

## State Estimation in Induction Motors Using the Closed Loop Observers

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### ABSTRACT

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This paper presents an estimation technique for the state variables of a three phase squirrel-cage induction motor such as stator current and, rotor flux components. Closed loop observers, which can be classified as full-order and reduced-order observers, are used for estimation. In observer design, the state-space d-q axis model of induction motor is used. Estimation algorithms given for either observer are implemented by using MATLAB software. In the estimation algorithms, some results for different observer gain matrices are obtained from input/output data of the state-space simulation of induction motor supplied by sinusoidal voltage sources. It is shown that estimation results are compatible with the simulation results of induction motor.

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## 1. Introduction

Induction motors are widely used in industry for speed and position control applications because of their low cost, high reliability and low maintenance [1]. With the advancement of solid state device technology in recent years, vector controlled drives for induction motors became popular [1,2,3]. This control is also known as the “field oriented control”, “flux oriented control” or “indirect torque control”. The most challenging and the limiting feature of the vector control, is to obtain rotor flux components. Depending on the obtaining method, the vector control is divided into two subcategories; direct and indirect vector control. In direct vector control, the flux measurement is done by using the flux sensing coils or the Hall devices. This adds to additional hardware cost and in addition, measurement is not highly accurate. Therefore, this method is not a very good control technique. The more common method is indirect vector control [1,4]. In this method, the flux components are not measured directly, but are estimated from the equivalent circuit model and from motor terminal measurements such as the stator currents and the voltages. Different estimators such as Kalman filter, sliding-mode estimator can be found in literature to obtain the state variables of induction motors [1,4,5]. In this study, full-order and reduced-order observers have been designed to estimate stator current and rotor flux components in d-q axis of an induction motor. In design, the continuous-time state space model of the induction motor is used. Estimation algorithms developed for full rank and reduced rank observers are programmed on MATLAB software.

## 2. The Induction Motor Model

A three-phase squirrel-cage induction motor can be represented by the following voltage equations in arbitrarily rotating reference frame where the motor is assumed to be symmetrical and flux distribution is sinusoidal Eq. 1 [1]:

$$v_{qs} = i_{qs} R_s + p \lambda_{qs} + \lambda_{ds} \omega_r \quad (1.a)$$

$$v_{ds} = i_{ds} R_s + p \lambda_{ds} - \lambda_{qs} \omega_r \quad (1.b)$$

$$v_{qr} = 0 = i_{qr} R'_r + p \lambda'_{qr} + (\lambda_{ds} - \lambda_{qs} \omega_r) \omega_r \quad (1.c)$$

$$v_{dr} = 0 = i'_{dr} R'_r + p \lambda'_{dr} - (\lambda_{qs} + \lambda_{ds} \omega_r) \omega_r \quad (1.d)$$

Likewise, flux equations of induction motor are presented as follows:

$$\lambda_{qs} = L_s i_{qs} + L_m i'_{qr} \quad (2.a)$$

$$\lambda_{ds} = L_s i_{ds} + L_m i'_{dr} \quad (2.b)$$

$$\lambda'_{qr} = L'_r i'_{qr} + L_m i_{qs} \quad (2.c)$$

$$\lambda'_{dr} = L'_r i'_{dr} + L_m i_{ds} \quad (2.d)$$

where

$v_{qs}, v_{ds}$  : stator voltages in d-q axes

$i_{qs}, i_{ds}$  : stator currents in d-q axes

$i'_{qr}, i'_{dr}$  : rotor currents referred to stator in d-q axes

$R_s, R'_r$  : stator resistance and rotor resistance referred to stator

$L_s, L'_r$  : stator inductance and rotor inductance referred to stator

$L_m$  : mutual inductance between stator and rotor

$\lambda_{qs}, \lambda_{ds}$  : stator flux components in d-q axes

$\lambda'_{qr}, \lambda'_{dr}$  : rotor flux components in d-q axes

$\omega_r$  : reference frame angular velocity

$\omega_r$  : rotor angular velocity

$p = \frac{d}{dt}$  : differential operator

Using these equations, state space model of squirrel-cage induction motor can be obtained as follows:

