THE STUDY OF CLASSICS PARTICLES' ENERGY AT ARCHIMEDEAN SOLIDS WITH CLIFFORD ALGEBRA

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ABSTRACT

Geometric algebras known as a generalization of Grassmann algebras complex numbers and quaternions are presented by Clifford (1878). Geometric algebra describing the geometric symmetries of both physical space and spacetime is a strong language for physics. Groups generated from `Clifford numbers` is firstly defined by Lipschitz (1886). They are used for defining rotations in a Euclidean space. In this work, Clifford algebra are identified. Energy of classic particles with Clifford algebra are defined. This calculations are applied to some Archimedean solids. Also, the vertices of Archimedean solids presented in the Cartesian coordinates are calculated.

Keywords: Clifford Algebra, Energy of Archimedean solids's, Archimedean solids

1. INTRODUCTION

The ideas and concepts of physics are best expressed in the language of mathematics. But this language is far from unique. Many different algebraic systems exist and are in use today, all with their own advantages and disadvantages. In this paper, we describe what we believe to be the most powerful available mathematical system developed to date. This is *geometric algebra,* which is presented as a new mathematical tool to add to your existing set as either a theoretician or experimentalist. Our aim is to introduce the new techniques via their applications, rather than as purely formal mathematics. These applications are diverse, and throughout we emphasize the unity of the mathematics underpinning each of these topics.

The history of geometric algebra is one of the more unusual tales in the development of mathematical physics. William Kingdom Clifford introduced his geometric algebra in the 1870s, building on the earlier work of Hamilton and Grassmann. It is clear from his writing that Clifford intended his algebra to describe the geometric properties of vectors, planes and higher-dimensional objects. But most physicists first encounter the algebra in the guise of the Pauli and Dirac matrix algebras of quantum theory. Few then contemplate using this unwieldy matrix for practical geometric computing. Indeed, some physicists come away from a study of Dirac theory with the view that Clifford's algebra is inherently quantum-mechanical [1].

Wessel, Argand, and Gauss used the complex numbers in the solutions of two-dimensional problems. The exponential form of complex numbers is useful in the theory of rotational motions. The quaternion algebra, which was defined by Sir W. R. Hamilton, was generalized for the three-dimensional complex numbers [2]. The quaternion algebra is the Clifford algebra of the two-dimensional anti-Euclidean space. Quaternions in the three-dimensional spaces have more useful appearances for the subalgebras of Clifford algebra. In the n-dimensional spaces, Grassmann carried on the studies for the multi-dimensional bodies and defined the central product, which includes both interior and exterior products. Grassmann's central product is the Clifford product of vectors. Clifford tried to combine Grassmann's

algebra and quaternions in a mathematical system. Then this study, which was entitled "Application of Grassmann's Extensive Algebra", was published [3].

Today, Clifford algebra has an important role in the investigations of the symmetry properties of systems, crystallography, molecular and solid-state physics.

The method of point groups in the multi-dimensional spaces is derived by transforming into the parameters of the reflections and possible rotational operations. In the three-dimensional spaces, Altmann showed that the Euler's angles are not useful for the rotational operations but Euler-Rodriques' parameters are more advantageous [4]. To know the rotational pole and angle for each rotational operations is necessary and enough. So, there is a similarity between proper and improper operations in.

The rest of the paper is organized within 4 sections. Section 2 reveals the quaternions and symmetry operations in. In section 3, the Clifford algebra and symmetry operations are shown. An application of the symmetry operations with Clifford algebra is given in section 4. Conclusions are drawn in the last section [5].

2. QUATERNIONS AND SYMMETRY OPERATIONS IN R^3

The abstract quaternion group, discovered by William Rowan Hamilton in 1843, is an illustration of group structure [6]. A quaternion is a quantity represented symbolically by **A** and it is defined through the following equations

$$\mathbf{A} = a1 + A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}, \tag{1}$$

or $\mathbf{A} = [a, \vec{A}]$, $\vec{A} = (\mathbf{A_x}, \mathbf{A_y}, \mathbf{A_z})$, where all a, A_x, A_y, A_z coefficients are the real numbers. The unitary quaternions **i, j, k** satisfy the multiplication rules as follows:

$$\mathbf{i\,j} = \mathbf{k}, \quad \mathbf{j\,i} = -\mathbf{k}, \mathbf{j\,k} = \mathbf{i}, \quad \mathbf{k\,j} = -\mathbf{i}, \quad \mathbf{i\,k} = -\mathbf{j}, \quad \mathbf{k\,i} = \mathbf{j}. \tag{2}$$

Also **i, j** and **k** can be written as e_1, e_2 and e_3, reciprocally. The vector quaternion **A** with components $[0, A_x, A_y, A_z]$ and a vector $\vec{A}$ of the Euclidean tridimensional space with components (A_x, A_y, A_z) are reciprocally associated [7].

