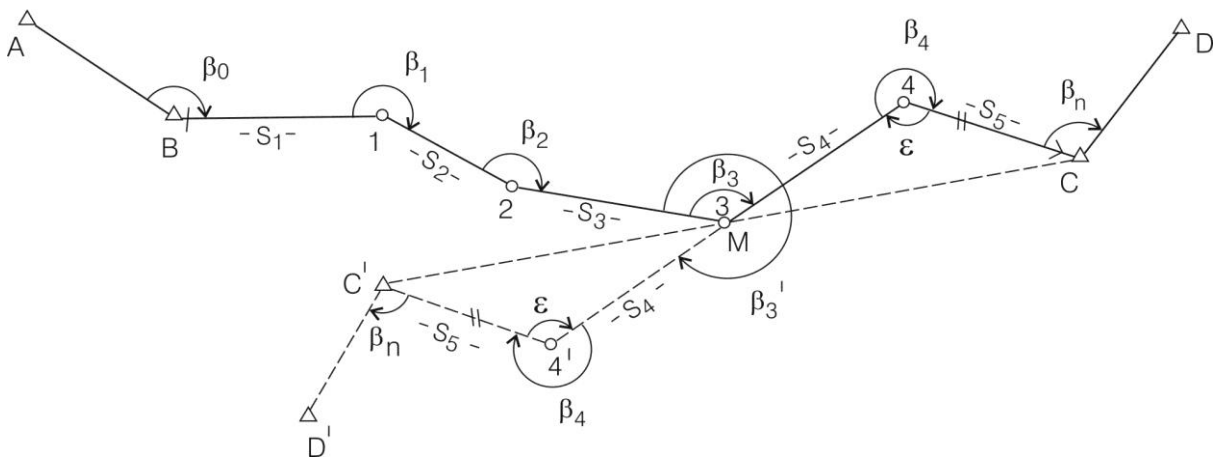


Figure 3- The effect of the error on the location of the following points when the angle on the connecting traverse route is taken as  $\beta_3' = \beta_3 + 200^{gon}$  at point 3.

The M coordinate of midpoint of CC' line is obtained from following equations.

$$Y_M = \frac{Y_{C'} + Y_C}{2}, \quad X_M = \frac{X_{C'} + X_C}{2} \quad (7)$$

To find the angle that is given in any value, the single calculation method developed by various researchers [5, 8-11, 14] is expressed as follows:

$$Y_P = Y_M - 0.5*(X_{C'} - X_C)/\tan(f_\beta/2) , \quad X_P = X_M + 0.5*(Y_{C'} - Y_C)/\tan(f_\beta/2) \quad (8)$$

### 2.2.2. Practical Investigation of The Point Where a Gross Error Angle Measurement is made (Manual Calculation)

According to Figure 3, when the direction of calculation is taken from  $B$  to  $C$ , and after the  $(CD)$  value calculated for control purposes using equation (6) is determined to be  $\pm 200^{gon}$  different from that calculated using equation (5), corrections made on angles using equation (3) are deleted. In the traverse calculation table, all bearings between the first and last connection points are calculated from following relationship.

$$(B1)' = \alpha_0 + \beta_0 \pm 200^{gon}$$

$$(12)' = (B1)' + \beta_1 \pm 200^{gon}$$

.

(9)

.

$$(3C)' = (23)' + \beta_3 \pm 200^{gon}$$

$$(CD)' = (3C)' + \beta_n \pm 200^{gon}$$

Using the calculated bearing and length values, the  $\Delta Y$  and  $\Delta X$  values are calculated, as follows:

$$\begin{aligned} \Delta Y_1 &= S_1(B_1)', & \Delta X_1 &= S_1 \cos(B_1)' \\ \Delta Y_2 &= S_2(12)', & \Delta X_2 &= S_2 \cos(12)' \\ &\vdots & &\vdots \\ \Delta Y_4 &= S_4(3C)', & \Delta X_4 &= S_4 \cos(3C)' \end{aligned} \quad (10)$$

