İstatistikçiler Dergisi: İstatistik & Aktüerya

Journal of Statisticians: Statistics and Actuarial Sciences

IDIA 12, 2019, 2, 58-71

Geliş/Received:01.08.2019, Kabul/Accepted: 08.10.2019

www.istatistikciler.org

Araştırma Makalesi / Research Article

A class of estimators in median ranked set sampling

Nursel Koyuncu
Hacettepe University, Department of Statistics
Beytepe, Ankara, TURKEY
nkoyuncu@hacettepe.edu.tr
0000-0003-1065-3411

Abstract

In this paper we have proposed a general class of estimators under median ranked set sampling (MRSS) both one auxiliary and multi-auxiliary variables are known. The mean square error (MSE) and bias formulas are derived by theoretically. We have conducted a simulation study to see the performance of proposed estimators. Also it is shown that the proposed estimators are always more efficient than existing estimators by theoretically. From simulation study we can say that suggested class of estimator performs better than Al-Omari [1] estimator and conclude that regression type estimator gives always more efficient results than Al-Omari [1] estimator.

Keywords: Auxiliary information, Efficiency, Median ranked set sampling, Monte Carlo simulation, Ratio estimator.

Öz

Bu çalışmada, medyan sıralı küme örneklemesi (MSKÖ) altında hem bir yardımcı değişken hem de birden çok yardımcı değişken bilinmesi durumunda genel bir tahmin edici sınıfı önerilmiştir. Hata kareler ortalaması (HKO) ve yanlılık formülleri teorik olarak elde edilmiştir. Önerilen tahmin edicilerin performansını görmek için bir benzetim çalışması yapılmıştır. Ayrıca önerilen tahmin edicilerin mevcut tahmin edicilerden her zaman daha etkin olduğu teorik olarak gösterilmiştir. Benzetim çalışmasından, önerilen tahmin edici sınıfının Al-Omari [1] tahmin edicisinden daha iyi performans gösterdiği söylenebilir ve regresyon tipi tahmin edicinin, Al-Omari[1] tahmin edicisinden her zaman daha etkin sonuçlar verdiği sonucuna varılabilir.

Anahtar sözcükler: *Yardımcı değişken, Etkinlik, Medyan sıralı küme örneklemesi, Monte Carlo benzetimi, Oransal Tahmin.*

1. Introduction

Ranked set sampling (RSS) is more efficient sampling plan than simple random sampling (SRS) when the measurements are difficult or expensive to obtain but ranking of units is relatively easy and cheap. Many authors developed and modified this sampling plan to estimate population parameters such as Al-Saleh and Al-Omari [2], Jemain and Al-Omari [3], Jemain et al. [4], Ozturk and Jafari Jozani [5], Amiri et al. [6] etc. Recently Al-Omari and Al-Nasser [7], used robust extreme ranked set sampling to estimate median. Al-Omari and Raqab [8] studied truncation-based ranked set samples to estimate population mean and

median. Haq et al. [9-10] introduced partial ranked set sampling and mixed ranked set sampling designs respectively. Some authors prefered various ranked set sampling designs to estimate parameters of distributions. Bhoj and Kushary [11-12] proposed a ranked set sampling procedure with unequal samples and unequal replications to estimate the population mean and have extended their study for skewed distributions. Samuh and Qtait [13] used median ranked set sampling to estimate parameters of exponentiated exponential distribution. Omar and Ibrahim [14] used extreme ranked set sampling to estimate shape and scale parameters of the Pareto distribution.
In sampling literature another way to get more efficient estimates is that using ratio estimator when the information of auxiliary variable is available. Moving this point, many authors proposed ratio estimators under different sampling desings (see [15-17]). But in median ranked set sampling (MRSS) there are a few studies related with ratio estimators. Al-Omari [1] first defined ratio estimators in MRSS and Koyuncu [18-19] proposed new ratio estimators in MRSS and extended ratio estimators to extreme ranked set and double robust extreme ranked set sampling.

In this study we have proposed more general class of estimators in MRSS when one auxiliary is known. Also we have extended our results for the case: multi-auxiliary variables are known. SRS and MRSS are defined in Section2 and Section3 respectively. In Section4, we have suggested a class of estimators in MRSS. Simulation study is given in Section5 and we have summarized our results in this section.

2. Simple Random Sampling

We have reviewed some well known estimators in SRS. Let $\mathrm{U}=(\mathrm{u}_1,\mathrm{u}_2,\ldots,\mathrm{u}_\mathrm{N})$ denote a finite population of size N. Let Y be the study; X and Z be two auxiliary variables associated with each unit $\mathrm{u}_\mathrm{j}(\mathrm{j}=1,\ldots,\mathrm{N})$ of the population. A sample of size n is drawn without replacement from the population. Let $(\mathrm{X}_1,\mathrm{Z}_1,\mathrm{Y}_1),(\mathrm{X}_2,\mathrm{Z}_2,\mathrm{Y}_2),\ldots,(\mathrm{X}_\mathrm{n},\mathrm{Z}_\mathrm{n},\mathrm{Y}_\mathrm{n})$ denote the observed values of Y, X and Z. Moreover let $\mu_y,\mu_x,\ \mu_z$, $\bar{y}_{SRS}=\frac{\sum_{i=1}^{n}Y_i}{n}$, $\bar{x}_{SRS}=\frac{\sum_{i=1}^{n}X_i}{n}$, $\bar{z}_{SRS}=\frac{\sum_{i=1}^{n}Z_i}{n}$ be the population and sample means of study and two auxiliary variables respectively.