If A and B quaternions are

$$A = a1 + A_xe_1 + A_ye_2 + A_ze_3 = [a, \vec{A}], \tag{3}$$

and

$$B = b1 + B_{xe1} + B_ye_2 + B_ze_3 = [b, \vec{B}], \tag{4}$$

the product of two quaternions, namely A and B, is given by $AB = [ab - \vec{A} \cdot \vec{B}, a\vec{B} + b\vec{A} + \vec{A} \times \vec{B}]$,

$$AB = (ab - A_xB_x - A_yB_y - A_zB_z) + e_1(A_xb + aB_x + A_yB_z - A_zB_y) +$$
$$e_2(aB_y + A_yb - A_xB_z + A_zB_x) + e_3(aB_z + A_zb + A_xB_y - A_yB_x), \tag{5}$$

where the result is a quaternion. It must be noted that the product of quaternions is not commutative, but associative. The product of **A** and **B** quaternions in the matrix form can be written.
For each quaternion A, its conjugate is [8]

$$A^* = a1 - A_x e_1 - A_y e_2 - A_z e_3. \tag{6}$$

The rotation of arbitrary points on an unit sphere ($\mathbf{R}(\gamma \mathbf{k})$) can be defined by γ angle around of the definite k-axis ($0 < \gamma < \Pi$). The poles of rotational operations are defined on the every half-sphere. In general, the rotations (counterclokwise) are accepted to be positive direction. If $\mathbf{R}(\beta \mathbf{l})$ and $\mathbf{R}(\alpha \mathbf{m})$ are the rotations with δ and α angles around of l and m axes, respectively, then the result of two rotational operations has to be equal to a new rotation with γ-angle around of k- axis, *i.e.*

$$\mathbf{R}(\beta \mathbf{l})\,\mathbf{R}(\alpha \mathbf{m}) = \mathbf{R}(\gamma \mathbf{k}), \tag{7}$$

where k, l, m are the axial vectors that correspond to the k, l, m rotation axes, respectively. Rodriques [9] defined the geometrical structure and algebra for the angles γ, β, α and the axes k, l, m in the following forms

$$\cos\frac{\gamma}{2} = \cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\,\mathbf{m}.\mathbf{l}, \tag{8}$$

$$\sin\frac{\gamma}{2}k = \sin\frac{\alpha}{2}\cos\frac{\beta}{2}m - \cos\frac{\alpha}{2}\sin\frac{\beta}{2}l + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}m \times l. \tag{9}$$

The rotational operations with the normalized quaternions can be written as

$$\mathbf{R}(\alpha \mathbf{m}) = [\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\mathbf{m}] \,, \tag{10}$$

$$\mathbf{R}(\beta \mathbf{l}) = [\cos\frac{\beta}{2}, \sin\frac{\beta}{2}\mathbf{l}] \,. \tag{11}$$

In the similar way, from Eqs. (7), (8), (9), $\mathbf{R}(\gamma \mathbf{k})$ can be defined by the following equations:

$$\mathbf{R}(\gamma \mathbf{k}) = [\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\mathbf{m}]\,[\cos\frac{\beta}{2}, \sin\frac{\beta}{2}\mathbf{l}] \,, \tag{12}$$

$$\mathbf{R}(\gamma \mathbf{k}) = [\cos\frac{\gamma}{2}, \sin\frac{\gamma}{2}\mathbf{k}] \,. \tag{13}$$

Any point represented by **R** quaternion transforms to the a new quaternionic point at the end of a rotation defined by **A** unit quaternion. This new quaternion is;

$$R' = \mathbf{A}\,\mathbf{R}\,\mathbf{A}^*, \tag{14}$$

where A* is complex conjugate of **A** [5].