Then the traverses and point of the  $C'$  coordinates are calculated as follows:

$$\begin{aligned}
 Y_1 &= Y_B + \Delta Y_1, & X_1 &= X_B + \Delta X_1 \\
 Y_2 &= Y_1 + \Delta Y_2, & X_2 &= X_1 + \Delta X_2 \\
 &\vdots & &\vdots \\
 Y_{C'} &= Y_3 + \Delta Y_4, & X_{C'} &= X_3 + \Delta X_4
 \end{aligned}
 \tag{11}$$

In the evaluation of the point at which the gross error angle is measured using the developed single calculation method (Figure 2 and Figure 3), equation (7) is used. The point coordinate value in the traverse calculation table, which is, nearly equal the values obtained from equation (7) is considered to be the point where the coarse error angle was measured.

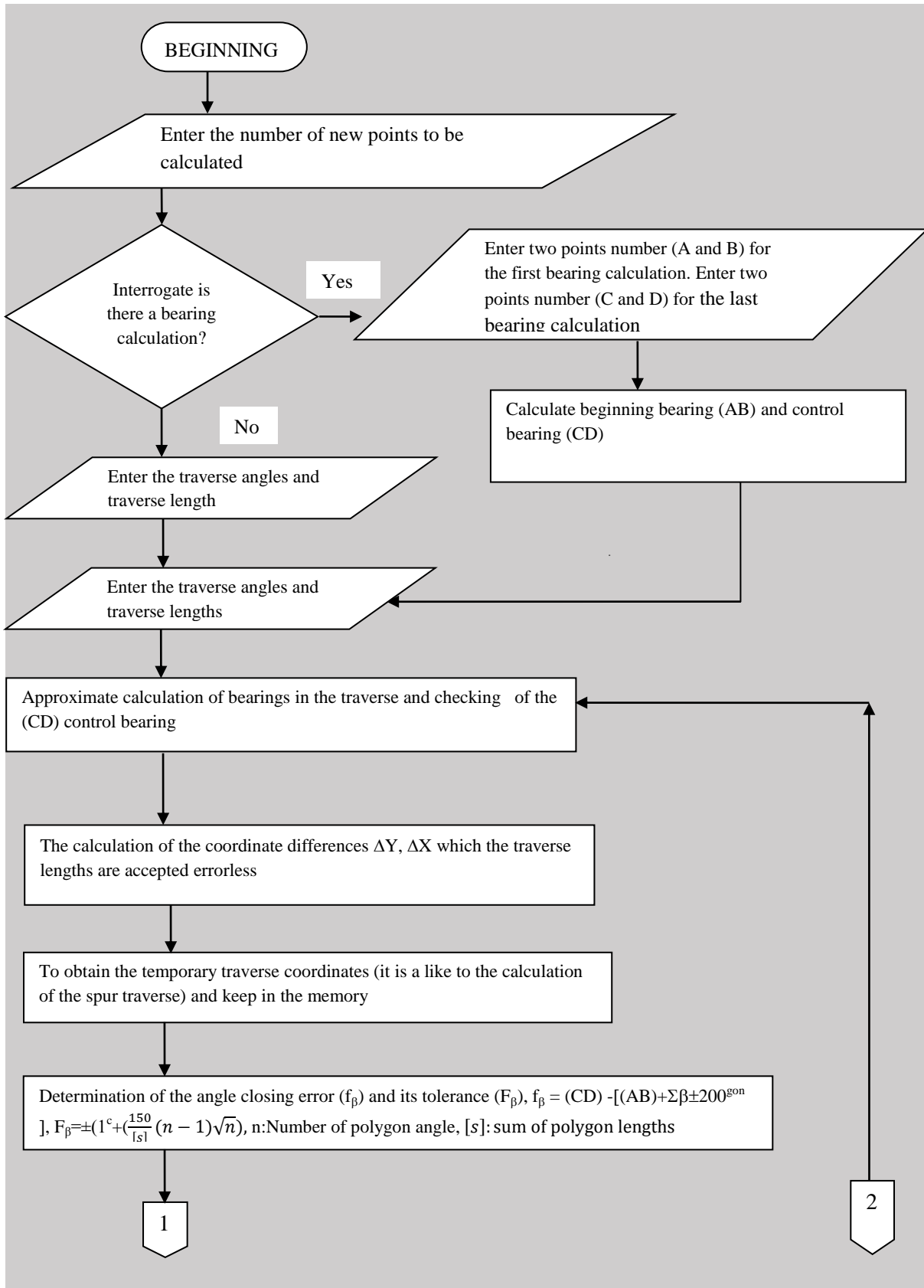
### 3. The Flow Chart of the Program

Recently the calculation of the traverse has been performed with CAD software. However, this software appeared on the market, the calculation of traverse and the researching of gross error angle and gross error length was achieved by programs written by researchers [16, 17] in Turkey in Basic or FORTRAN.

The investigation of gross error angle was carried out in two ways in this study. The first way was to use the classical formula (8) to calculate the coordinate of the point with a measured gross error angle of any value (8). The second way was to find the coordinate of the point with measured angle with a  $\pm 200^{gon}$  after the incorrect formula (7) was used. In addition another study [18-20] concerning the investigation of the gross error length was taken into the consideration.

The flow chart for the program was originally created for scientific calculators and it is given below (Fig. 4).