The well known ratio estimator using SRS is defined by

$$\bar{y}_{R\_SRS}=\bar{y}_{SRS}\frac{\mu_x}{\bar{x}_{SRS}} \tag{1}$$

where $\bar{x}_{SRS}\neq 0$. If the quantiles of auxiliary variable, q_1,q_3 are known Al-Omari [1] defined ratio type estimators in SRS as

$$\bar{y}_{R1\_SRS}=\bar{y}_{SRS}\frac{\mu_x+q_1}{\bar{x}_{SRS}+q_1} \tag{2}$$

$$\bar{y}_{R2\_SRS}=\bar{y}_{SRS}\frac{\mu_x+q_3}{\bar{x}_{SRS}+q_3} \tag{3}$$

The regression-type estimator of population mean using one auxiliary variable is defined by

$$\bar{y}_{\mathrm{Re}g1\_SRS}=\bar{y}_{SRS}+b_{SRS1}\left(\mu_x-\bar{x}_{SRS}\right) \tag{4}$$

where $b_{SRS1} = \frac{s_{SRS\_yx}}{s^2_{SRS\_x}}$ denotes the estimator of regression coefficient and $s^2_{SRS\_x} = \frac{\sum_{i=1}^{n}(x_i - \bar{x}_{SRS})^2}{n-1}$,

$s_{SRS\_yx} = \frac{\sum_{i=1}^{n}(y_i - \bar{y}_{SRS})(x_i - \bar{x}_{SRS})}{n-1}$, [20]).

If two auxiliary variables are available the ratio type and regression-type estimators are, respectively, defined as

$$\bar{y}_{R3\_SRS} = \bar{y}_{SRS} \frac{\mu_x}{\bar{x}_{SRS}} \frac{\mu_z}{\bar{z}_{SRS}} \tag{5}$$

$$\bar{y}_{\mathrm{Re}g2\_SRS} = \bar{y}_{SRS} + b_{SRS1}(\mu_x - \bar{x}_{SRS}) + b_{SRS2}(\mu_z - \bar{z}_{SRS}) \tag{6}$$

where $b_{SRS2} = \frac{s_{SRS\_yz}}{s^2_{SRS\_z}}$ denotes the estimator of regression coefficient for second auxiliary variable and

$s^2_{SRS\_z} = \frac{\sum_{i=1}^{n}(z_i - \bar{z}_{SRS})^2}{n-1}$, $s_{SRS\_yz} = \frac{\sum_{i=1}^{n}(y_i - \bar{y}_{SRS})(z_i - \bar{z}_{SRS})}{n-1}$, [20]) .

3. Median Ranked Set Sampling

For the sake of brevity we follow Al-Omari [1] sampling design and notations given by

1. Randomly select n samples each of size n multivariate units from the population.

2. The units within each sample are ranked with respect to a variable of interest.

3. If n is odd, select the $\left(\frac{n+1}{2}\right)$th-smallest ranked unit auxiliary variables together with the associated study variable from each set, i.e., the median of each set. If n is even, from the first $\frac{n}{2}$ sets select the $\left(\frac{n}{2}\right)$ th ranked unit auxiliary variables together with the associated study variable and from the other $\frac{n}{2}$ sets select the $\left(\frac{n+2}{2}\right)$ th ranked unit auxiliary variables together with the associated study variables.

4. The above procedure can be repeated m times to obtain a sample of size nm units.

To extend our results to multivariate case we assume that Y is study variable and X and Z are auxiliary variable. In application to ranked the samples we prefer higly correlated auxiliary variable with study variable.

Let $\left(X_{i(1)}, Z_{i(1)}, Y_{i[1]}\right), \left(X_{i(2)}, Z_{i(2)}, Y_{i[2]}\right), \ldots, \left(X_{i(n)}, Z_{i(n)}, Y_{i[n]}\right)$ be the order statistics of $X_{i1}, X_{i2}, \ldots, X_{in}$, $Z_{i1}, Z_{i2}, \ldots, Z_{in}$ and the judgement order of $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ $(i = 1, 2, \ldots, n)$, where $(\ )$ and $[\ ]$ indicate that the ranking of X, Z is perfect and ranking of Y has errors. For odd and even sample sizes the units measured using MRSS are denoted by MRSSO and MRSSE, respectively.

