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SOLITON SOLUTIONS OF GURSEY MODEL WITH BICHROMATIC FORCE

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ABSTRACT. Gursey proposed a spinor field equation which is similar to Heisenberg's nonlinear generalization of Dirac's equation. This equation is the first nonlinear conformal invariant wave equation. In this paper, we investigate the soliton solutions in Gursey wave equation held in a tilted bichromatic force by constructing their Poincaré sections in phase space depending on the system parameters.

1. INTRODUCTION

Gursey proposed a spinor field equation after the successful interpretation of electron and positron by Dirac's nonlinear spinor field wave equation. Gursey Lagrangian is conformal invariant [1]. Gursey had to use a nonpolynomial form to be able to write this Lagrangian. Recently, many studies have been done on Gursey model to understand the quantum properties and dynamics. [2-5]. Also it is known that, solitons are the solutions of nonlinear wave equations and a special kind of localized waves with particle-like behaviours [6]. Soliton type solutions of Gursey model have been found by the use of soler ansatz [7,8]. In this paper, we construct the Poincaré sections of Gursey solitons against bicromatic force may provide us some insight on the subject.

2. MODEL

Gursey spinor wave equation [1] with the positive coupling constant as

$$i\partial\psi = -\frac{4}{3}g(\bar{\psi}\psi)^{\frac{1}{3}} + m\psi \tag{2.1}$$

If we consider the Soler ansatz [7] to find soliton type solutions of Gursey wave equation

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$$\psi = \begin{bmatrix} g(r)\begin{bmatrix}1\\0\end{bmatrix} \\ if(r)\begin{bmatrix}cos(\theta)\\ e^{i\phi}sin(\theta)\end{bmatrix}\end{bmatrix} e^{-i\omega t} \tag{2.2}$$

Inserting Eq. (2) into Eq. (1), one can obtain

$$i\gamma_\mu \partial_{mu}\psi = \begin{bmatrix} \bar{\omega}g(r) - f'(r) - \frac{2}{r}f(r) \\ 0 \\ -icos(\theta)(\bar{\omega}f(r) + g'(r)) \\ -isin(\theta)e^{i\theta}(\bar{\omega}f(r) + g'(r)) \end{bmatrix} e^{-i\omega t} \tag{2.3}$$

and

$$(\bar{\psi}\psi) = g^2(r) - f^2(r) \tag{2.4}$$

with

$$\begin{aligned} \bar{\psi}(\psi^*)^T\gamma^0 &= \begin{bmatrix}\begin{bmatrix} g(r) \\ 0 \\ -if(r)cos(\theta) \\ -if(r)e^{i\phi}sin(\theta)\end{bmatrix} e^{-i\omega t}\end{bmatrix}^T \\ \bar{\psi} = \begin{bmatrix} g(r) & 0 & -if(r)cos(\theta) & -if(r)e^{-i\phi}sin\theta\end{bmatrix} & e^{i\omega t}\gamma^0 \\ \bar{\psi} = \begin{bmatrix} g(r) & 0 & if(r)cos(\theta) & if(r)e^{-i\phi}sin\theta\end{bmatrix} & e^{i\omega t} \end{aligned} \tag{2.5}$$

Substituting these expressions, the differential equations system can be written as

$$(\bar{\omega} - m)g(r) - f'(r) + \frac{2}{r} + \frac{4}{3}\alpha g(r)(g^2(r) - f^2(r))^{\frac{1}{3}} = 0 \tag{2.6}$$

$$\begin{aligned} -icos(\theta)(\bar{\omega}f(r) + g'(r)) + \frac{4}{3}\alpha(g^2(r) - f^2(r))^{\frac{1}{3}} if(r)cos(\theta) \\ -mf(r)cos(\theta) = 0 \end{aligned} \tag{2.7}$$

$$\begin{aligned} -isin(\theta)e^{i\phi}(\bar{\omega}f(r) + g'(r)) + \frac{4}{3}\alpha(g^2(r) - f^2(r))^{\frac{1}{3}} if(r)e^{i\phi}sin(\theta) \\ -mf(r)e^{i\phi}sin(\theta) = 0 \end{aligned} \tag{2.8}$$

By the transformations given in Ref. [7] with $r = \frac{\rho}{m+\bar{\omega}}$ and $\nu = \frac{m-\bar{\omega}}{m+\bar{\omega}}$, we achieve the dimensionless form of the nonlinear differential equation system [8] as

$$\frac{dF(\rho)}{d\rho} + \frac{2}{\rho}F(\rho) + \nu G(\rho) - (G^2(\rho) - F^2(\rho))^{\frac{1}{3}} G(\rho) = 0 \tag{2.9}$$

$$\frac{dG(\rho)}{d\rho} + F(\rho) - (G^2(\rho) - F^2(\rho))^{\frac{1}{3}} F(\rho) = 0 \tag{2.10}$$

If we define the externally forced system under the bichromatic force

$$\frac{dF(\rho)}{d\rho} + \frac{2}{\rho}F(\rho) + \nu G(\rho) - (G^2(\rho) - F^2(\rho))^{\frac{1}{3}} G(\rho) = 0 \tag{2.11}$$

$$\frac{dG(\rho)}{d\rho} + F(\rho) - (G^2(\rho) - F^2(\rho))^{\frac{1}{3}} F(\rho) \\ = A_1 cos^2(w_1 H(\rho)) + A_2 cos^2(w_2 H(\rho)) \tag{2.12}$$

$$H(\rho)) = \Omega \tag{2.13}$$

with a $H(\rho)$ function of ρ adding an extra dimension for numerical calculations.