$$p \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{L_s} + \frac{1-\sigma}{T_r}\right) & 0 & \frac{L_m}{L_s L'_r T_r} & -\frac{L_m \omega_r}{L_s L'_r} \\ 0 & -\left(\frac{R_s}{L_s} + \frac{1-\sigma}{T_r}\right) & \frac{L_m \omega_r}{L_s L'_r} & \frac{L_m}{L_s L'_r T_r} \\ \frac{L_m}{T_r} & 0 & -\frac{1}{T_r} & -(\omega_r) \\ 0 & \frac{L_m}{T_r} & (\omega_r) & -\frac{1}{T_r} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} \quad (3.a)$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ ' \\ ' \\ dr \end{bmatrix} \tag{3.b}$$

where

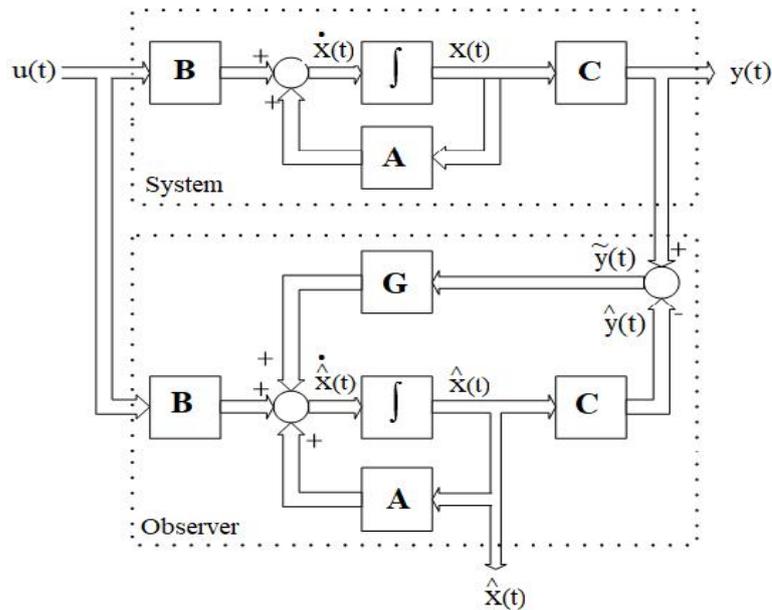
$$= 1 - \frac{L_m^2}{L_s L_r'} : \text{Inductance leakage coefficient}$$

$$T_r = \frac{L_r'}{R_r'} : \text{Rotor time constant}$$

We can apply either the synchronous or stationary reference frames if all voltages are balanced and continuous. In this paper, we use stationary reference frame by choosing  $\omega = 0$ .

### 3. Full Order Observers

If the closed loop observer estimates all state variables of the system, regardless of whether some state variables are available for direct measurement, it is called a full-order observer. That is to say, the order of the observer that will be discussed here is the same as that of the system. The simulation diagram of the full-order observer as presented by D.G. Luenberger is given as follows [11,13]:



**Figure 1.** Simulation Diagram of the Full Order Observer

According to the diagram, the system can be defined by following equations.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{4.a}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{4.b}$$

Likewise, the state and output equations of full-order observer is written as:

$$\dot{\hat{\mathbf{x}}}(t) = (\mathbf{A} - \mathbf{G}\mathbf{C})\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{y}(t) \tag{5.a}$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}\hat{\mathbf{x}}(t) \quad (5.b)$$

To obtain the observer error equation, let us subtract Equation (5.a) from Equation (4.a).

$$\dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t) = (\mathbf{A} - \mathbf{G}\mathbf{C})(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) \quad (6.a)$$

$$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \quad (6.b)$$

$$\dot{\mathbf{e}}(t) = (\mathbf{A} - \mathbf{G}\mathbf{C})\mathbf{e}(t) \quad (6.c)$$

If the system is completely observable, then it can be proved that it is possible to choose matrix  $\mathbf{G}$  such that  $(\mathbf{A} - \mathbf{G}\mathbf{C})$  has arbitrarily desired eigenvalues [12,13,25]. That is, the observer gain matrix  $\mathbf{G}$  can be determined to yield the desired error dynamic.

#### 4. Reduced Order Observers

In practice, some of the state variables may be accurately measured. Thus, measurable state variables need not estimating.

