On Some Hyperideals in Ordered Semihypergroups

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Abstract — In this paper, we study ordered hyperideals in ordered semihypergroups. Also, we study \((m, n)\)-regular ordered semihypergroups in terms of ordered \((m, n)\)-hyperideals. Furthermore, we obtain some ideal theoretic results in ordered semihypergroups.

Keywords — Ordered semihypergroup, regular ordered semihypergroup, ordered bi-hyperideal, ordered \((m, n)\)-hyperideal

1. Introduction and Basic Definitions

The concept of the hypergroup introduced by the French Mathematician Marty at the 8th Congress of Scandinavian Mathematicians [1]. The concept of a semihypergroup is a generalization of the concept of a semigroup. Algebraic hyperstructures are a standard generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Many authors studied different aspects of semihypergroups, for instance, Davvaz [2], De Salvo et al. [3], Fasino and Freni [4], Gutan [5]. The monograph on application of hyperstructures to various area of study has been written by Corsini and Leoreanu [6]. Heideri and Davvaz studied ordered hyperstructures [7]. For semihypergroups, we refer [2, 8, 9]. Hila et al. studied quasi-hyperideals of ordered semihypergroups [10]. Corsini also studied hypergroup theory [11], [12]. Changphas and Davvaz [13] studied properties of hyperideals in ordered semihypergroups. Most recently, Basar et al. [14–16] investigated different types of hyperideals in ordered hypersemigroups, ordered LA-Γ-semigroups and LA-Γ-semihypergroups.

Let \(H\) be a nonempty set, then the mapping \(\circ : H \times H \rightarrow H\) is called hyperoperation or join operation on \(H\), where \(P^*(H) = P(H) \setminus \{0\}\) is the set of all nonempty subsets of \(H\). Let \(A\) and \(B\) be two nonempty sets. Then, a hypergroupoid \((S, \circ)\) is called a semihypergroups if for every \(x, y, z \in S\),

\[
x \circ (y \circ z) = (x \circ y) \circ z
\]

i.e.,

\[
\bigcup_{u \in yz} x \circ u = \bigcup_{v \in xy} v \circ z
\]

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A semihypergroup \((S, \circ)\) together with a partial order “\( \leq \)” on \(S\) that is compatible with semihypergroup operation such that for all \(x, y, z \in S\), we have

\[
x \leq y \Rightarrow z \circ x \leq z \circ y
\]

and

\[
x \circ z \leq y \circ z
\]
is called an ordered semihypergroup. For subsets \(A, B\) of an ordered semihypergroup \(S\), the product set \(A \circ B\) of the pair \((A, B)\) relative to \(S\) is defined as below:

\[
A \circ B = \{ a \circ b : a \in A, b \in B \}
\]

and for \(A \subseteq S\), the product set \(A \circ A\) relative to \(S\) is defined as \(A^2 = A \circ A\). For \(M \subseteq S\), \((M) = \{ s \in S \mid s \leq m \text{ for some } m \in M \}\). Also, we write \((s]\) instead of \((\{s\}]\) for \(s \in S\). Let \(A \subseteq S\). Then for a non-negative integer \(m\), the power of \(A\) is defined by \(A^m = A \circ A \circ A \circ \cdots\), where \(A\) occurs \(m\) times. Note that the power vanishes if \(m = 0\). So, \(A^0 \circ S = S \circ A^0\). In what follows we denote ordered semihypergroup \((S, \leq)\) by \(S\) unless otherwise specified.

Suppose \(S\) is an ordered semihypergroup and \(I\) is a nonempty subset of \(S\). Then, \(I\) is called an ordered right (resp. left) hyperideal of \(S\) if

\[
\begin{align*}
(i) & \quad I \circ S \subseteq I (\text{resp. } S \circ I \subseteq I) \\
(ii) & \quad a \in I, b \leq a \text{ for } b \in S \Rightarrow b \in I
\end{align*}
\]

**Definition 1.1.** Suppose \(B\) is a sub-semihypergroup (resp. nonempty subset) of an ordered semihypergroup \(S\). Then \(B\) is called an (resp. generalized) \((m, n)\)-hyperideal of \(S\) if

\[
(i) \quad B^m \circ S \circ B^n \subseteq B,
\]

and for (ii) \(b \in B, s \in S, s \leq b \Rightarrow s \in B\).