For odd sample size let

$$\left(X_{1\left(\frac{n+1}{2}\right)}, Z_{1\left(\frac{n+1}{2}\right)}, Y_{1\left[\frac{n+1}{2}\right]}\right), \left(X_{2\left(\frac{n+1}{2}\right)}, Z_{2\left(\frac{n+1}{2}\right)}, Y_{2\left[\frac{n+1}{2}\right]}\right), \ldots, \left(X_{n\left(\frac{n+1}{2}\right)}, Z_{n\left(\frac{n+1}{2}\right)}, Y_{n\left[\frac{n+1}{2}\right]}\right)$$

denote the observed units by MRSSO. $\bar{x}_{MRSSO} = \frac{1}{n}\sum_{i=1}^{n} X_{i\left(\frac{n+1}{2}\right)}$, $\bar{z}_{MRSSO} = \frac{1}{n}\sum_{i=1}^{n} Z_{i\left(\frac{n+1}{2}\right)}$ and $\bar{y}_{MRSSO} = \frac{1}{n}\sum_{i=1}^{n} Y_{i\left[\frac{n+1}{2}\right]}$ be the sample mean of X, Z and Y respectively.

For even sample size let

$$\left(X_{1\left(\frac{n}{2}\right)}, Z_{1\left(\frac{n}{2}\right)}, Y_{1\left[\frac{n}{2}\right]}\right), \left(X_{2\left(\frac{n}{2}\right)}, Z_{2\left(\frac{n}{2}\right)}, Y_{2\left[\frac{n}{2}\right]}\right), \ldots, \left(X_{\frac{n}{2}\left(\frac{n}{2}\right)}, Z_{\frac{n}{2}\left(\frac{n}{2}\right)}, Y_{\frac{n}{2}\left[\frac{n}{2}\right]}\right), \left(X_{\frac{n+2}{2}\left(\frac{n+2}{2}\right)}, Z_{\frac{n+2}{2}\left(\frac{n+2}{2}\right)}, Y_{\frac{n+2}{2}\left[\frac{n+2}{2}\right]}\right)$$

$\left(X_{\frac{n+4}{2}\left(\frac{n+2}{2}\right)}, Z_{\frac{n+4}{2}\left(\frac{n+2}{2}\right)}, Y_{\frac{n+4}{2}\left[\frac{n+2}{2}\right]}\right), \ldots, \left(X_{n\left(\frac{n+2}{2}\right)}, Z_{n\left(\frac{n+2}{2}\right)}, Y_{n\left[\frac{n+2}{2}\right]}\right)$ denote the observed units by MRSSE.

$$\bar{x}_{MRSSE} = \frac{1}{n}\left(\sum_{i=1}^{\frac{n}{2}} X_{i\left(\frac{n}{2}\right)} + \sum_{i=\frac{n+2}{2}}^{n} X_{i\left(\frac{n+2}{2}\right)}\right), \quad \bar{z}_{MRSSE} = \frac{1}{n}\left(\sum_{i=1}^{\frac{n}{2}} Z_{i\left(\frac{n}{2}\right)} + \sum_{i=\frac{n+2}{2}}^{n} Z_{i\left(\frac{n+2}{2}\right)}\right)$$

and $\bar{y}_{MRSSE} = \frac{1}{n}\left(\sum_{i=1}^{\frac{n}{2}} Y_{i\left[\frac{n}{2}\right]} + \sum_{i=\frac{n+2}{2}}^{n} Y_{i\left[\frac{n+2}{2}\right]}\right)$ be the sample mean of X, Z and Y respectively. Let rewrite the sample means as $\bar{x}_{MRSS(j)}, \bar{z}_{MRSS(j)}, \bar{y}_{MRSS(j)}$ where $j = (E, O)$ denote the sample size even or odd.

In MRSS we can re-write estimators in defined in section two as given by, respectively

$$\bar{y}_{R\_MRSS} = \bar{y}_{MRSS(j)} \frac{\mu_x}{\bar{x}_{MRSS(j)}} \tag{7}$$

$$\bar{y}_{R1\_MRSS} = \bar{y}_{MRSS(j)} \frac{\mu_x + q_1}{\bar{x}_{MRSS(j)} + q_1} \tag{8}$$

$$\bar{y}_{R2\_MRSS} = \bar{y}_{MRSS(j)} \frac{\mu_x + q_3}{\bar{x}_{MRSS(j)} + q_3} \tag{9}$$

$$\bar{y}_{\mathrm{Re}g1\_MRSS} = \bar{y}_{MRSS(j)} + b_{MRSS1}\left(\mu_x - \bar{x}_{MRSS(j)}\right) \tag{10}$$

where b_{MRSS1} denotes the estimator of regression coefficient under MRSS

$$b_{MRSS1(j)} \cong \begin{cases} \dfrac{\rho_{xy\left[\frac{n+1}{2}\right]} s_{yx\left[\frac{n+1}{2}\right]}}{s^2_{x\left(\frac{n+1}{2}\right)}}, \ n \ is \ odd \\ \dfrac{\left(\rho_{xy\left[\frac{n}{2}\right]} + \rho_{xy\left[\frac{n+2}{2}\right]}\right)\left(s_{yx\left[\frac{n}{2}\right]} + s_{yx\left[\frac{n+2}{2}\right]}\right)}{\left(s^2_{x\left(\frac{n}{2}\right)} + s^2_{x\left(\frac{n+2}{2}\right)}\right)}, n \, is \, even \end{cases}$$