Suppose that state vector  $\mathbf{x}$  is a  $n$ -vector and  $m$  state variables need not be estimated. We need to estimate only  $n-m$  state variables. Then the reduced order observer becomes an  $(n-m)$  th-order observer. Consider the system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (7.a)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \quad (7.b)$$

where the state vector  $\mathbf{x}$  can be partitioned into two parts  $\mathbf{x}_m$  and  $\mathbf{x}_u$ . Here  $\mathbf{x}_u$  is the unmeasurable portion of the state vector, whereas  $\mathbf{x}_m$  is the portion that consists of directly measurable states. Then the partitioned state and output equations become [25].

$$\begin{bmatrix} \dot{\mathbf{x}}_m(t) \\ \dot{\mathbf{x}}_u(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{mm} & \mathbf{A}_{mu} \\ \mathbf{A}_{um} & \mathbf{A}_{uu} \end{bmatrix} \begin{bmatrix} \mathbf{x}_m(t) \\ \mathbf{x}_u(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_m \\ \mathbf{B}_u \end{bmatrix} \mathbf{u}(t) \quad (8.a)$$

$$\mathbf{y}_m(t) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_m(t) \\ \mathbf{x}_u(t) \end{bmatrix} \quad (8.b)$$

In addition to these equations, when we define

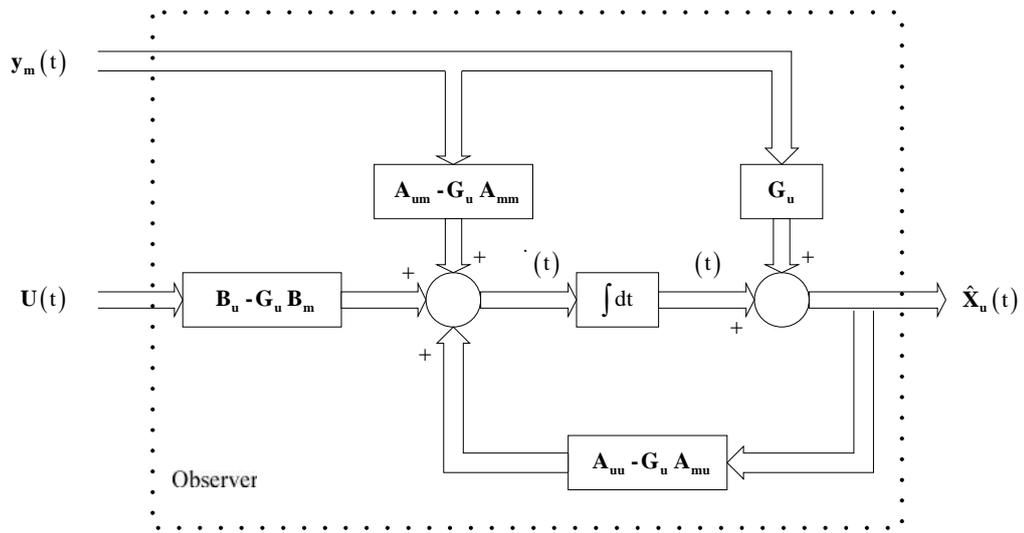
$$\hat{\mathbf{x}}_u(t) = \mathbf{x}_u(t) + \mathbf{G}_u \mathbf{y}_m(t) \quad (9)$$

The state and output equations of reduced-order observer can be written as:

$$\dot{\hat{\mathbf{x}}}_u(t) = (\mathbf{A}_{uu} - \mathbf{G}_u \mathbf{A}_{mu}) \hat{\mathbf{x}}_u(t) + \left[ (\mathbf{B}_u - \mathbf{G}_u \mathbf{B}_m) \quad \mathbf{M}((\mathbf{A}_{uu} - \mathbf{G}_u \mathbf{A}_{mu})\mathbf{G}_u + \mathbf{A}_{um} - \mathbf{G}_u \mathbf{A}_{mm}) \right] \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{y}_m(t) \end{bmatrix} \quad (10.a)$$

$$\hat{\mathbf{x}}_u(t) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \hat{\mathbf{x}}_u(t) + \begin{bmatrix} \mathbf{0} & \mathbf{M}\mathbf{G}_u \end{bmatrix} \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{y}_m(t) \end{bmatrix} \quad (10.b)$$

Based on equations [10.a] and [10.b], the simulation diagram of the reduced-order observer is constructed like below:



**Figure 2.** Simulation Diagram of the Reduced Order Observer [25].

We need to obtain the observer error equation.

$$\mathbf{e}_u(t) = \mathbf{x}_u(t) - \hat{\mathbf{x}}_u(t) \tag{11.a}$$

$$\dot{\mathbf{e}}_u(t) = (\mathbf{A}_{uu} - \mathbf{G}_u \mathbf{A}_{mu}) \mathbf{e}_u(t) \tag{11.b}$$

For a completely observable system, we can choose matrix  $\mathbf{G}_u$  such that  $(\mathbf{A}_{uu} - \mathbf{G}_u \mathbf{A}_{mu})$  has arbitrarily desired eigenvalues. That means we can achieve the desired error dynamics.