Note that in the above Definition 1.1, if we set \(m = n = 1\), then \(B\) is called a (generalized) bi-hyperideal of \(S\).

**Definition 1.2.** Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \(m, n\) are nonnegative integers. Then \(S\) is called \((m, n)\)-regular if for any \(s \in S\), there exists \(x \in S\) such that \(s \leq s^m \circ x \circ s^n\). Equivalently: \((S, \circ, \leq)\) is \((m, n)\)-regular if \(s \in (s^m \circ S \circ s^n)\) for all \(s \in S\).

**2. Preliminary**

We begin with the following:

**Lemma 2.1.** Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \(s \in S\). Let \(m, n\) be non-negative integers. Then, the intersection of all ordered (generalized) \((m, n)\)-hyperideals of \(S\) containing \(s\), denoted by \([s]_{m,n}\), is an ordered (generalized) \((m, n)\)-hyperideal of \(S\) containing \(s\).

**Proof.** Let \(\{A_i : i \in I\}\) be the set of all ordered (generalized) \((m, n)\)-hyperideals of \(S\) containing \(s\). Obviously, \(\bigcap_{i \in I} A_i\) is a sub-semihypergroup of \(S\) containing \(s\). Let \(j \in I\). As, \(\bigcap_{i \in I} A_i \subseteq A_j\), we have

\[
\left( \bigcap_{i \in I} A_i \right)^m \circ S \circ \left( \bigcap_{i \in I} A_i \right)^n \subseteq A_j^m \circ S \circ A_j^n \subseteq A_j
\]

Therefore, \(\left( \bigcap_{i \in I} A_i \right)^m \circ S \circ \left( \bigcap_{i \in I} A_i \right)^n \subseteq \bigcap_{i \in I} A_i\) as \(\bigcap_{i \in I} A_i\) is a sub-semihypergroup of \(S\) containing \(s\). Let \(a \in \bigcap_{i \in I} A_i\) and \(b \in S\) so that \(b \leq a\). Therefore, \(b \in \bigcap_{i \in I} A_i\). Hence, \(\bigcap_{i \in I} A_i\) is an ordered (generalized) \((m, n)\)-hyperideal of \(S\) containing \(s\).

**Theorem 2.2.** Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \(s \in S\). Then, we have the following:

\[
(i) \quad [s]_{m,n} = (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)
\]

for any positive integers \(m, n\).
(ii) \([s]_{m,0} = (\bigcup_{i=1}^{m} s^i \cup s^m \circ S)\) for any positive integer \(m\)

(iii) \([s]_{0,n} = (\bigcup_{i=1}^{n} s^i \cup s^n)\) for any positive integer \(n\)

**Proof.** (i) \((\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\) \(\neq \emptyset\). Let \(a, b \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\) be such that \(a \leq x\) and \(b \leq y\) for some \(x, y \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\). If \(x, y \in s^m \circ S \circ s^n\) or \(x \in s^m \circ S \circ s^n\), \(y \in s^m \circ S \circ s^n\) or \(x \in s^m \circ S \circ s^n\) then \(x \circ y \subseteq s^m \circ S \circ s^n\), and therefore,

\[
x \circ y \subseteq \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n
\]

It follows that \(a \circ b \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\). Let \(x, y \in (\bigcup_{i=1}^{m+n} s^i)\). Then \(x = s^p\), \(y = s^q\) for some \(1 \leq p, q \leq m + n\).

Now two cases arise: If \(1 \leq p + q \leq m + n\), then \(x \circ y \subseteq (\bigcup_{i=1}^{m+n} s^i)\).

If \(m + n < p + q\), then \(x \circ y \subseteq s^m \circ S \circ s^n\). So, \(x \circ y \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\). This implies that \((\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\) is a sub-semihypergroup of \(S\). Moreover, we have

\[
(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m+n} = (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S) \circ S
\]

\[
\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S) \circ S
\]

\[
\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-1} \circ (s \circ S)
\]

\[
= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S) \circ (s \circ S)
\]

\[
\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S) \circ (s \circ S)
\]

\[
\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^{m-2} \circ (s^2 \circ S)
\]

\[
\subseteq (s^m \circ S)
\]

In a similar fashion, we have

\[
S \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n) \subseteq (S \circ s^n)
\]

Therefore,

\[
(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)^{m+n} \circ S \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n) \subseteq (s^m \circ S \circ s^n)
\]

\[
\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)
\]

So, \((\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\) is an \((m, n)\)-hyperideal of \(S\) containing \(s\); hence, \([s]_{m,n} \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\).