3. NUMERICAL RESULTS

In this paper, we set $m = 1.00007 \times 10^{-9}$, $\bar{\omega} = 9.7 \times 10^{-10}$. Gursey solitons exhibits stable behaviours in phase space without external force as seen in the Figure 1.

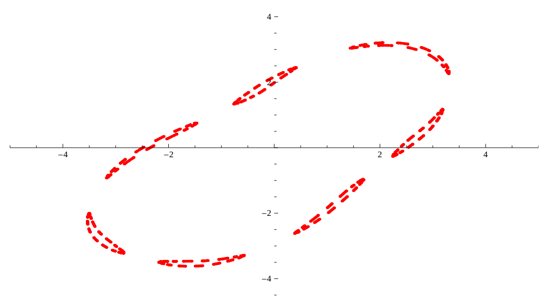

FIGURE 1. Poincaré section diagram without force

In Figure 2, the Poincaré sections are given for $w_1 = 2\pi$, $w_2 = 3\pi$ and the initial conditions are $F(0) = 0.3725$, $G(0) = -0.1652$. The obtained Poincaré sections show the vanishing of the stability of Gursey solitons depending on the parameter values and they exhibit chaotic bahaviours under the bicromatic force.

4. CONCLUSION

In summary, we use the techniques from the viewpoint of nonlinear dynamics in this paper to get more information on spinor type Gursey solitons with bichromatic force. From the obtained numerical results, we can say that the system shows chaotic behaviours depending on the parameter values. As we increase the forcing, the system exhibits more chaotic region. It is known that the study of chaos in soliton physics based on chaos criterion is required interest. We can positively contribute to the work of researchers on this subject as obtaining more results.

5. ACKNOWLEDGMENT

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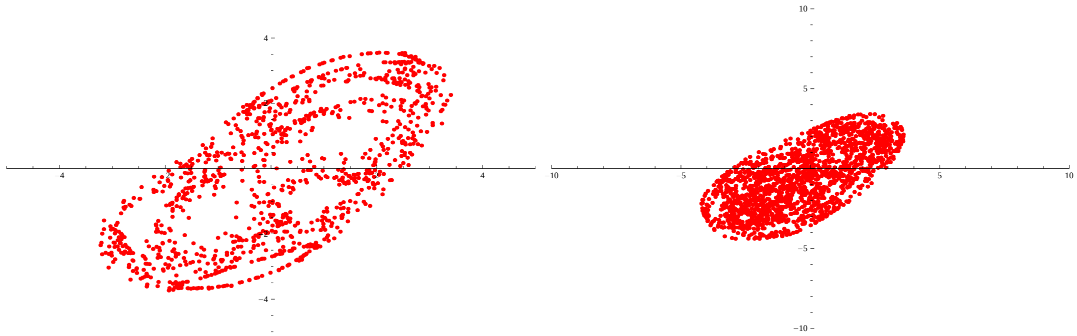

FIGURE 2. Poincaré section diagrams of forcing system for (a) $A_1 = 0.1$, $A_2 = 0.2$ (b) $A_1 = 0.5$, $A_2 = 0.7$

REFERENCES

[1] F. Gursey, On a conform-invariant spinor wave equation, Nuovo Cimento, 1956 3, 988.

[2] F. Aydogmus, Dynamics of Excited Instantons in the System of Forced Gursey Nonlinear Differential Equations, Journal of Experimental and Theoretical Physics, 2015, 120, 2, 210-216.

[3] F. Aydogmus and E. Tosyali, Common Behaviours of Spinor Type Instantons in 2-D Thirring and 4-D Gursey Fermionic Models, Advances in High Energy Physics, 148375 (1-11), (2014).

[4] F. Aydogmus, Chaos in a 4D dissipative nonlinear fermionic model, J. Mod. Phys. C, 2015, 26(7), 1550083.

[5] F. Aydogmus, Unstable Behaviors of Classical Solutions in Spinor-type Conformal Invariant Fermionic Models, Journal of Experimental and Theoretical Physics, 2017, 125, 5, 719-727.

[6] M. Dunajski, Solitons, Instantons, and Twistors, Oxford University Press, New York, 2010.

[7] M. Soler, Ciassical, Stable, Nonlinear Spinor Field with Positive Rest Energy, Phys. Rev. D1, 1970, 2766.

[8] S. Sagaltici, M.S. Thesis, Istanbul University, Institute of Science, Physics Department, Istanbul, Turkey, 2004.

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