3. SYMMETRY OPERATIONS WITH CLIFFORD ALGEBRA

Clifford algebra is used for the studies of symmetry in physics. There are three basic units e_i *(i=1, 2, 3)* in Clifford algebra such that

$$\mathbf{e_i e_j + e_j e_i = 2\delta_{ij},} \tag{15}$$

which are equivalent to

$$e_i e_j = 1, \tag{16}$$

$$e_i e_j = -e_j e_i. \tag{17}$$

The easiest way to understand the geometric product is by example, so consider a two-dimensional space (a plane) spanned by two orthonormal vectors $\mathbf{e_1}$ and $\mathbf{e_2}$. These basis vectors satisfy

$$e_1^2 = e_2^2 = 1 \tag{18}$$

$$e_1.e_2 = 0 \tag{19}$$

The final entity present in the algebra is the bi-vector $e_1 \Lambda e_2$. This is the highest grade element in the algebra, since the outer product of a set of dependent vectors is always zero. The Clifford product of two vectors, namely $\vec{a}$ and $\vec{b}$, with components

$$\vec{a} = a_1 e_1 + a_2 e_2, \tag{20}$$

and

$$\vec{b} = b_1 e_1 + b_2 e_2, \tag{21}$$

is given by

$$\vec{a}\vec{b} = \vec{a}.\vec{b} + \vec{a} \Lambda \vec{b} = a_1 b_1 + a_2 b_2 + (a_1 b_2 - a_2 b_1)\mathbf{e_{12}}, \tag{22}$$

where "Λ" is called as the wedge product [10]. To study the properties of the bivec- tor $e_1 \Lambda e_2$ we first recall that for orthogonal vectors the geometric product is a pure bivector:

$$e_1 e_2 = e_1.e_2 + e_1 \Lambda e_2 = e_1 \Lambda e_2 = e_{12} \tag{23}$$
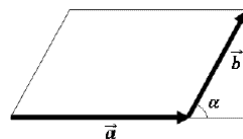
Figure 1. The geometrical meaning of a Λ b [11].

and that orthogonal vectors anticommute:

$$e_2 e_1 = e_2 \Lambda e_1 = -e_1 \Lambda e_2 = -e_1 e_2 \tag{24}$$

We can now form products in which $e_2 e_1$ multiplies vectors from the left and the right . First from the left we find that

$$(e_1 \Lambda e_2)e_1 = (-e_2 e_1)e_1 = -e_2 e_1 e_1 = -e_2 \tag{25}$$

and

$$(e_1 \Lambda e_2)e_2 = (e_1 e_2)e_2 = e_1 e_2 e_2 = e_1 \tag{26}$$

If we assume that e_1 and e_2 form a right-handed pair, we see that left multiplication by the bivector rotates vectors 90° clockwise(i.e. in a negative sense).

Similarly, acting from the right

$$e_1(e_1e_2) = e_2, \tag{27}$$
$$e_2(e_1e_2) = -e_1, \tag{28}$$

So right multiplication rotates 90° anticlockwise -a positive sense.
The final product in the algebra to consider is the square of the bivector $e_1 \wedge e_2$:

$$(e_1 \wedge e_2)^2 = e_1e_2e_1e_2 = -e_1e_1e_2e_2 = -1. \tag{29}$$

Geometric consideration have led naturally to quantity which square to -1. This fits with the fact that two successive left (or right) multiplication of a vector by $e1e_2$ rotates the vector through 180°, which is equivalent to multiplying by -1. 1, e_1, e_2, e_{12} form the basis of the Clifford algebra Cl_2 of the vector plane R^2.

The Clifford algebra Cl_2 is the four-dimensional linear space and its basis elements have the multiplication table as follows:

Table 1. The basis of Clifford Algebra

	e_1	e_2	e_{12}
e_1	1	e_{12}	e_2
e_2	$-e_{12}$	1	$-e_1$
e_{12}	$-e_2$	e_1	-1

The reflection of $\vec{r}$ across the line $\vec{a}$, namely the mirror image $\vec{r}$ of $\vec{r}$ with respect to $\vec{a}$ is given by

$$\vec{r} = \vec{a}\,\vec{r}\,\vec{a}^{-1} \tag{30}$$

Equation (30) can be directly obtained from using the commutation properties of Clifford algebra [10]. The composition of two reflections, first across $\vec{a}$ and then across $\vec{b}$, is given by

$$\vec{r} = \vec{b}\,\vec{r}\,\vec{b}^{-1} = \vec{b}(\vec{a}\,\vec{r}\,\vec{a}^{-1})\,\vec{b}^{-1} = (\vec{b}\,\vec{a})\,\vec{r}\,(\vec{b}\,\vec{a})^{-1} \tag{31}$$

The composite of these two reflections is a rotation by twice the angle between $\vec{a}$ and $\vec{b}$.
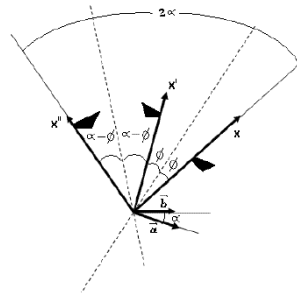
Figure 2. When **x** is subjected two successive reflection first with respect to a plane perpendicular to a and then with to a plane perpendicular to b, the result is a rotation of **x** about an axis in the direction axb.