For the reverse inclusion, suppose \(a \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\) is such that \(a \leq t\) for some \(t \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\). If \(t = s^j\) for some \(1 \leq j \leq m + n\), then \(t \in [s]_{m,n}\), therefore, \(a \in [s]_{m,n}\). If \(t \in s^m \circ S \circ s^n\), then \(t \in [s]_{m,n}\); hence, \(a \in [s]_{m,n}\). This implies that \((\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n) \subseteq [s]_{m,n}\). Hence, \([s]_{m,n} = (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)\).

(ii) and (iii) can be proved in a similar fashion.
Lemma 2.3. Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \(s \in S\). Suppose \(m, n\) are positive integers. Then, we have the following:

(i) \(((s_{m,0})^m \circ S \subseteq (s^m \circ S)\)

(ii) \(S \circ ([s_{0,n}]^n \subseteq (S \circ s^n)\)

(iii) \(((s_{m,n})^m \circ S \circ ([s_{m,n}]^n \subseteq (s^m \circ S \circ s^n)\)

Proof. (i) Using Theorem 2.2, we have

\[
([s_{m,0}]^m \circ S = \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S)^m \circ S
\]

\[
= \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ S
\]

\[
\subseteq \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ \bigcup_{i=1}^{m+n} s^i \circ S \circ s^m \circ S \circ S
\]

\[
\subseteq \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ \bigcup_{i=1}^{m+n} s^i \circ S \circ s^m \circ S
\]

\[
\subseteq (s^m \circ S)
\]

Hence, \(((s_{m,0})^m \circ S \subseteq (s^m \circ S)\). (ii) can be proved similarly as (i).

(iii) Applying Theorem 2.2, we have

\[
([s_{m,n}]^m \circ S = \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n)^m \circ S
\]

\[
= \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n \circ \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n \circ S
\]

\[
\subseteq \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n \circ \bigcup_{i=1}^{m+n} s^i \circ S \circ s^m \circ S \circ s^n \circ S
\]

\[
= \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n \circ \bigcup_{i=1}^{m+n} s^i \circ S \circ s^m \circ S \circ s^n \circ S
\]

\[
\subseteq (s^m \circ S)
\]

Therefore, \(((s_{m,n})^m \circ S \subseteq (s^m \circ S)\). In a similar way, \(S \circ ([s_{m,n}]^n \subseteq (S \circ s^n)\). Therefore,

\[
([s_{m,n}]^m \circ S \circ ([s_{m,n}]^n \subseteq (s^m \circ S) \circ ([s_{m,n}]^n
\]

\[
\subseteq (s^m \circ S) \circ ([s_{m,n}]^n
\]

\[
\subseteq (s^m \circ (S \circ ([s_{m,n}]^n))
\]

\[
\subseteq (s^m \circ (S \circ s^n))
\]

\[
\subseteq (s^m \circ S \circ s^n)
\]

Hence, (iii) holds.

Theorem 2.4. Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \(m, n\) are positive integers. Let \(R_{(m,0)}\) and \(L_{(0,n)}\) be the set of all ordered \((m, 0)\)-hyperideals and the set of all ordered \((0, n)\)-hyperideals of \(S\), respectively. Then:

(i) \(S\) is \((m, 0)\)-regular if and only if for all \(R \in R_{(m,0)}\), \(R = (R^m \circ S)\)
Consider the following four cases:

**Case (i):** Suppose \( S \) is \((m, 0)\)-regular. Then,
\[
\forall s \in S, s \in (s^m \circ S).
\]  \hspace{1cm} (1)

Suppose \( R \in R_{(m, 0)} \). As, \( R^m \circ S \subseteq R \) and \( R = (R) \), we have \((R^m \circ S) \subseteq R \). If \( s \in R \), by (1), we obtain \( s \in (s^m \circ S) \subseteq (R^m \circ S) \), therefore, \( R \subseteq (R^m \circ S) \). So, \((R^m \circ S) = R \).

