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LIBERATING SCHOOL MATHEMATICS FROM PROCEDURAL VIEW

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ÖZET:

Matematiğe işlemsel olarak yaklaşan görüş matematiği doğrudan doğruya aktarılabilen hazır bir bilgi kabul eder. Buna karşın, matematiğe kavramsal açıdan bakan görüş, matematiksel bilginin doğrudan öğretmen tarafından öğrenciye aktarılabileceğine karşı çıkarak gerçek matematiksel anlamanın bizzat öğrencinin kendi aktivitelerinden meydana gelebileceğini savunur. Çalışmanın birinci bölümünde matematiğe kavramsal açıdan bakmanın ne anlama geldiği açıklanıyor. İkinci bölümde ise mevcut matematik öğretmenliği eğitimi ele alınarak kavramsal görüşün Türk eğitim sisteminde uygulanabilirliği incelenmektedir. Son bölümde matematik öğretmenliği programları için yapılabilecek öneriler tartışıldı.

ANAHTAR SÖZCÜKLER: Matematik eğitimi, hizmetöncesi öğretmen eğitimi.

ABSTRACT:

Procedural view of mathematics assumes that the teacher is the possessor of ready made mathematical knowledge is to be conveyed to the student. On the other hand, "conceptual view of mathematics" challenges the assumption that teachers transmit knowledge to students and argues that teacher's conceptualizations cannot be given directly to the learner and conceptual understanding in mathematics must be constructed by the learner. The first section of the paper starts by explaining what is meant by the term "conceptual view of mathematics". The second section deals with the current pattern of teacher education and problems in training secondary school mathematics teachers, and examines the possibilities of liberating the Turkish mathematics education from the procedural view of mathematics. In the last section, some possible implications for mathematics teacher education are discussed.

KEY WORDS : Mathematics education, preservice teacher education.

1. CONCEPTUAL VIEW OF MATHEMATICS

In order to understand what is going on in school mathematics and how student teachers learn and change, we have to identify the distinction between conceptual and procedural view proposed in various ways [1, 2, 3]. The distinction is problematic because it is not always clear enough to be applied precisely to all situations in mathematics. However, the widespread perception of mathematics as a collection of disparate rules and procedures has been well documented in empirical research on teachers' beliefs [4, 5].

Carter and Yackel describe learners with procedural view as a rule and procedure-driven or rule memoriser [3]. For these learners, mathematics is a series of rules, to learn mathematics means one must learn the rules, usually through memorization. One must also learn the situations to which the rules apply. This perspective reflects a procedural view of mathematics where there is an authority, such as the teacher or textbook author who know the rules and procedures and transfers them to the pupil.

In contrast, Cartel and Yackel describe learners with conceptual view of mathematics as problem solvers who can use their creativity in solving problems and generating mathematical knowledge. From this description it can be said that the conceptual view of mathematics values learning in terms of understanding and not merely in terms of producing the teacher's mathematics and algorithms. It tends to view mathematics as a group of related concepts and ideas, and suggests the need for the provision of opportunities for pupils to construct these concepts and ideas for themselves.

DiSessa deals with two main views of learning and teaching which effect students' learning experiences in the school system. One prevalent view is a procedural one, that the learning of mathematics and science is simply acquiring new knowledge specifically located in the rules, principles and equations of textbooks. These rules, principles and equations are understood essentially on the surface level of knowing the principles by the name and statement, and the equations by letters [1]. DiSessa calls the students who hold this view of learning "result men". Learning, for these students, consists in the matching of problems to equations in order to produce an answer. They must substitute the facts or data given in the problem statement into the proper variables in the appropriate equation. They then perform the indicated algebraic and arithmetic operations to produce a correct answer which is true by virtue of all

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the presented facts. The answer is often verified in the back of the book. These students rely heavily on authority as a source of mathematical knowledge and tend to solve mathematical problems by employing rules and procedures which have been explicitly taught by the teacher.

The conceptual view, according to DiSessa, describes a rare, but certainly a more powerful way of understanding mathematics and science. Students who hold this view of learning realize that in order to "get the point", their intuitions must be substantially reorganized; and that they can come to see a mathematical task as a group of related concepts and ideas. They search for the structure of the problem as small pieces that fit together rather than look at the type of the problem in order to identify superficial clues and produce algorithms.

These two contrasting views play a crucial role in the classroom. They determine not only what students believe they are learning, but also how they must proceed to learn it. Therefore, a student's epistemology shapes the attitude towards, and conception of, both the content and process of learning, and it determines whether the student is a rule memoriser or a conceptualiser [1]. In fact, student mathematics teachers do not come to the faculty of education as empty vessels; they have conceptions and views about mathematics and its teaching which have been constructed throughout their school years. As this is the mathematics they will teach, what they have learnt about the subject matter in lycée and faculty classes turns out to be a significant component of their preparation for teaching in lycée. They have learned that mathematics is a fixed body of rules and procedures, an uninteresting subject best taught through memorization and repetition.