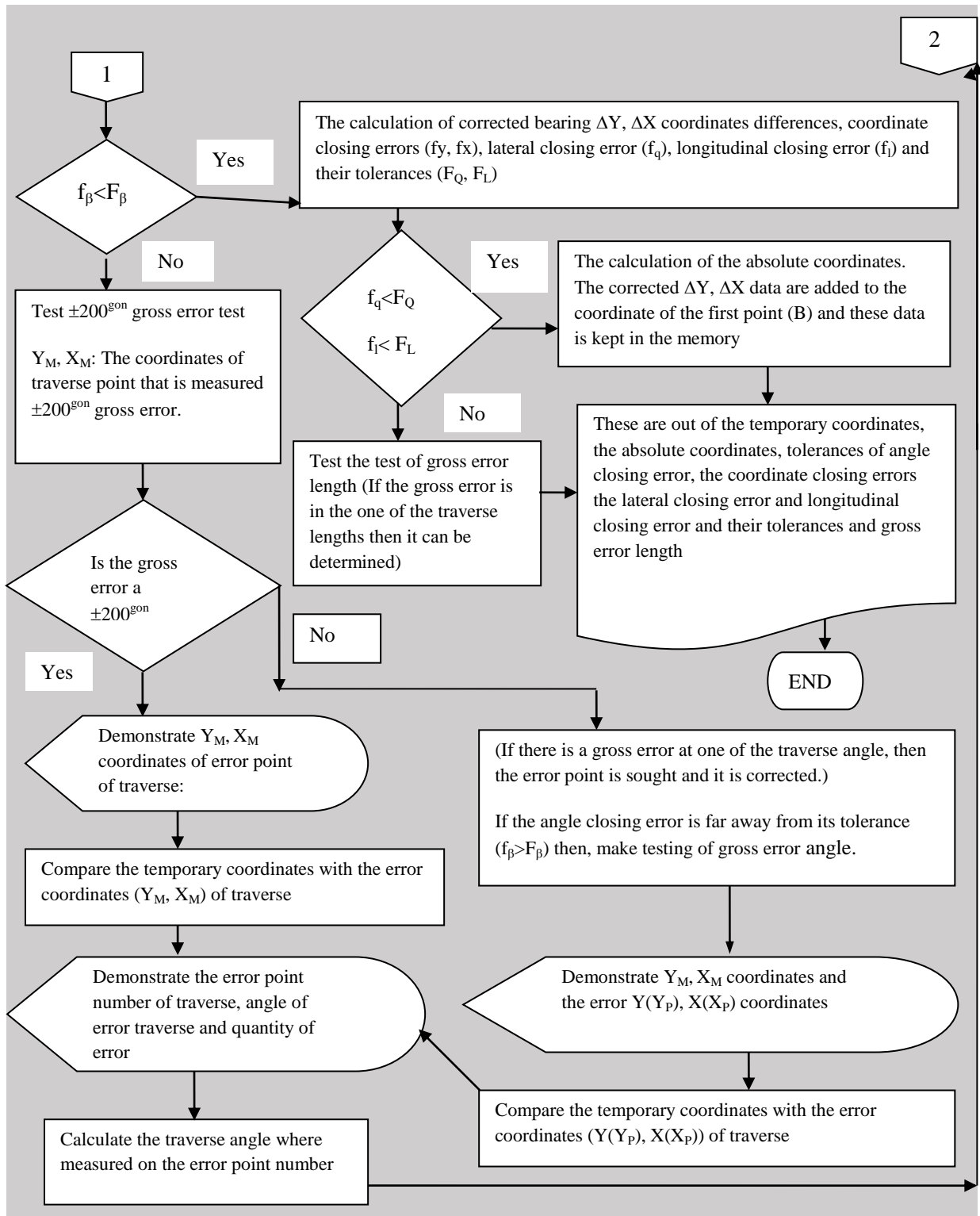


Figure 4- Flow chart of the program

#### 4. An Application

For a numerical application, the traverse route without a gross error in which the lateral and longitudinal closing errors are smaller than their tolerances was taken into the consideration. The data of traverse route is given in Table 1. Assuming that the angle at point 2 in Figure 5 is measured as  $115.2756^{gon}$  by mistake;

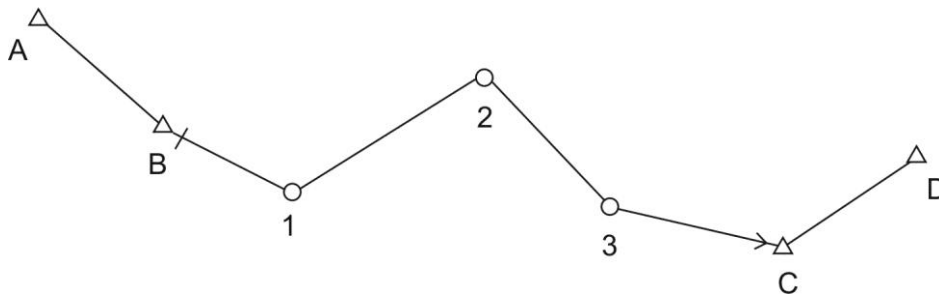


Figure 5- A traverse route placed without gross error

Table 1. The data of the traverse net without gross error angle

Point Number	Y	X	Traverse Angles (gon)	Lengths (meter)	Bearings (gon)
A	2701.38	4945.10	$\beta_0=175.4253$	B1=150.27	$\alpha_0=(AB)=185.9613$
B	2750.62	4725.43	$\beta_1=101.3347$	12=180.56	$\alpha_n=(CD)=84.1350$
C	3140.10	4382.63	$\beta_2=315.2756$	23= 195.42	
D	3383.10	4444.47	$\beta_3=185.4877$	3C=160.90	
			$\beta_n=120.6385$		