Conversely, suppose
\[
\forall R \in R_{(m, 0)}, R = (R^m \circ S)
\]  \hspace{1cm} (2)

Suppose \( s \in S \). Therefore, \([s]_{m, 0} \in R_{(m, 0)} \). By (2), we obtain
\[
[s]_{m, 0} = (([s]_{m, 0})^m \circ S]
\]
Applying Lemma 2.3, we obtain
\[
[s]_{m, 0} \subseteq (s^m \circ S)
\]
Therefore, \( s \in (s^m \circ S) \). Hence, \( S \) is \((m, 0)\)-regular.

**Case (ii):** It can be proved analogously.

**Theorem 2.5.** Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \( m, n \) are non-negative integers. Suppose \( A_{(m, n)} \) is the set of all ordered \((m, n)\)-hyperideals of \( S \). Then,
\[
S \text{ is } (m, n) \text{-regular } \iff \forall A \in A_{(m, n)}, A = (A^m \circ S \circ A^n)
\]  \hspace{1cm} (3)

**Proof.** Consider the following four conditions:

Case (i): \( m = 0 \) and \( n = 0 \). Then (3) implies \( S \) is \((0, 0)\)-regular \( \iff \forall A \in A_{(0, 0)}, A = S \) because \( A_{(0, 0)} = \{S\} \) and \( S \) is \((0, 0)\)-regular.

Case (ii): \( m = 0 \) and \( n \neq 0 \). Therefore, (3) implies \( S \) is \((0, n)\)-regular \( \iff \forall A \in A_{(0, n)}, A = (S \circ A^n) \). This follows by Theorem 2.4(ii).

Case (iii): \( m \neq 0 \) and \( n = 0 \). This can be proved applying Theorem 2.4(i).

Case (iv): \( m \neq 0 \) and \( n \neq 0 \). Suppose \( S \) is \((m, n)\)-regular. Therefore,
\[
\forall s \in S, s \in (s^m \circ S \circ s^n)
\]  \hspace{1cm} (4)

Let \( A \in A_{(m, n)} \). As \( A^m \circ S \circ A^n \subseteq A \) and \( A = (A) \), we obtain \((A^m \circ S \circ A^n) \subseteq A \). Suppose \( s \in A \). Applying (4), \( s \in (s^m \circ S \circ s^n) \subseteq (A^m \circ S \circ A^n) \). Therefore, \( A \subseteq (A^m \circ S \circ A^n) \). Hence, \( A = (A^m \circ S \circ A^n) \).

Conversely, suppose \( A = (A^m \circ S \circ A^n) \) for all \( A \in A_{(m, n)} \). Suppose \( s \in S \). As \([s]_{m, n} \in A_{(m, n)} \), we have
\[
[s]_{m, n} = (([s]_{m, n})^m \circ S \circ ([s]_{m, n})^n)
\]
Applying Lemma 2.3(iii), we obtain \([s]_{m, n} \subseteq (s^m \circ S \circ s^n) \), therefore, \( s \in (s^m \circ S \circ s^n) \). Hence, \( S \) is \((m, n)\)-regular.

**Theorem 2.6.** Suppose \((S, \circ, \leq)\) is an ordered semihypergroup and \( m, n \) are nonnegative integers. Suppose \( R_{(m, 0)} \) and \( L_{(0, n)} \) is the set of all \((m, 0)\)-hyperideals and \((0, n)\)-hyperideals of \( S \), respectively. Then,
\[
S \text{ is } (m, n) \text{-regular ordered semihypergroup } \iff \forall R \in R_{(m, 0)}, \forall L \in L_{(0, n)}, R \cap L = (R^m \circ S \cap R \circ L^n)
\]  \hspace{1cm} (5)

**Proof.** Consider the following four cases:

Case (i): \( m = 0 \) and \( n = 0 \). Therefore, (5) implies \( S \) is \((0, 0)\)-regular \( \iff \forall R \in L_{(0, 0)}, \forall L \in L_{(0, 0)}, R \cap L = (L \cap R) \) because \( L_{(0, 0)} = L_{(0, 0)} = \{S\} \) and \( S \) is \((0, 0)\)-regular.