$$s_{xy\left[\frac{n+1}{2}\right]} = \rho_{xy\left[\frac{n+1}{2}\right]} s_{x\left(\frac{n+1}{2}\right)} s_{y\left[\frac{n+1}{2}\right]}, \quad \left(s_{xy\left[\frac{n}{2}\right]} + s_{xy\left[\frac{n+2}{2}\right]}\right) = \left(\rho_{xy\left[\frac{n}{2}\right]} + \rho_{xy\left[\frac{n+2}{2}\right]}\right)\left(s_{x\left[\frac{n}{2}\right]} + s_{x\left[\frac{n+2}{2}\right]}\right)\left(s_{y\left[\frac{n}{2}\right]} + s_{y\left[\frac{n+2}{2}\right]}\right)$$

and $\rho_{xy\left[\frac{n+1}{2}\right]}, \rho_{xy\left[\frac{n}{2}\right]}$ represent correlation coefficient between study and auxiliary variable for odd and even sample sizes.

If the two auxiliary variables are known, following estimators can be used

$$\bar{y}_{R3\_MRSS} = \bar{y}_{MRSS(j)} \frac{\mu_x}{\bar{x}_{MRSS(j)}} \frac{\mu_z}{\bar{z}_{MRSS(j)}} \tag{11}$$

$$\bar{y}_{\mathrm{Re}g2\_MRSS} = \bar{y}_{MRSS(j)} + b_{MRSS1}\left(\mu_x - \bar{x}_{MRSS(j)}\right) + b_{MRSS2}\left(\mu_z - \bar{z}_{MRSS(j)}\right) \tag{12}$$

where b_{MRSS2} denotes the estimator of regression coefficient of second auxiliary variable under MRSS

$$b_{MRSS2(j)} \cong \begin{cases} \dfrac{\rho_{zy\left[\frac{n+1}{2}\right]} s_{yz\left[\frac{n+1}{2}\right]}}{s^2_{z\left(\frac{n+1}{2}\right)}}, \ n \ is \ odd \\ \dfrac{\left(\rho_{zy\left[\frac{n}{2}\right]} + \rho_{zy\left[\frac{n+2}{2}\right]}\right)\left(s_{yz\left[\frac{n}{2}\right]} + s_{yz\left[\frac{n+2}{2}\right]}\right)}{\left(s^2_{z\left(\frac{n}{2}\right)} + s^2_{z\left(\frac{n+2}{2}\right)}\right)}, n \, is \, even \end{cases}$$

$$s_{zy\left[\frac{n+1}{2}\right]} = \rho_{zy\left[\frac{n+1}{2}\right]} s_{z\left(\frac{n+1}{2}\right)} s_{y\left[\frac{n+1}{2}\right]},$$

$$\left(s_{zy\left[\frac{n}{2}\right]} + s_{zy\left[\frac{n+2}{2}\right]}\right) = \left(\rho_{zy\left[\frac{n}{2}\right]} + \rho_{zy\left[\frac{n+2}{2}\right]}\right)\left(s_{z\left[\frac{n}{2}\right]} + s_{z\left[\frac{n+2}{2}\right]}\right)\left(s_{y\left[\frac{n}{2}\right]} + s_{y\left[\frac{n+2}{2}\right]}\right).$$

4. Suggested Classes of Estimators

Following Srivastava and *Jhajj* [21] and Koyuncu and Kadilar [22], we have defined a class of estimators of population mean using MRSS as

$$t_g = \bar{y}_{MRSS(j)} H(u) \tag{13}$$

where $u = \dfrac{\bar{x}_{MRSS(j)}}{\mu_x}$ and $H(.)$ is a function of u satisfies the following regularity conditions:

a) $H(1) = 1$

b) The first and second order partial derivatives of H with respect to u exist and are known constants at a given point $u = 1$. Note that the estimators are defined in (7)-(9) are members of t_g. Let us define following expectations under MRSS as follows

$$\varepsilon_{0(j)} = \frac{\bar{y}_{MRSS(j)} - \mu_y}{\mu_y},\ \varepsilon_{1(j)} = \frac{\bar{x}_{MRSS(j)} - \mu_x}{\mu_x},$$

such that $E\left(\varepsilon_{0(j)}\right) = E\left(\varepsilon_{1(j)}\right) = 0$ and $j = (E, O)$ denote the sample size even or odd.

If sample size n is odd we can write

$$E\left(\varepsilon^2_{1(o)}\right) = \frac{1}{n\mu_x^2}\sigma^2_{x\left(\frac{n+1}{2}\right)},\ E\left(\varepsilon^2_{0(o)}\right) = \frac{1}{n\mu_y^2}\sigma^2_{y\left(\frac{n+1}{2}\right)},\ E\left(\varepsilon_{0(o)}\varepsilon_{1(o)}\right) = \frac{1}{n\mu_y\mu_x}\sigma_{xy\left(\frac{n+1}{2}\right)}$$