### 5. Determination Of The State Observer Gain Matrix

The problem of designing a state observer means the problem of determining the observer gain matrix. There are a few approaches for this purpose such as Ackermann, direct substitution etc. [14,25]. These are especially available for single-output systems. For multiple-output systems, we shall carry out some extra calculations. If the state matrix  $\mathbf{A}$  is cyclic, the computations get simpler. In such case, the gain matrix  $\mathbf{G}$ , for a system with  $n$  states and  $r$  outputs, can be written as a product of two different vectors as follows:

$$\mathbf{G}_{n \times r} = \mathbf{N}_{n \times 1} \cdot \mathbf{R}_{1 \times r} \tag{12}$$

Where  $\mathbf{R}$  is arbitrary and  $\mathbf{N}$  is obtained from one of the single output approaches. How it is possible is stated below:

$$(\mathbf{A} - \mathbf{G}\mathbf{C}) \rightarrow (\mathbf{A} - \mathbf{N}\mathbf{R}\mathbf{C}) \tag{13.a}$$

$$\mathbf{C}_m = \mathbf{R}\mathbf{C} \tag{13.b}$$

$$(\mathbf{A} - \mathbf{G}\mathbf{C}) \rightarrow (\mathbf{A} - \mathbf{N}\mathbf{C}_m) \tag{13.c}$$

In single-output approaches, we calculate the vector  $\mathbf{N}$  as the gain matrix by using  $\mathbf{C}_m$  instead of  $\mathbf{C}$ . Then we determine the observer gain matrix  $\mathbf{G}$  according to the equation [11].

### 6. Cyclic matrices

Cyclic matrices is a special form of non-negative matrices. Each different eigenvalue of these matrices has only one jordan block. We firstly need to diagonalize the matrix to see if it is cyclic. Below are two examples for understanding:

$$A = \begin{bmatrix} 2 & 1 & 0 & \vdots & 0 & 0 \\ 0 & 2 & 1 & \vdots & 0 & 0 \\ 0 & 0 & 2 & \vdots & 0 & 0 \\ \dots & \dots & \dots & \vdots & \dots & \dots \\ 0 & 0 & 0 & \vdots & -3 & 1 \\ 0 & 0 & 0 & \vdots & 0 & -3 \end{bmatrix}, \text{ the matrix A is cyclic}$$

$$A = \begin{bmatrix} 2 & 1 & \vdots & 0 & \vdots & 0 & 0 \\ 0 & 2 & \vdots & 0 & \vdots & 0 & 0 \\ \dots & \dots & \vdots & \dots & \vdots & \dots & \dots \\ 0 & 0 & \vdots & 2 & \vdots & 0 & 0 \\ \dots & \dots & \vdots & \dots & \vdots & \dots & \dots \\ 0 & 0 & \vdots & 0 & \vdots & -3 & 1 \\ 0 & 0 & \vdots & 0 & \vdots & 0 & -3 \end{bmatrix}, \text{ the matrix A is not cyclic.}$$

### 7. Algorithm

MATLAB program that simulates and estimates the state variables of the induction motor can be written according to the flowchart below:

