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Research Article

# Modeling Hyperelastic Materials by MATLAB

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**Abstract:**

The finite element studies of hyperelastic materials always need founding a mathematical model describes the behavior of their elements. Several constitutive models differ in matching accuracy, can describe the behavior of hyperelastic material, such as Neo-Hookean, Yeoh, and Mooney-Rivlin, which are all derived from the strain energy density function.
Founding a mathematical model describing some hyperelastic material's behavior means the determination of the constitutive model's invariants, which are considered material parameters.
In this work, the two-parameter Mooney-Rivlin model was chosen to demonstrate the procedure of forming the mathematical model that describes the mechanical behavior of an incompressible hyperelastic material. Comparing with those results taken from Abaqus, obtained results were very close and exhibited a lower absolute error. This procedure can be considered as a general method to describe the hyperelastic materials by the other polynomial constitutive models.

## 1. Introduction

The finite element method used to study the material's mechanical performance is subject to various difficulties, starting with finding an appropriate mathematical model that describes element performance. As it is known, with a more accurate mathematical model, more realistic results are obtained.
Hooke's law is accurately characterized by linear elastic materials, but this law is not appropriate to describe many materials such as hyperelastic materials, which perform nonlinearly. Specific constitutive models are used in Finite element studies of hyperelastic materials, like New-Hookean, Mooney-Rivlin [1], Arruda-Boyce [2], and Ogden model. The constitutive model is chosen to respect the kind of data and the strain working range [3]. Moony-Rivlin model is a special case of stress-energy density function [4], which has two terms associated with the material's shear flexibility and compressibility.

$$W = \sum_{i+j=1}^{N} C_{ij}(I_1 - 3)^i (I_2 - 3)^j + \sum_{i=1}^{N} \frac{1}{D_i}(J_{el} - 1)^{2i}$$

Where $C_{ij}, D_i$ Material's parameters, $I_1$, $I_2$ Strain invariants and $J_{el}$ elastic volume ratio. The hyperelastic materials, in general, including the rubber-like materials, have a little compressibility compering with its shear flexibility. This little compressibility is not considerable for 2D elements or when the element is not highly restricted, but it should be considered in the 3D problems or in a highly restricted case [5]. The Mooney-Rivlin model is a linear combination of two strain invariants of the left Cauchy-Green deformation tensor [6]. It's derived by taking the first term of shear flexibility (N=1), and considering the incompressibility of hyperelastic materials ($J_{el} = 1$). It is known that the Mooney-Rivlin model gives accurate results up to 200% of strains.

$$W = C_1 (I_1 - 3) + C_2 (I_2 - 3)$$

The specimen chosen in the experiments has a quiet little thickness compared with its high and width, so it can be considered a shell that is processed as an incompressible material.

The materials parameters in the Mooney-Rivlin model have a close relationship with the second shear modulus G expressed by,

$$G = 2\,(C_1 + C_2)$$

## 2. Determining material parameters

The hyperelastic materials can be defined by determining its parameters, so the hyperelastic model characterizing the material's performance will be determined. In general, the parameters of the hyperelastic models derived from strain energy density function (i.e., C1, C2, C3…) are determined by statistical analysis utilizing one of the codding programs, such as Matlab or Python (Matlab was chosen in this paper). Depending on experimental points, an equation system will be formed. Then this system will be solved to acquire the common solutions, which are the material's parameters. The chosen common solution should dedicate an objective function, which is mostly the least-squares criterion [7].

$$\text{min: } S = \sum_{q=1}^{4} w_q \left(\sum_{i=1}^{n_q} \left((f_q)_i - (\sigma_q)_i\right)^2\right)$$

The wq, nq are weight of different experiment types and the number of experimental data for each experiment, respectively. The number of experiment types is q. Here, the number of tests is 4. $(f_q)_i$ , $(\sigma_q)_i$ notations are the predicted value by the equation and experimental value, respectively.

Notice that the model found is valid to predict the results of the tests considered before when the model's parameters were acquired, i.e., the model found considering uniaxial tension test only is invalid for a biaxial or planar test [8]. So to obtain a more general model, all kinds of tests should be considered [7]. In general, the determination of material's parameters requires implantation of four kinds of tests, uniaxial tension, biaxial tension, planer (pure shear) test, and volumetric tension (Volumetric tension test is only required when the compressibility can't be ignored in the 3D analyses or unrestrained objects, so it is neglected in this paper).

Rivlin [9] formed the differential equations to connect nominal stresses and principal stretch ration λ (λ=1+$\mathcal{E}$).

For the uniaxial tensile test:

$$T=2(1-\lambda^{-3})(\lambda\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2})$$

For the equibiaxial tensile test:

$$T=2(\lambda-\lambda^{-5})(\frac{\partial W}{\partial I_1} + \lambda^2\frac{\partial W}{\partial I_2})$$

For the planar or pure share test:

$$T=2(\lambda-\lambda^{-3})(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2})$$

The hyperelastic constitutive model must dedicate Drucker's stability postulate [10], which decides that if a material is stable or not. According to Drucker, the typical material curves must have no softening region at the end of the tension test, but if it has, it is considered as unstable material. So Drucker's stability condition is expressed as follows:

$$\sum_i \partial\sigma_i \partial\varepsilon_i \geq 0$$

Where $\partial\sigma_i$ is presented an increment in the principal Cauchy stress. Also, an increment of the corresponding strain is described by $\partial\varepsilon_i$.