1	the scalar
$e_1, e_2, e_3.$	Vectors
e_1e_2, e_1e_3, e_2e_3	bivectors	(32)
$e_1e_2e_3$	a volume element

An arbitrary element in Cl_3 is a sum of a scalar, a vector, a bivector and a volume element, and can be written as

$$u = u_0 + u_1e_1 + u_2e_2 + u_3e_3 + u_{12}e_{12} + u_{13}e_{13} + u_{23}e_{23} + u_{123}e_{123}. \tag{33}$$

The Clifford units, ei 's, are identified with orthogonal reflections (mirrors)

$$e_1 \leftrightarrow \sigma_{yz}, \ e_2 \leftrightarrow \sigma_{xz}, \ e_3 \leftrightarrow \sigma_{xy}. \tag{34}$$

The mappings between the Clifford bivectors e_ie_j and the corresponding quaternion units are defined

$$e_3e_2 \ \leftrightarrow \ \sigma_{xy}\sigma_{xz} = C_{2x} \leftrightarrow [0, (1, 0, 0)], \tag{35}$$
$$e_1e_3 \leftrightarrow \ \sigma_{yz}\sigma_{xy} = C_{2y} \leftrightarrow [0, (0,1, 0)], \tag{36}$$
$$e_2 e_1 \ \leftrightarrow \ \sigma_{xz}\sigma_{yz} = C_{2z} \leftrightarrow [0, (0, 0,1)]. \tag{37}$$

The inversion, which is a product of three reflections, is obtained by a trivector as follows:

$$e_1e_2e_3 \ \leftrightarrow \ \sigma_{yz}\sigma_{xz}\sigma_{xy} = i, \tag{38}$$

where $e_1e_2e_3 \in Cl_3$ is performed [5].

4. AN APPLICATION OF THE SYMMETRY OPERATIONS WITH CLIFFORD ALGEBRA

The reflection and rotation operations in the solid state physics and molecular physics play an important role.

Table 2. Numbers for the five Platonic solids

	Number of faces	*Number of edges*	*Number of vertices*	*Edges per face*
Tetrahedron	4	6	4	3
Cube	6	12	8	4
Octahedron	8	12	6	3
Icosahedron	20	30	12	3
Dodecahedron	12	30	20	5

The symmetry operations can be easily applied on the regular polyhedra, which are called *Platonic Solids.,* and semi-regular poly-hedra, which are called *Archimedean Solids.* The Platonic solids are tetrahedron, cube, octahedron, icosahedron and dodecahedron. Some important numbers for the Platonic solids are shown in Table 2. The Archimedean solids are truncated tetrahedron, cuboctahedron, truncated cube, truncated octahedron, rhombicuboctahedron (or small rhombicuboctahedron), truncated cuboctahedron (or great rhombicuboctahedron), snub cube (or snub hexahedron), icosidodecahedron, truncated dodecahedron, truncated icosahedron, rhombicosidodecahedron (or small rhombicosi-dodecahedron), truncated icosidodecahedron (or great rhombicosidodecahedron), snub dodecahedron (or snub icosidodecahedron). Some important numbers for the Platonic solids are shown in Table 3 [14].

Table 3. Numbers for the thirteen Archimedean solids

	Number of faces	Number of edges	Number of
Truncated Tetrahedron	8(4 triangles,4 hexagons)	18	12
Cuboctahedron	14 (8 triangles,6 squares)	24	12
Truncated Cube	14 (8 triangles,6 octagons)	36	24
Truncated Octahedron	14 6 squares,8 hexagons)	36	24
Rhombicuboctahedron	26 (8 triangles,18 squares)	48	24
Truncated Cuboctahedron	26 (12 squares,8 hexagons,6 octagons)	72	48
Snub Cube	38 (32 triangles,6 squares)	60	24
Icosidodecahedron	32 (20 triangles,12 pentagons)	60	30
Truncated Dodecahedron	32 (20 triangles,12 decagons)	90	60
Truncated Icosahedron	32 (12 pentagons,20 hexagons)	90	60
Rhombicosidodecahedron	62 (20 triangles,30 squares,12 pentagons)	120	60
Truncated Icosidodecahedron	62 (30 squares,20 hexagons,12 decagons)	180	120
Snub Dodecahedron	92 (80 triangles,12 pentagons)	150	60