If sample size n is even we can write

$$E\left(\varepsilon^2_{1(E)}\right) = \frac{1}{2n\mu_x^2}\left(\sigma^2_{x\left(\frac{n}{2}\right)} + \sigma^2_{x\left(\frac{n+2}{2}\right)}\right),\ E\left(\varepsilon^2_{0(E)}\right) = \frac{1}{2n\mu_y^2}\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right)$$

$$E\left(\varepsilon_{0(E)}\varepsilon_{1(E)}\right) = \frac{1}{2n\mu_x\mu_y}\left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right)$$

Expanding $H(u)$ in a second order Taylor's series we have

$$H(u) = H\left(1 + (u-1)\right) = H(1) + (u-1)\left.\frac{\partial H}{\partial u}\right|_{u=1} + (u-1)^2 \frac{1}{2}\left.\frac{\partial^2 H}{\partial u^2}\right|_{u=1} + \dots \tag{14}$$

Note that $|u-1| < 1$ thus the higher order terms can be neglected. Putting (14) in (13) we have

$$t_g = \bar{y}_{MRSS(j)}\left[1 + (u-1)H_1 + (u-1)^2 H_2 + \dots\right] \tag{15}$$

where $H_1 = \left.\frac{\partial H}{\partial u}\right|_{u=1}$, $H_2 = \left.\frac{1}{2}\frac{\partial^2 H}{\partial u^2}\right|_{u=1}$. Expressing (15) with ε terms and extracting μ_y from both sides we have

$$t_g - \mu_y = \mu_y \left[\varepsilon_{1(j)} H_1 + \varepsilon_{1(j)}^2 H_2 + \varepsilon_{0(j)} + \varepsilon_{0(j)}\varepsilon_{1(j)} H_1 + ... \right] \tag{16}$$

Taking expectation on both sides, the bias of class is given by

$$B\left(t_g\right) \cong \begin{cases} \frac{1}{n\mu_x}\left[\frac{\mu_y}{\mu_x}\sigma^2_{x\left(\frac{n+1}{2}\right)} H_2 + \sigma_{xy\left(\frac{n+1}{2}\right)} H_1\right] & if\ odd \\ \frac{1}{2n\mu_x}\left[\frac{\mu_y}{\mu_x}\left(\sigma^2_{x\left(\frac{n}{2}\right)} + \sigma^2_{x\left(\frac{n+2}{2}\right)}\right) H_2 + \left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right) H_1\right] & if\ even \end{cases} \tag{17}$$

Squaring both sides of (16) and taking expectation we obtain the MSE is given by

$$\left(t_g - \mu_y\right)^2 \cong \mu_y^2 \left[\varepsilon_{0(j)}^2 + \varepsilon_{1(j)}^2 H_1^2 + 2\varepsilon_{0(j)}\varepsilon_{1(j)} H_1\right]$$

$$MSE\left(t_g\right) \cong \begin{cases} \frac{1}{n}\left[\sigma^2_{y\left(\frac{n+1}{2}\right)} + \frac{\mu_y^2}{\mu_x^2}\sigma^2_{x\left(\frac{n+1}{2}\right)} H_1^2 + 2\frac{\mu_y}{\mu_x}\sigma_{xy\left(\frac{n+1}{2}\right)} H_1\right] & if\ odd \\ \frac{1}{2n}\left[\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right) + \frac{\mu_y^2}{\mu_x^2}\left(\sigma^2_{x\left(\frac{n}{2}\right)} + \sigma^2_{x\left(\frac{n+2}{2}\right)}\right) H_1^2 + 2\frac{\mu_y}{\mu_x}\left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right) H_1\right] & if\ even \end{cases} \tag{18}$$

Differentiating (18) with respect to H_1 and putting again we obtain minimum MSE as

$$MSE_{\min}\left(t_g\right) \cong \begin{cases} \frac{1}{n}\sigma^2_{y\left(\frac{n+1}{2}\right)}\left(1 - \rho^2_{xy\left(\frac{n+1}{2}\right)}\right), & n\ is\ odd \\ \frac{1}{2n}\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right)\left(1 - \left(\rho^2_{xy\left(\frac{n+1}{2}\right)} + \rho^2_{xy\left[\frac{n+2}{2}\right]}\right)\right), & n\, is\, even \end{cases} \tag{19}$$

Note that this is also MSE of $\bar{y}_{\text{Re}g1\_MRSS}$. (see Koyuncu [19] for details)

Srivastava [23] defined a wider class in simple random sampling. Secondly taking motivation from Srivastava [23] we have suggested wider class of estimators as

$$t_w = H\left(\bar{y}_{MRSS(j)}, u\right) \tag{20}$$

where $H\left(\bar{y}_{MRSS(j)}, u\right)$ is a function of $\bar{y}_{MRSS(j)}$ and u satisfy the following regularity conditions:

a) The point $\left(\bar{y}_{MRSS(j)}, u\right)$ assumes the value in a bounded, closed convex subset R_2 of two-dimensional real space containing the point $\left(\bar{Y}, 1\right)$.

b) The function $H\left(\bar{y}_{MRSS(j)},u\right)$ is continuous and bounded in R_2.

c) $H\left(\bar{Y},1\right)=\bar{Y}$ and $H_0\left(\bar{Y},1\right)=1$ which is first order partial derivative of H with respect to $\bar{y}_{MRSS}$.

d) The first and second order partial derivatives of $H\left(\bar{y}_{MRSS},u\right)$ exist and are continuous and bounded in R_2.