**Manual Calculation;**

**Table 2. Calculation of the coordinates of the traverse without a gross error angle**

Point Num.	Traverse Angles ( <sup>gon</sup> )	Bearing ( <sup>gon</sup> )	Length (m)	Y (m) ΔY (m)	X (m) ΔX (m)
A		$\alpha_0=185.9613$			
B	$175.4253^{+23cc}$			2750.62	14725.43
1	$101.3347^{+24}$	161.3889	150.27	$85.65^{-3cm}$	$-123.47^{-2cm}$
				2836.24	14601.94
2	$315.2756^{+24}$	62.7260	180.56	$150.49^{-3}$	$99.78^{-2}$
				2986.70	14701.70
3	$185.4877^{+24}$	178.0040	195.42	$66.18^{-4}$	$-183.87^{-2}$
				3052.84	14517.81
C	$120.6385^{+24}$	163.4941	160.90	$87.29^{-3}$	$-135.16^{-2}$
				3140.10	14382.63
D	$\alpha_{0+\Sigma\beta}= 1084.1231$ $-1000.0000$ $\alpha_n'=84.1231^{gon}$	$\alpha_n= 84.1350$ $-\alpha_n'=-84.1231$ $f_\beta=0.0119^{gon}$ $F_\beta= \pm 0.0295^{gon}$		$\Delta Y=389.48$ $-[\Delta Y]=-389.61$ $f_y = -0.13 m$ $f_q=0.15m$ $F_Q= \pm 0.17 m$	$\Delta X=-342.80$ $-[\Delta X]=-(-342.72)$ $f_x = -0.08 m$ $f_l=0.04m$ $F_L= \pm 0.23 m$

**Table 3. Calculation of the coordinates of the traverse with a gross error angle of  $\pm 200^{gon}$  value**

Point Num.	Angle ( <sup>gon</sup> )	Bearing ( <sup>gon</sup> )	Length (m)	Y (m) ΔY (m)	X (m) ΔX (m)
A		$\alpha_0=185.9613$			
B	175.4253			2750.62	14725.43
1	101.3347	161.3866	150.27	85.66	-123.47
				2836.28	14601.96
		62.7213	180.56	150.48	99.79

2	115.2756			<b>2986.76</b>	<b>14701.75</b>
3	185.4877	377.9969	195.42	-66.20	183.86
				2920.56	14885.61
C	120.6385	363.4866	160.90	-87.31	135.15
				3140.10	14382.63
D	$\alpha_{0+\Sigma\beta}=884.1231$ 800.0000	$\alpha_n= 84.1350$ $\alpha_n' = (284.1231)$ $f_\beta=200.0119^{\text{gon}}$		$Y_C'=(2833.25)$	$X_C'=(15020.76)$

The coordinate of the midpoint,  $M$ , were computed with equation (7) as follows.

$$Y_M = (2833.25 + 3140.10) / 2 = 2986.67 \text{ m.}$$

$$X_M = (15020.76 + 143.63) / 2 = 14701.70 \text{ m.}$$

These  $Y_M$  and  $X_M$  values are similar to the coordinate values for point 2, indicating that the gross angular error occurred at point 2.

## 5. Results and Discussion

a) The investigation of gross error angle can be done in two ways:

- In the first way, the classical formula (8) can be used to calculate the coordinate of an angle with any value.
- As a second way, the formula (7) was developed to calculate the coordinate of a point with an angle with an error of  $\pm 200^{\text{gon}}$  from the normal value.

b) The formulas in equation (7) are easier to use and simpler than classical formula given in equation (8) for this special case.

c) If software used to calculate the traverse has been coded without taking into consideration this special case, then the program application fails. This means that a special program must be created to cope with this error situation.

d) A study of gross error angle and gross error length on the traverse have been investigated used the software developed in this study. This software can even be used with scientific calculators such as Casio calculators.

e) To use the formula (7) in manual calculations, the following points should be noted:

- The use of  $\pm 200^{gon}$  different value of an angle on calculation of the traverse coordinate appears when checking the control bearing.
- The corrections on the traverse angles have to be cancelled to find the gross error point and the coordinates of the points must be calculated using temporary bearings.
- The gross error point has to be determined by comparing the calculated value obtained by formula (7) with the coordinates calculated on the traverse calculation table.