Case (ii): \( m = 0 \) and \( n \neq 0 \). Therefore, (5) implies \( S \) is \((0, n)\)-regular \( \iff \forall R \in R_{(0, n)}, \forall L \in L_{(0, n)}, R \cap L = (L \cap R \circ L^n) \). Suppose \( S \) is \((0, n)\)-regular. Suppose \( R \in R_{(0, n)} \) and \( L \in L_{(0, n)} \). By Theorem 2.4(ii), \( L = (S \circ L^n) \). As \( R \in R_{(0, n)} \), we have \( R = S \), therefore, \( R \cap L = L \). Therefore,
\[
(L \cap R \circ L^n) = (L \cap S \circ L^n) = ((S \circ L^n) \cap S \circ L^n) = (S \circ L^n) = L \cap L
\]
Conversely, suppose
\[ \forall R \in \mathcal{R}_{(0,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L = (L \cap R \circ L^n). \]  
(6)

If \( R \in \mathcal{R}_{(0,0)} \), then \( R = S \). If \( L \in \mathcal{L}_{(0,n)} \), \( S \circ L^n \subseteq L \) and \( L = (L \circ S). \) Therefore, (6) implies
\[ \forall L \in \mathcal{L}_{(0,n)}, L = (S \circ L^n] \]
Applying Theorem 2.4(ii), \( S \) is \((0,n)\)-regular.

Case (iii): \( m \neq 0 \) and \( n = 0 \). This can be proved as before.

Case (iv): \( m \neq 0 \) and \( n \neq 0 \). Suppose that \( S \) is \((m,n)\)-regular. Suppose \( R \in \mathcal{R}_{(m,0)} \) and \( L \in \mathcal{L}_{(0,n)} \).

To prove that \( R \cap L \subseteq (R^m \circ L] \cap (R \circ L^n] \), suppose \( s \in R \cap L \). We have
\[ s \in (s^m \circ S \circ s^n] \subseteq (s^m \circ L] \subseteq (R^m \circ L] \]
and
\[ s \in (s^m \circ S \circ s^n] \subseteq (R \circ s^n] \subseteq (R \circ L^n] \]
Hence, \( R \cap L \subseteq (R^m \circ L] \cap (R \circ L^n] \). As
\[ (R^m \circ L] \subseteq (R^m \circ S] \subseteq (R] = R \]
and
\[ (R \circ L^n] \subseteq (S \circ L^n] \subseteq (L] = L \]
This implies that \( (R^m \circ L] \cap (R \circ L^n] \subseteq R \cap L \), therefore, \( R \cap L = (R^m \circ L] \cap (R \circ L^n] \).

Conversely, suppose
\[ \forall R \in \mathcal{R}_{(m,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L = (R^m \circ L \cap R \circ L^n)] \]
(7)
Suppose \( R = [s]_{m,0} \) and \( L = S \). Applying (7), we obtain \([s]_{m,0} \subseteq (([s]_{m,0})^m \circ S]. \) Applying Lemma 2.3, we obtain
\[ [s]_{m,0} \subseteq (s^m \circ S] \]
(8)
In a similar fashion, we obtain
\[ [s]_{0,n} \subseteq (S \circ s^n] \]
(9)
As \( R^m \subseteq R \) and \( L^n \subseteq L \), by (7), we have
\[ \forall R \in \mathcal{R}_{(m,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L \subseteq (R \circ L] \]
As \((s^m \circ S] \in \mathcal{R}_{(m,0)} \) and \((S \circ s^n] \in \mathcal{L}_{(0,n)}, \) we obtain
\[ (s^m \circ S] \cap (S \circ s^n] \subseteq ((s^m \circ S] \circ (S \circ s^n]) \subseteq (s^m \circ S \circ s^n] \]
Applying (8) and (9), we obtain
\[ [s]_{m,0} \cap [s]_{0,n} \subseteq (s^m \circ S \circ s^n] \]
Hence, \( S \) is \((m,n)\)-regular.

3. Conclusion

In this article, we investigated ordered hyperideals in ordered semihypergroups. Also, we studied \((m,n)\)-regular ordered semihypergroups in terms of ordered \((m,n)\)-hyperideals. Moreover, we characterized ordered semihypergroups by some results based on ideal theory.

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