Expanding $H\left(\bar{y}_{MRSS(j)},u\right)$ about point of $\left(\bar{Y},1\right)$ in a second order Taylor series we have

$$t_w = H\left(\bar{y}_{MRSS(j)},u\right) = H\left[\left(\bar{Y}+\left(\bar{y}_{MRSS(j)}-\mu_y\right)\right),\left(1+\left(u-1\right)\right)\right] \tag{21}$$

$$t_w \cong \bar{y}_{MRSS(j)} + \left(u-1\right)H_1 + \left(u-1\right)^2 H_2 + \left(\bar{y}_{MRSS(j)}-\mu_y\right)\left(u-1\right)H_3 + \left(\bar{y}_{MRSS(j)}-\mu_y\right)^2 H_4$$

where

$$H_1 = \left.\frac{\partial H}{\partial u}\right|_{\bar{y}_{MRSS}=\mu_y,u=1},\ H_2 = \left.\frac{1}{2}\frac{\partial^2 H}{\partial u^2}\right|_{\bar{y}_{MRSS}=\mu_y,u=1},\ H_3 = \left.\frac{1}{2}\frac{\partial^2 H}{\partial \bar{y}_{MRSS}\partial u}\right|_{\bar{y}_{MRSS}=\mu_y,u=1},\ H_4 = \left.\frac{1}{2}\frac{\partial^2 H}{\partial \bar{y}^2}\right|_{\bar{y}_{MRSS}=\mu_y,u=1}$$

Extracting μ_y from both sides of (21) we obtain

$$t_w - \mu_y \cong \mu_y \varepsilon_{0(j)} + \varepsilon_{1(j)}H_1 + \varepsilon_{1(j)}^2 H_2 + \varepsilon_{0(j)}\varepsilon_{1(j)}H_3 + \varepsilon_{0(j)}^2 H_4 \tag{22}$$

The bias of t_w is, given by

$$B\left(t_w\right) \cong \begin{cases} \dfrac{1}{n}\left(\dfrac{1}{\mu_x^2}\sigma^2_{x\left(\frac{n+1}{2}\right)}H_2 + \dfrac{1}{\mu_y\mu_x}\sigma_{xy\left(\frac{n+1}{2}\right)}H_3 + \dfrac{1}{\mu_y^2}\sigma^2_{y\left(\frac{n+1}{2}\right)}H_4\right) & \textit{if odd} \\ \dfrac{1}{2n}\left(\dfrac{1}{\mu_x^2}\left(\sigma^2_{x\left(\frac{n}{2}\right)} + \sigma^2_{x\left(\frac{n+2}{2}\right)}\right)H_2 + \dfrac{1}{\mu_x\mu_y}\left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right)H_3 + \dfrac{1}{\mu_y^2}\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right)H_4\right) & \textit{if even} \end{cases} \tag{23}$$

The MSE of t_w is, given by

$$MSE\left(t_w\right) \cong \begin{cases} \dfrac{1}{n}\left(\sigma^2_{y\left(\frac{n+1}{2}\right)} + \dfrac{1}{\mu_x^2}\sigma^2_{x\left(\frac{n+1}{2}\right)}H_1^2 + 2\dfrac{1}{\mu_x}\sigma_{xy\left(\frac{n+1}{2}\right)}H_1\right) & \textit{if odd} \\ \dfrac{1}{2n}\left(\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right) + \dfrac{1}{\mu_x^2}\left(\sigma^2_{x\left(\frac{n}{2}\right)} + \sigma^2_{x\left(\frac{n+2}{2}\right)}\right)H_1^2 + 2\dfrac{1}{\mu_x}\left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right)H_1\right) & \textit{if even} \end{cases} \tag{24}$$

After differentiating with respect to H_1 and putting it (24) we obtain the $MSE_{\min}\left(t_w\right)$ is equal to $MSE_{\min}\left(t_g\right)$. This means that the usual linear regression estimator is special case of wider class of estimator.

Now we define a general class of estimators using multivariate auxiliary variable as

$$\bar{y}_s = \bar{y}_{MRSS(j)} H\left(u_1, u_2, ..., u_p\right) = \bar{y}_{MRSS(j)} H\left(\underline{u}\right) \tag{25}$$

where $u_k = \dfrac{\bar{x}_{MRSS(j)k}}{\mu_{xk}}$ $\quad k = 1, 2, ..., p$ and $H(\underline{u})$ is a parametric function such that $H(\underline{\in}) = 1$ for

$\underline{\in} = (1,1,...,1)_{1*p}$, satisfying certain regularity condition such as the first and second order partial derivatives of H with respect to $\underline{u}$ exist and are known. Expanding $H(\underline{u})$ around the $\underline{\in}$ by using second order Taylor's series we have

$$H(\underline{u}) = H[\underline{\in} + (u - \underline{\in})] = H(\underline{\in}) + (\underline{u} - \underline{\in})\frac{\partial H}{\partial \underline{u}}\bigg|_{\underline{u}=\underline{\in}} + \frac{1}{2}(\underline{u} - \underline{\in})' \frac{\partial^2 H}{\partial \underline{u}'\underline{u}}\bigg|_{\underline{u}=\underline{\in}} (\underline{u} - \underline{\in}) + ... \tag{26}$$