## REFERENCES

- [1] Bullock, B., (2004). GPS Antenna Placement Analysis for Receivers, [www.commsdesign.com/main/9802fe.3htm](http://www.commsdesign.com/main/9802fe.3htm)
- [2] Tsakiri M. and Stewart M., (1998). A Future Vehicle Positioning System For Data Capture In The Urban Canyon, *Cartography- The Journal*, June, Vol.27, No.1.
- [3] Allan, A. L., Hollwey, J. R. and Maynes, J. H. B., (1968). *Practical Field Surveying and Computations*, Heinemann, London.
- [4] Bannister, A., Raymond, S. and Backer, R., (1992). *Surveying*, John Willey & Sons. Inc. 605 third Avenue, New York.
- [5] Kavanagh, B.F. and Bird, S. J. G., (2000). *Surveying Principles and Applications*, Upper Saddle River, New Jersey 07458.
- [6] Kissam. P., (1966). *Surveying Practice the Fundamentals Of Surveying*, McGraw- Hill, New York.
- [7] Uren, J. and Price, W.F., (1986). *Surveying For Engineers*, Second Edition, ELBS Macmillan and London.
- [8] Pinkwart,, (1941). Study On The Gross Error Angles Of The Conecting Traverse Calculations (in German: Die Auffindung eines groben Winkelfehlers im Polygonzug ohne Abschlurichtung.) *Zfv*. p. 227.
- [9] Rennebach,, (1941). The Determination of Gross Error Angles And Error Points In The Traverse Calculation (in German: Ermittlung eines groben Winkelfehlers und des Fehlerpunktes im Streckenzuge (Polygonzuge) ohne Abschlurichtung). *Zfv*, p.81.
- [10] Hamacher, W., (1963). Study on the Error Point of the Gross Error Angels of the Traverse Calculation (in German: Das Auffinden des Fehlerpunktes bei einem groben Winkelfehler im Polygonzug), *Zfv*, Nr, 3/1963, p.130-132.
- [11] Kavanagh, B. F., (2003). *Geomatics*, Pearson Education Inc., New Jersey.
- [12] Wolf, P. R., and Ghilani, C. D., (2008). *Elementary Surveying an Introduction to Geomatics*. 12th Edition, Upper Saddle River, New Jersey: Pearson Prentice-Hall.
- [13] Kavanagh, B. F., (2009). *Surveying principles and application*. 8th ed. Columbus: Pearson Education Inc.,

- [14] Brönnimann,, (1888). Study On The Gross Error Angles Of Traverse Calculations (in German : Auffindung eines groben Winkelfehlers in einem Polygonzug) Zfv. p.524.
- [15] URL\_1:www.nedcad.com.tr
- [16] Banger, G., (1995). Basics of Computer Programming, Basic As Programming, Language, General Programming Structures, Sample Programs Volume 2 (in Turkish), K.T.Ü. Faculty of Engineering & Architecture, Faculty Press No: 52, Trabzon, Turkey.
- [17] Pektekin, A., (1978). Calculating The Coordinates Of A Traverse By Computers And Searching of Gross Error Angle And Length (in Turkish), Y.T.Ü. Text Nots / Monographies, İstanbul, Turkey.
- [18] İnce, H., (1999). The Proposals In The Researching By Calculation Method Of The Length Which Has Gross Error On The Dependent Traverse And Series Triangulation (in Turkish) K.T.Ü. Faculty of Engineering & Architecture, Faculty Press No: 24, Trabzon, Turkey.
- [19] Mao, Y., et al. Model of Gross Error Elimination for Monitoring Data Collected by A Surveying Robot, Journal of Engineering Science and Technology Review 7 (4) (2014) 13-17.
- [20] Mao Y. et al. A new method data caper method for rejecting the deformation monitoring data with the gross errors, Journal of Northeastern University, 2011(7), pp.1020-1023.