In Turkey the main characteristic of the traditional way of teaching mathematics is consistent with the procedural view of mathematics set out here. The teachers' priority is to follow the textbooks, to spend the majority of their time lecturing to students using the blackboard, to stress algorithms, rules, definitions, axioms, and formulas to be memorised, and to provide sample problems to be used as examples for the solution of nearly identical problems.

This traditional way frequently leads to students replicating mathematical routines without developing conceptual understanding. Of course, this is consistent with the main goal of their mathematics learning, which is to answer correctly a sufficient number of questions in order to get a high score in the exam -- particularly in the university entrance exam. However, as a consequence of focusing on the development of procedural view, students come to view mathematics as a list of unrelated rules and procedures that must be memorised.

Large number of students therefore, graduate from lycée and enter university programmes with serious deficiencies in their conceptual understanding. It is evident that a significant number of students who were successful at mathematics in lycée failed to study more advanced mathematics at university [6]. Therefore, the traditional method of learning mathematics becomes inadequate when students enrolled for undergraduate mathematics courses which required them to think mathematically rather than merely memorise formulas and manipulate mathematical symbols. As a result, the continued decline in the number and percentage of mathematics students who persist to the PhD degree becomes one of the most urgent problems facing Turkish mathematics today. This reflects the current failures of the traditional approach to the learning and teaching of mathematics in Turkey.

Today, procedural view of mathematics and the transmission model of learning and teaching are dominant at all levels in the education system. This dominant pattern undoubtedly reinforces prospective teachers' conceptions about learning and teaching: primarily that teaching is telling and learning is reproducing what the teacher says. The teacher training programmes do not provide prospective teachers with an opportunity to experience alternative methods and approaches to the learning and teaching of mathematics. Thus, when they become teachers, they are not able to make informed decisions about what and how their students learn, and they are not able to take responsibility for shaping the content and process underlying a task to meet the needs of their students.

If prospective mathematics teachers' conceptions of mathematics and its teaching formed in the traditional system are to be changed, there needs to be a break in the cycle of "as we were taught, so do we teach". In other words, to develop more realistic and healthy beliefs about mathematics and learning, we need to change what occurs in the mathematics classroom. This can only happen through an interaction between what prospective teachers see in an alternative educational environment and what they bring to this new environment where they are confronted with relevant and meaningful events which require them to reflect on their past experiences, and to modify their previous conceptions of mathematics and its teaching.

2. PROBLEMS IN TRAINING MATHEMATICS TEACHERS

The outcome of mathematics education at secondary levels is far from satisfactory because schools

are not able to develop positive attitudes towards mathematics, and students are not provided with opportunities to develop scientific thinking skills and problem solving skills [7].

> American students' performance will not improve much if the quality of teaching is not improved. And teaching will not improve much without dramatic improvements in the field of teacher education [8]

If this statement is true for the US, it is even more applicable to Turkey. In any society, any improvement in education needs a similar improvement in teacher education. It is evident in the recent literature that because of the failure to account for the central role of teachers in the educational development process, many innovation have failed. As a result, the teacher's crucial role in educational change is becoming increasingly accepted [4]. This indicates that significant changes in school mathematics will only be achieved if there are marked changes in teacher's conceptions(1) about mathematics and its teaching.

On the other hand, the Turkish teacher education programme, as in other levels of the mathematics curriculum, is very traditional and not readily open to any innovative approaches. Although teacher educators in the Higher Educational Council in the Ministry of Education sometimes speak of preservice teacher education as the first stage in learning to teach conceptually, nothing could be further from the truth.

Throughout my investigation of secondary preservice programmes, I noticed that prospective mathematics teachers not only need to understand the practical and theoretical characteristics of the various teaching methods, but also they need instructional models [6]. This has not been fully appreciated by the educators who are in office at the Faculties of Education. However, most of them admit that secondary mathematics teacher preparation programmes are not adequately preparing preservice teachers, but they are also quick to add that many of the reasons for the inadequate programmes are completely out of their hands. There are, in fact, other institutional forces that maintain the status quo in teacher education and these forces may be overwhelming.

Even subject application courses are not under the control of teacher educators but are taught by the subject matter professors who may or may not have any background in teaching methods. These courses are based on a traditional pattern: listening to lectures, with no independent activity on the part of the attendants. Thus, student mathematics teachers through subject application courses are not even aware of viable instructional alternatives. They only see one type of teacher as a model and one type of class as an instructional model, and they tend to rely on teacher-centered task organizations and believe that they will teach best when direct instruction occurs. Thus, student-centered instructional approaches remain unknown and seem risky and untenable in prospective mathematics teachers' eyes.