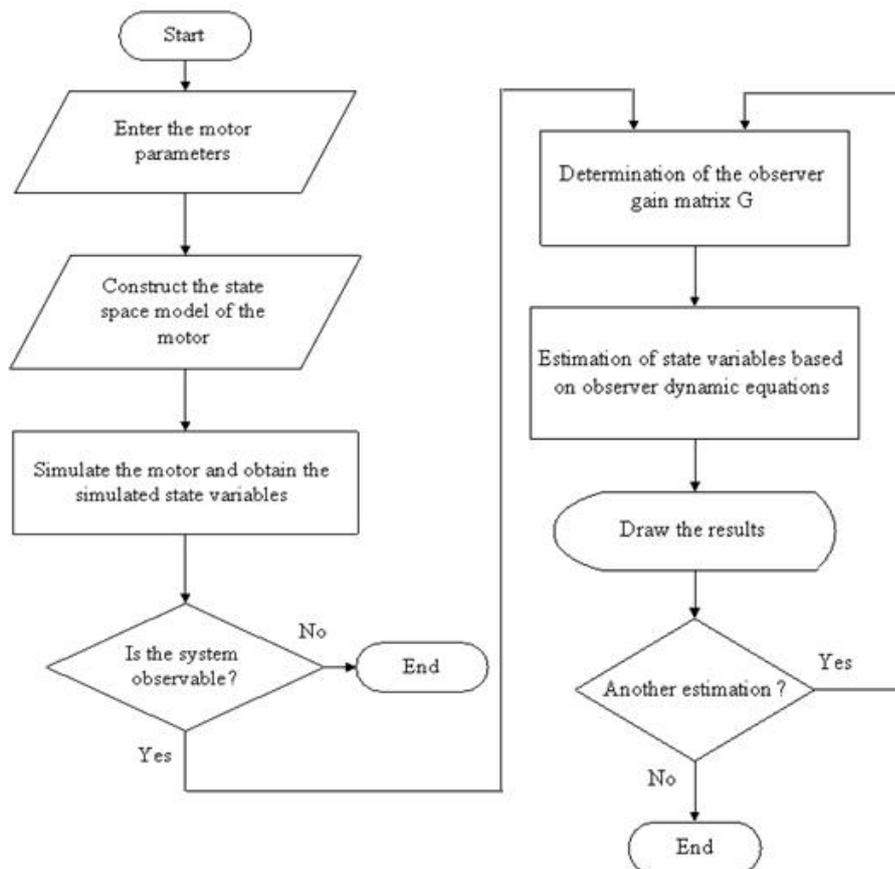


Figure 3. Flowchart of the Matlab Algorithm

### 8. Simulation Results

Using the Matlab software, which is written according to flowchard in Figure 3, the simulation and estimation results were obtained for the full rank and reduced rank observers under different operation conditions. The motor parameters given in Aacknowledgment are used in the simulations. The d-q axes components of the sinusoidal supply applied to the motor are given in figure 4. Figure 5 and Figure 6 show the  $i_{qs}$  and  $i_{ds}$  estimation and simulation curves for the full-range observer. Estimation values catch up with simulation values in about 15 ms. for observer eigenvalues  $(-500 \pm j250, -1000 \pm j50)$ ; while in about 25 ms. for observer eigenvalues  $(-150 \pm j250, -150 \pm j50)$ . The  $i_{qr}$  and  $i_{dr}$  estimation and simulation curves for the full-range observer is given in Figure 7 and Figure 8. Estimation values for rotor flux components converges to simulation values in approximately 15 ms. for observer eigenvalues  $(-500 \pm j250, -1000 \pm j50)$ ; while in about 25 ms. for observer eigenvalues  $(-150 \pm j250, -150 \pm j50)$ .

In reduced order observer is given in Figure 9 and Figure 10, estimation values catch up with simulation values in about 80 ms. for observer eigenvalues;  $-50 \pm j315$ ; while in about 200 ms. for observer eigenvalues;  $-20 \pm j315$ . We can clearly see that the performance of full order observer is better than that of reduced order observer. Because it catches up with the simulation values more quickly. In addition, the further the real parts of observer eigenvalues go to the left-hand side on the s-domain, the better the each observer performance gets.