Putting (26) in (25) we have

$$\bar{y}_s = \mu_y \left[1 + \varepsilon_{0(j)} + (\underline{u} - \underline{\in})\underline{H_1} + \frac{1}{2}(\underline{u} - \underline{\in})' \underline{H_2}(\underline{u} - \underline{\in}) + \varepsilon_{0(j)}(\underline{u} - \underline{\in})\underline{H_1} + ...\right] \tag{27}$$

where $\underline{H_1} = \dfrac{\partial H}{\partial \underline{u}}\bigg|_{\underline{u}=\underline{\in}}$ and $\underline{H_2} = \dfrac{\partial^2 H}{\partial \underline{u}'\underline{u}}\bigg|_{\underline{u}=\underline{\in}}$ are the matrices consisting of first and second order partial derivatives of the function H with respect to $\underline{u}$ and evaluated at $\underline{u} = \underline{\in}$.

$$B(\bar{y}_s) = \mu_y E\left[\frac{1}{2}(\underline{u} - \underline{\in})' H_2 (\underline{u} - \underline{\in}) + \varepsilon_{0(j)}(\underline{u} - \underline{\in})\underline{H_1} + ...\right] \tag{28}$$

$$B(\bar{y}_s) \cong \begin{cases} \dfrac{1}{n}\mu_y\left[\sum_{k=1}^{p}\dfrac{1}{\mu_{xk}^2}\sigma^2_{xk\left(\frac{n+1}{2}\right)}H_{2k} + \sum_{k=1}^{p}\dfrac{1}{\mu_{xk}\mu_y}\sigma_{yxk\left(\frac{n+1}{2}\right)}H_{1k}\right] & \text{if odd} \\ \dfrac{1}{2n}\mu_y\left[\sum_{k=1}^{p}\dfrac{1}{\mu_{xk}^2}\left(\sigma^2_{xk\left(\frac{n}{2}\right)} + \sigma^2_{xk\left(\frac{n+2}{2}\right)}\right)H_{2k} + \sum_{k=1}^{p}\dfrac{1}{\mu_{xk}\mu_y}\left(\sigma_{yx\left[\frac{n}{2}\right]} + \sigma_{yx\left[\frac{n+2}{2}\right]}\right)H_{1k}\right] & \text{if even} \end{cases} \tag{29}$$

where H_{1j} and H_{2j} denote j^{th} diagonal element of H_2 and j^{th} component of H, respectively.

$$MSE(\bar{y}_s) \cong \mu_y^2 E\left[\varepsilon_0^2 + H_1'(\underline{u} - \underline{\in})'(\underline{u} - \underline{\in})\underline{H_1} + 2\varepsilon_0(\underline{u} - \underline{\in})\underline{H_1}\right] \tag{30}$$

$$MSE(\bar{y}_s) \cong \begin{cases} \dfrac{\mu_y^2}{n}\left[\dfrac{1}{\mu_y^2}\sigma^2_{y\left(\frac{n+1}{2}\right)} + 2\dfrac{1}{\mu_y\mu_{xt}}\sum_{t=1}^{k}\rho_{yxt}\sigma_y\sigma_{xt}H_{1t} + \dfrac{1}{\mu_{xt}\mu_{xs}}\sum_{t=1}^{k}\sum_{s=1}^{k}H_{1t}H_{1s}\rho_{xt}\rho_{xs}\sigma_{xt}\sigma_{xs}\right] & if\ odd \\ \dfrac{\mu_y^2}{2n}\left[\begin{array}{l}\dfrac{1}{\mu_y^2}\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right) + 2\dfrac{1}{\mu_y\mu_{xt}}\sum_{t=1}^{k}\left(\sigma_{yxt\left[\frac{n}{2}\right]} + \sigma_{yxt\left[\frac{n+2}{2}\right]}\right)H_{1t} \\ + \dfrac{1}{\mu_{xt}\mu_{xs}}\sum_{t=1}^{k}\sum_{s=1}^{k}H_{1t}H_{1s}\rho_{xt}\rho_{xs}\left(\sigma_{xt\left[\frac{n}{2}\right]} + \sigma_{xt\left[\frac{n+2}{2}\right]}\right)\left(\sigma_{xs\left[\frac{n}{2}\right]} + \sigma_{xs\left[\frac{n+2}{2}\right]}\right)\end{array}\right] & if\ even \end{cases} \tag{31}$$