In essence, the Faculties of Education with this current pattern train and prepare mathematicians not mathematics teachers. It is evident in the initial training programme that the mathematics component is strong but not relevant for secondary school mathematics, and the pedagogical content knowledge component is virtually non-existing. Of course it is true that mathematics teachers should know sufficient mathematics beyond the level that they are supposed to teach. But the mathematics they need is at the same time different in content as well as significance from that needed by the mathematicians.

On the other hand, mathematics teachers need special training in developing a sound understanding of certain mathematical content which is particularly important for teaching at secondary schools. The current teacher education programme does not provide prospective mathematics teachers with an opportunity to study and learn some mathematical topics which they will teach in secondary schools. For example, basic concepts in Euclidean geometry, trigonometry, geometric progressions, systems of linear equations, and some concepts in modern mathematics are not taught in the current teacher training programme, while they are critically important for secondary mathematics teachers. In this regard, there is a need to rethink and reevaluate the content of the subject courses run in the existing mathematics education departments in the country in order to decide what mathematical content is necessary and appropriate for the prospective mathematics teacher. The unfortunate part of this situation is that so little effort is really being made to help Turkish prospective secondary school mathematics teachers.

It is evident in the literature that the years spent in mathematics classes, watching teachers and being students contributes to the continuity of a traditional cycle of "as we were taught-so we teach" in mathematics education [9]. Similarly, in Turkish education faculties, the time spent in mathematics classrooms as students gives prospective mathematics teachers a specialized apprenticeship of observation. By watch-

(1) We use the term "conceptions" to include beliefs, attitudes and views.

ing teachers through school days and paying attention to their own experiences and teaching tactics and strategies, prospective mathematics teachers develop ideas about the teacher's role, form beliefs about what works in mathematics learning and teaching, and acquire a repertoire of strategies for teaching specific content. This leads to a vicious circle which is reproduced continuously: rule-memoriser university teacher, rule-memoriser secondary school teacher, rule-memoriser prospective teacher, rule-memoriser student and so on.

Consequently, prospective mathematics teachers' experiences within the current teacher training programmes influence and shape their conceptions and views about what they do with their students in the classroom. The vicious circle is not easily broken through current inservice activities because inservice teachers are never involved in any different experience during their professional lives. So, they carry their procedural view of mathematics throughout their professional career and even though they may encounter different teaching activities they always approach them with great skepticism. For instance, Baki reported that teachers commonly share the idea that when they state a series of procedures, theorems, and proofs correctly and clearly, and solve problems with plenty of symbols, they must necessarily be understood [10]. Furthermore, if these teachers were successful in mathematics at secondary school and in their teacher education programme, they are likely to approve of the patterns they saw in the past and perhaps they may not be interested in alternative ways of teaching. Even if they are critical of their own past teachers for teaching uncreatively, they may lack alternative models.

3. SUMMARY AND CONCLUSIONS

This study illustrates that the existing pattern in teacher education has proved its inadequacy for providing alternative approaches to mathematics teaching. Trainee teachers are never involved in any different experience during their training, and they carry their procedural view throughout their professional lives. This contributes to the reproduction of rule-memoriser teachers and students in mathematics education. The other obstacle for conceptual teaching and learning is the University Entrance Examination which measures only procedural view, and does not provide enough scope for students to learn mathematics through exploration.

Learning to teach is a developmental process which should include opportunities for the development of knowledge structures that support teachers' professional development. Therefore, the feasibility of "breaking with tradition" seems dependent upon an achievement in changing what occurs in the classroom. From the point of view of the present work, this proposed change in mathematics education started from the vantage points of teacher training. The present work addresses the shortcomings of the current pattern followed pattern followed in the preparation of mathematics teachers, and offers an alternative approach to the learning and teaching of mathematics. This approach aims to provide a foundation which addresses these educational shortcomings. It prioritizes pattern followed in the preparation of mathematics teachers, and offers an alternative approach to the learning and teaching of mathematics.

The necessity to involve teachers in the type of learning experiences and environments that they are expected to implement in their teaching, has been advocated by various researchers [11, 12]. They propose that teachers should have practical experiences with approaches, materials and activities that they are expected to employ when they will eventually be teaching. This means that teachers need to experience the role of mathematics learner, consistent with the conceptual view of mathematics, before they are ready to facilitate such learning among their students. With this in mind, new courses should be designed and implemented as an alternative to traditional types of teacher education courses in some significant ways.

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