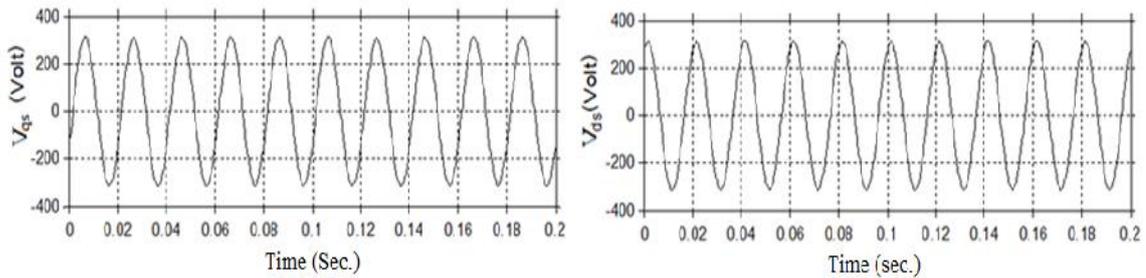
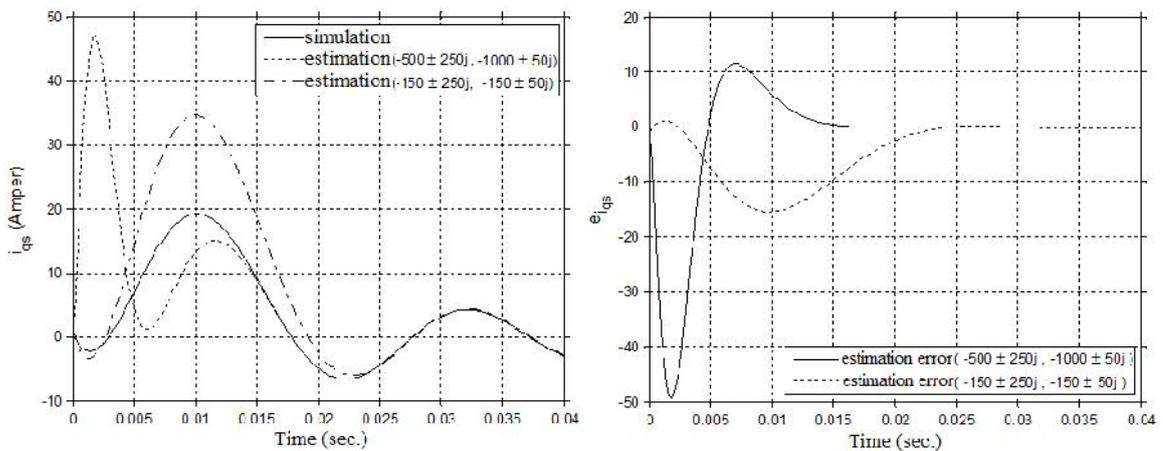


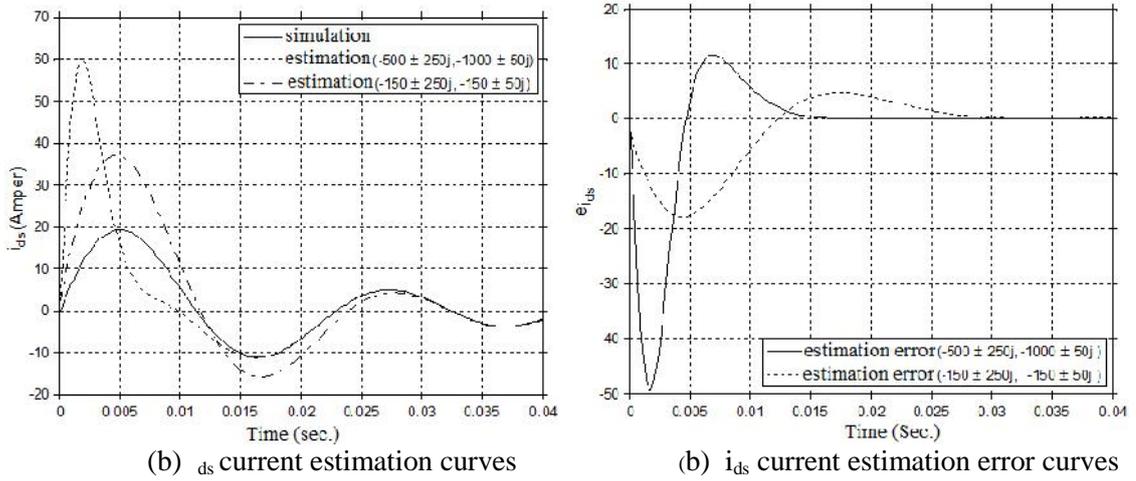
Figure 4. d-q axes components for Sinusoidal voltage supply



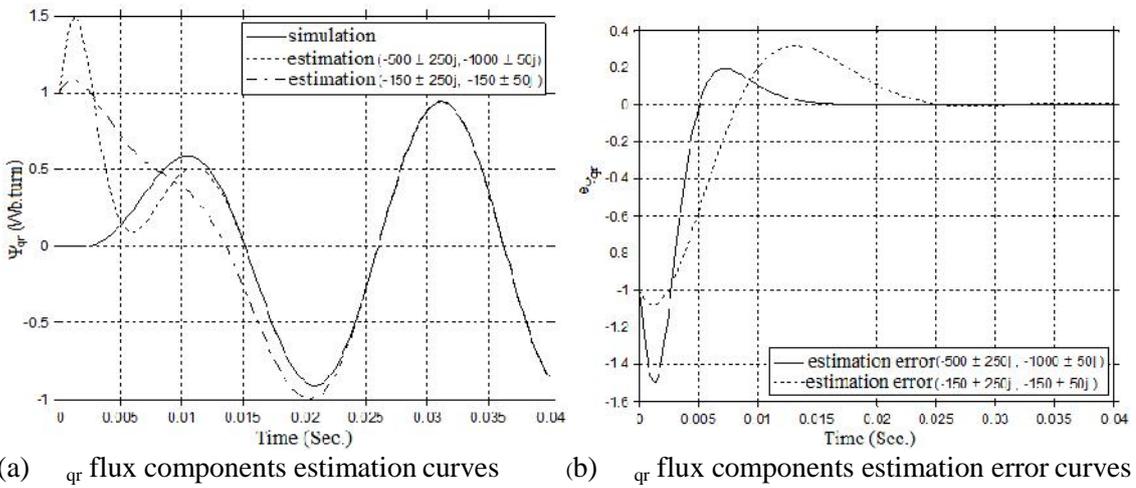
(a)  $i_{qs}$  current estimation curves