Table 4. For the model, shown in Fig. 1, the vertices of a rhombicuboctahedron

corner	x	y	z
1	a	-a	1,85
2	a	a	1,85
3	-a	a	1,85
4	-a	-a	1,85
5	a	-1,85	a
6	1,85	-a	a
7	a	-1,85	-a
8	1,85	-a	-a
9	-a	1,85	a
10	-1,85	a	a
11	-a	1,85	-a
12	-1,85	a	-a
13	1,85	a	a
14	a	1,85	a
15	1,85	a	-a
16	a	1,85	-a
17	-1,85	-a	a
18	-a	-1,85	a
19	-1,85	-a	-a
20	-a	-1,85	-a
21	a	-a	-1,85
22	a	a	-1,85
23	-a	a	-1,85
24	-a	-a	-1,85

A rhombicuboctahedron has twenty four-vertices. The vertices of a dodecahedron, whose origin was chosen at the centre of body, are indexed as cartesian coordinates in Table 3.

Assuming the distances of midpoints of all the edges to the origin of the rhombicuboctahedron are normalized. We suppose that edge lenght is 2a.

Now, for example, from eqs.(12), (13) and (21), the C2 rotation of 13th vertex around the z-axis is

$R' = A_z R_{13} a_z^*$

$=$

$$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1,85 & -a & -a \\ 1,85 & 0 & -a & -a \\ a & a & 0 & -1,85 \\ a & a & 1,85 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1,85 \\ -a \\ a \end{bmatrix}$$

where $\mathbf{A_z}$, R_{13} and a_z^* are the (4x4)-matrix representations of rotation z-axis and 13th vertex, (4×1)-column matrix representation of the (4×4) matrix of A_z^* respectively. This result of the matrix operation defines the 17th vertex of the rhombicuboctahedron.

This method is the conventional geometrical method of the rotation operation for rhombicuboctahedron. Using quaternions, the same calculation can be obtained as follows:
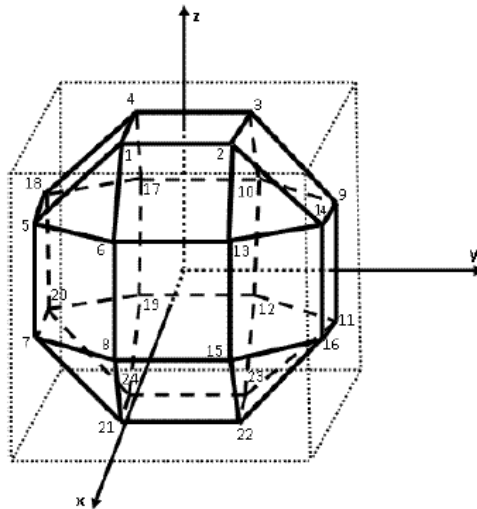

Figure 3. Rhombicuboctahedron

$R' = a_z R_{13} a_z^*$
$= (1e_{12})(\mathbf{1,85e_1 + ae_2 + ae_3})(-le_{12})$
$= (-1,85e_1 + -ae_2 + ae_3)$

This result is equal to the quaternionic definition of 17th vertex, as well. Now we investigate another rotation operation of 17th z-axis around the straight line.

The rotation 90° of 17th vertex z-axis around this straight line.

$$R' = a_z R_{17} a_z^{\,*}$$
$$= (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} e_{12})(-1{,}85e_1 - ae_2 + ae_3)(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} e_{12})$$
$$= (—ae_1 + 1{,}85e_2 + ae_3).$$

This result is equal to the quaternionic definition of 9th vertex. The rotation 45° of 9th vertex z-axis around this straight line

$$R' = a_z R_9 a_z^{\,*}$$
$$= (\frac{\sqrt{\sqrt{2}+2}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} e_{12})(-ae_1 + 1{,}85e_2 + ae_3) (\frac{\sqrt{\sqrt{2}+2}}{2} - \frac{\sqrt{2-\sqrt{2}}}{2} e_{12}) \qquad (56)$$
$$= (ae_1 + 1{,}85e_2 + ae_3).$$