The materials' parameters described above can be obtained by the following procedure explained step by step.

- Choosing a hyperelastic model depending on the objective of this model and the working strain range.
- Using Rivlin's relations between stretch ratios and nominal stresses for uniaxial, biaxial, and pure shear tests.
- Derivation of the chosen strain energy density function respects the strain invariants to obtain stress-strain relations for each test type.
- Substituting the experimental points in the relations leads to forming an equation system, whose unknowns are the material's parameters.
- Searching for common solutions by implementing statistical analyses, using the least square criterion by Matlab or Python.
- Investigating Ducker's stability conditions.

## 3. The validation of the procedure

To ensure that the procedure and written code were implemented correctly, an example of a rubber (incompressible material) with known material parameters was chosen [5].

The experimental results corresponding to the uniaxial tension test, the biaxial tension test, and the planer test are shown in tables 1, 2, and 3. The comparison between Abaqus and Matlab code results is shown in Figures 1, 2, 3. The comparison showed high matching quality for all test kinds. Table 4 shows the Similarity between the material's parameters acquired by Abaqus and Matlab.

***Table 1**. Uniaxial tension test*

| Stress (Pa) | Strain |
|---|---|
| 0.054E6 | 0.0380 |
| 0.152E6 | 0.1338 |
| 0.254E6 | 0.2210 |
| 0.362E6 | 0.3450 |
| 0.459E6 | 0.4600 |
| 0.583E6 | 0.6242 |
| 0.656E6 | 0.8510 |
| 0.730E6 | 1.4268 |

***Table 2.** Equibiaxial tension test*

| Stress (Pa) | Strain |
|---|---|
| 0.089E6 | 0.0200 |
| 0.255E6 | 0.1400 |
| 0.503E6 | 0.4200 |
| 0.958E6 | 1.4900 |
| 1.703E6 | 2.7500 |
| 2.413E6 | 3.4500 |

***Table 3.** Planer tension test*

| Stress (Pa) | Strain |
|---|---|
| 0.055E6 | 0.0690 |
| 0.324E6 | 0.2828 |
| 0.758E6 | 1.3862 |
| 1.269E6 | 3.0345 |
| 1.779E6 | 4.0621 |

***Table 4.** Material's parameters*

| | Abaqus | Matlab |
|---|---|---|
| C1 | 176050 | 173740 |
| C2 | 4330 | 4590 |
| Absolute error | 1,1757e+11 | 1,1090e+11 |

## Conclusion:

The mechanical performance of a hyperelastic material was modeled by Matlab, considering three

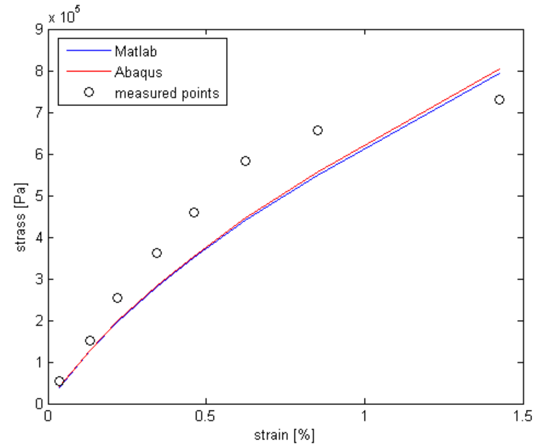


***Figure 1.** Uniaxial tension test*

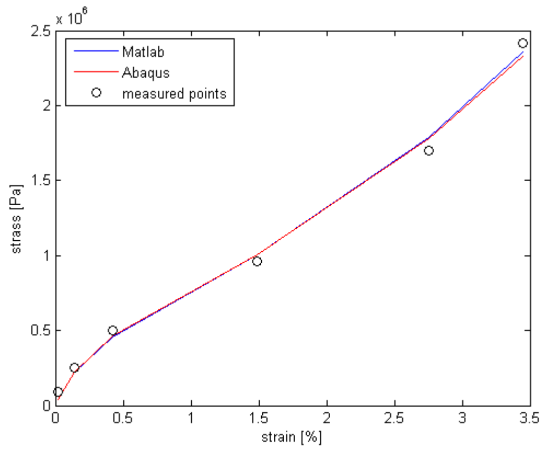


***Figure 2.** Equibiaxial tension test*

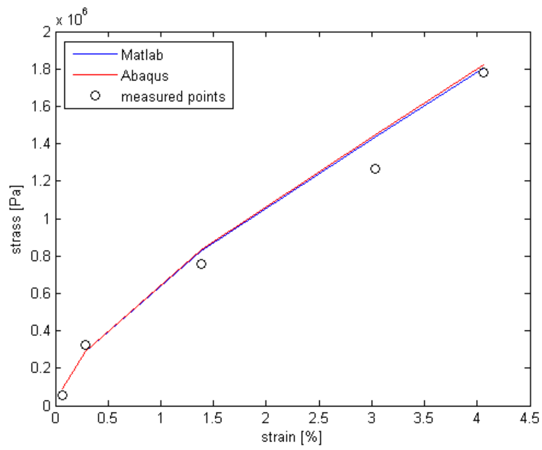


***Figure 3.** Planer shear test*

kinds of tests uniaxial, equibiaxial, and planer tension tests. The results predicted by Matlab code well matched both of the experimental points, and those results were taken from Abaqus, so the procedure used produced a mathematical model describing the material's performance successfully. The results obtained by the Matlab code had a less absolute error.

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