(b)  $i_{qs}$  current estimation error curves

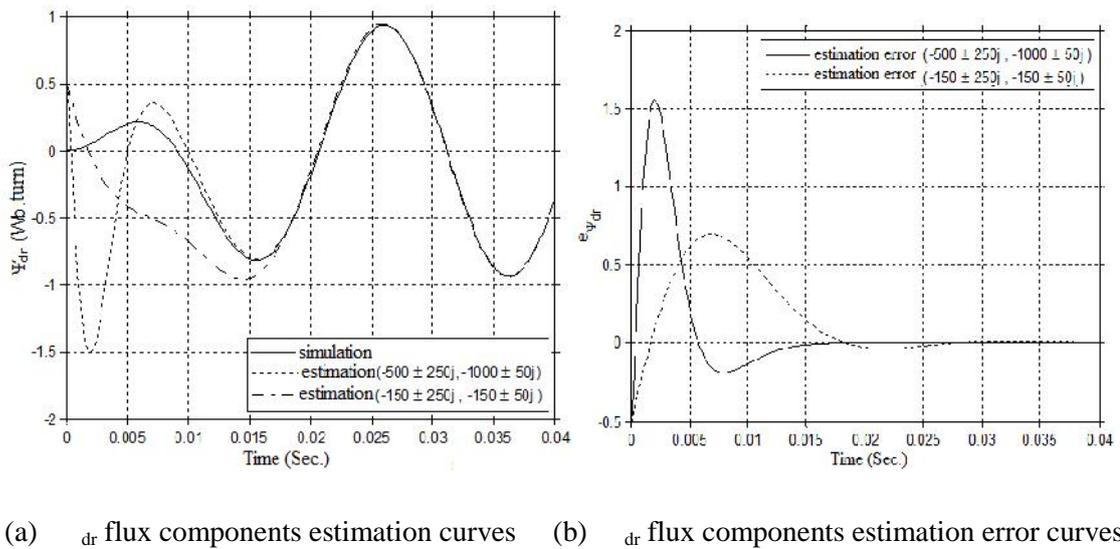
Figure 5. Stator current q-axes components simulation, estimation and error curves for full order observer



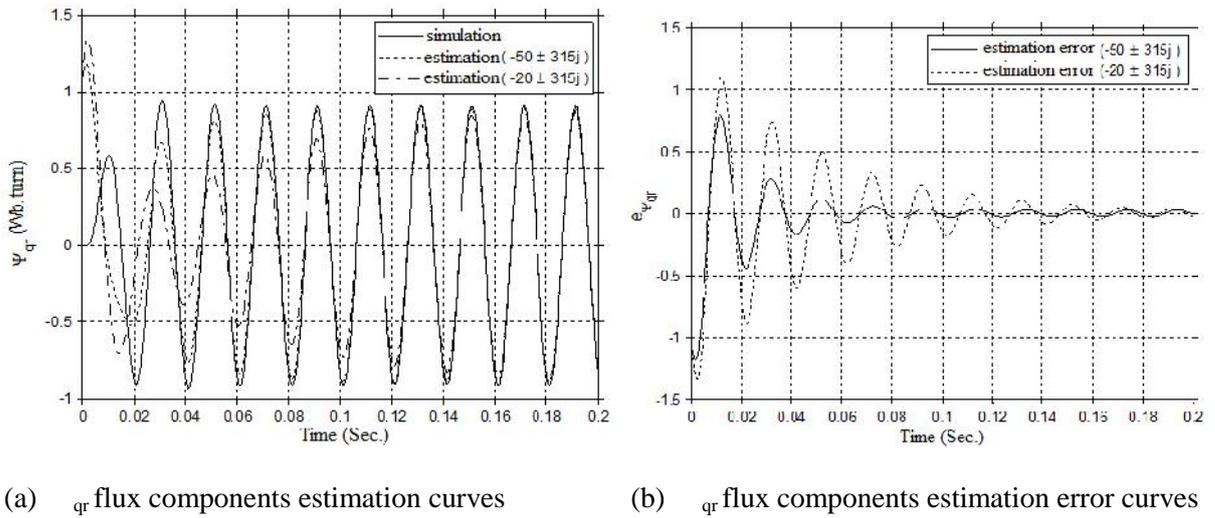
**Figure 6.** Stator current d-axes components simulation, estimation and error curves for full order observer