This result is equal to the quaternionic definition of 14th vertex. The reflection operations can be also defined by the Clifford algebra elements. According to the xz-plane, the reflection of 2th vertex with Clifford algebra is

$$R' = \sigma_{xz} R_{10} \sigma_{xz}^{-1}$$
$$= e_{13}(ae_1 + ae_2 + 1{,}85e_3)e_{13}$$
$$= ae_1 - ae_2 + 1{,}85e_3 ,$$

where σ_{xz}^{-1} is the inverse of σ_{xz} operation. The reflection of 10th vertex on the xz-plane is equal to the 1th vertex. After this reflection of the 2th vertex on the xz-plane, the reflection of 1th vertex on the xy-plane can be written as

$$R'' = \sigma_{xy} R' \sigma_{xy}^{-1}$$
$$= \sigma_{xy} (\sigma_{xz} R_2 \sigma_{xy}^{-1}) \sigma_{xy}^{-1}$$
$$= (\sigma_{xy}\sigma_{xz})R_2(\sigma_{xz}\sigma_{xy})$$
$$= ae_1 - ae_2 - 1{,}85e_3.$$

This result is equal to the quaternionic definition of 21th vertex in Table 4.

The rotations 45° (C_8), 90° (C_4), 180° (C_2) of around axis which are perpendicular to the this surface, from the midpoint of each square face and the rotations 60° (C_6) of around axis which are perpendicular to the this surface, from the midpoint of each face of triangle are possible. 3 reflection in a plane perpendicular to a 3-fold axis and the rotoreflections 180° (S_2) of around the axis drawn from each corner to the other corner are also possible. O_h (*432) group; achiral octahedral symmetry or full octahedral symmetry; is made up of all this operations of rotation and reflection.

5. CLASSICS PARTICLES' ENERGY AT ARCHIMEDEAN SOLIDS WITH CLIFFORD ALGEBRA

The reflection and rotation operations in the solid state physics and molecular physics play an important role. The symmetry operations can be easily applied on the regular polyhedra, which are called Platonic Solids. At the same time the operations can be apply to The Archimedean solids.

Any point represented by **R** quaternion transforms to the a new quaternionic point at the end of a rotation defined by **A** unit quaternion. This new quaternion is

R´=A R A*,
where **A*** is complex conjugate of **A**.
The reflection of **r** across the line **a**, namely the mirror image **r'** of **r** with respect to **a**, is given by
r'=ara^{-1}

For example rhombicuboctahedron's 17th vertices's replasement and potential energy with Clifford algebra:

$$R' = a_z R_{17} a_z{}^*$$
$$= (\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2} e_{12})(-1{,}85e_1 - ae_2 + ae_3)(\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2} e_{12})$$
$$= (-ae_1 + 1{,}85e_2 + ae_3).$$

This result is equal to the quaternionic definition of 14th vertex.
We can calculate difference of potential energy for this two points. The potential energy of this points are equal.

14th vertex's coordiante is ($ae_1 + 1{,}85e_2 + ae_3$) and 21th vertex's coordiante is $ae_1 - ae_2 - 1{,}85e_3$.
$R_{21}'-R_{14}= (ae_3)-(-1{,}85e_3.)= 2{,}85e_3$
The difference of potential energy for this points is
U=mg(2,85).

Table 5. Potential energies of rhombicuboctahedron's edges
(m is mass, g is gravitational acceleration)

Vertices number's	z
1	1,85 gh
2	1,85 gh
3	1,85 gh
4	1,85 gh
5	a gh
6	a gh
7	-a gh
8	-a gh
9	a gh
10	a gh
11	-a gh
12	-a gh
13	a gh
14	a gh
15	-a gh
16	-a gh
17	a gh
18	a gh
19	-a gh
20	-a gh
21	-1,85 gh
22	-1,85 gh
23	-1,85 gh
24	-1,85 gh

6. CONCLUSIONS

In this study we apply Clifford Algebra to Archimedean Solids. They are semi-symmetric solids. The geometrical methods and matrices are used for the investigation of symmetry operations of the symmetric solid too [5]. Clifford algebras are algebras of geometries and quaternions are hypercomplex numbers [15]. In this study, Clifford algebra and quaternions are used for the symmetry operations. When these operations are made with Clifford algebra and quaternions, it is obvious that the calculations are easy and compact. The quaternions and Clifford algebra are much simpler to apply to the symmetry operations than the conventional methods of molecular symmetry. This method can be applied to the more complex structures.

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