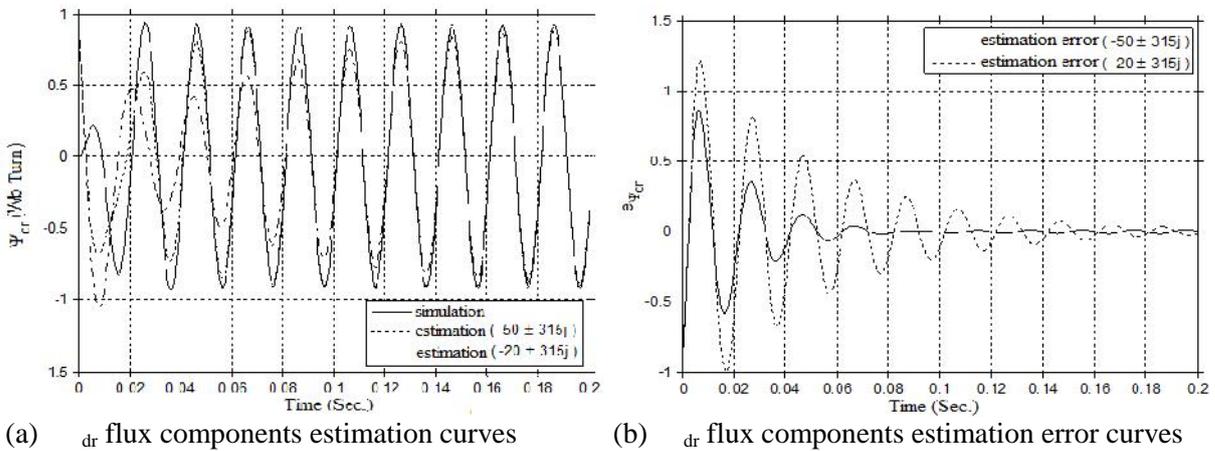
**Figure 7.** Rotor flux q-axes components simulation, estimation and error curves for full order observer



**Figure 8.** Rotor flux d-axes components simulation, estimation and error curves for full order observer



**Figure 9.** Rotor flux q-axes components simulation, estimation and error curves for reduced order observer



**Figure 10.** Rotor flux d-axes components simulation, estimation and error curves for reduced order observer

## 9. Conclusions

This paper takes a comparative look at closed loop estimators. When we investigate the simulation results, we can clearly see that the performance of full order observer is better than that of reduced order observer. Because it catches up with the simulation values more quickly. In addition, the further the real parts of observer eigenvalues go to the left-hand side on the s-domain, the better the observer performance gets. When it comes to full order observer, we can more freely select the poles (eigenvalues) we desire, whereas it requires some effort to find appropriate poles for reduced order observer. In this paper, we ignored the system and sensor noises that we need to take into account in implementation. For this purpose, adaptive observers such as Kalman Filter are recommended. Closed loop observers are sensitive to variation of the parameters in mathematical model. That's why the induction motor speed, which is a model parameter, is kept constant. But in practice, we often encounter variable-speed applications. In this case, we can obtain several gain matrices for different points on speed curve. Then, observer can operate at variable speeds by using interpolation techniques. Besides, in every sample interval, state matrix of the model must be re-computed. Today's microcontrollers or DSPs can easily handle these tasks.

## Acknowledgment

### Motor parameters :

$P=1.1$  KW , Number of Poles =2,  
 $f=50$  Hz ,  $R_r=4.3$  ,  $R_s=6.37$  ,  
 $L_s=0.26$  H,  $R_r=0.26$  H.  $L_m=0.24$  H,  
 $\omega_r=314$  rad/s,  $\omega_s=0$  rad/s.

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