On differentiating with respect to $\underline{H_1} = (H_{11},...,H_{1p})^t$ and equating to zero, we will obtain a set of p equations, as

$$\begin{bmatrix} \dfrac{1}{\mu_{x1}^2}\sigma^2_{x1\left(\frac{n+1}{2}\right)} & \dfrac{\sigma_{x1x2\left(\frac{n+1}{2}\right)}}{\mu_{x1}\mu_{x2}} & \cdots & \dfrac{\sigma_{x1xp\left(\frac{n+1}{2}\right)}}{\mu_{x1}\mu_{xp}} \\ \dfrac{\sigma_{x1x2\left(\frac{n+1}{2}\right)}}{\mu_{x1}\mu_{x2}} & \dfrac{1}{\mu_{x2}^2}\sigma^2_{x2\left(\frac{n+1}{2}\right)} & \cdots & \dfrac{\sigma_{x2xp\left(\frac{n+1}{2}\right)}}{\mu_{x2}\mu_{xp}} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\sigma_{x1xp\left(\frac{n+1}{2}\right)}}{\mu_{x1}\mu_{xp}} & \dfrac{\sigma_{x2xp\left(\frac{n+1}{2}\right)}}{\mu_{x2}\mu_{xp}} & \cdots & \dfrac{1}{\mu_{xp}^2}\sigma^2_{xp\left(\frac{n+1}{2}\right)} \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ \vdots \\ H_{1p} \end{bmatrix} = \begin{bmatrix} -\mu_y\dfrac{\sigma_{yx1\left(\frac{n+1}{2}\right)}}{\mu_y\mu_{x1}} \\ -\mu_y\dfrac{\sigma_{yx2\left(\frac{n+1}{2}\right)}}{\mu_y\mu_{x2}} \\ \vdots \\ -\mu_y\dfrac{\sigma_{yxp\left(\frac{n+1}{2}\right)}}{\mu_y\mu_{xp}} \end{bmatrix}$$

or $\underline{A}\underline{H_1} = C$. This equation can easily be solved for unknown parameters as $\underline{H_1} = \underline{A}^{-1}C$

$$MSE(\bar{y}_s)_{\min} \cong \begin{cases} \dfrac{1}{n\mu_y^2}\sigma^2_{y\left(\frac{n+1}{2}\right)}\left[1-R^2_{y.x1x2...xp}\right] & if\ odd \\ \dfrac{1}{2n\mu_y^2}\left(\sigma^2_{y\left[\frac{n}{2}\right]} + \sigma^2_{y\left[\frac{n+2}{2}\right]}\right)\left[1-R^2_{y.x1x2...xp}\right] & if\ even \end{cases} \tag{32}$$

Note that this MSE is same with MSE of multivariate regression estimator.

Corollary: A wider class of estimators for estimating population mean μ_y using p auxiliary variables $\mu_{x1},...,\mu_{xp}$ can easily be defined as

$$\bar{y}_w = H\left(\bar{y}_{MRSS(j)}, \underline{u}\right)$$

where $H(.,.)$ is a parametric function such that $H(\bar{Y}, \in) = 1$, satisfying certain regularity conditions. It is easy to show that a wider class of estimators $\bar{y}_w$ has the same asymtotic mean square error as that of the general class of ratio type estimators of population mean defined at $\bar{y}_{MRSS(j)}H(\underline{u})$.

5. Simulation Study

In this section, we conducted a simulation study to investigate the properties of proposed estimators. In the simulation study, we track following steps:

1. Generate correlated four finite populations of size $N = 10000$ using bivariate normal distribution using "mvrnorm" function in R programme. In the simulation, we considered $\mu_x = 2$, $\mu_z = 3$, $\mu_y = 4$. To see the efficiency of correlation we consider different correlations. To use the ratio type estimators we have taken into account only positive correlations. We have generated populations with different covariance matrices as given below:

Population I

$$\sigma^2 = \begin{bmatrix} 18 & 3 & 2.9 \\ 3 & 3 & 1.1 \\ 2.9 & 1.1 & 4 \end{bmatrix} \qquad \rho_{yx} = 0.4082 \qquad \rho_{yz} = 0.3418 \qquad \rho_{xz} = 0.3175$$

Population II

$$\sigma^2 = \begin{bmatrix} 14 & 3 & 2.9 \\ 3 & 2 & 1.1 \\ 2.9 & 1.1 & 2 \end{bmatrix} \qquad \rho_{yx} = 0.5669 \qquad \rho_{yz} = 0.5480 \qquad \rho_{xz} = 0.55$$

Population III

$$\sigma^2 = \begin{bmatrix} 10 & 3 & 2.9 \\ 3 & 2 & 1.1 \\ 2.9 & 1.1 & 2 \end{bmatrix} \qquad \rho_{yx} = 0.6708 \qquad \rho_{yz} = 0.6485 \qquad \rho_{xz} = 0.55$$

Population IV

$$\sigma^2 = \begin{bmatrix} 6 & 3 & 2.9 \\ 3 & 2 & 1.1 \\ 2.9 & 1.1 & 2 \end{bmatrix} \qquad \rho_{yx} = 0.8660 \qquad \rho_{yz} = 0.8372 \qquad \rho_{xz} = 0.55$$

2. Select samples size from bivariate normal distributions using SRS and MRSS for $n = 5, 6, 7, 8$ on the basis of 60.000 replications respectively.

3. Compute mean estimation using $\bar{y}_{R\_SRS}$, $\bar{y}_{R1\_SRS}$, $\bar{y}_{R2\_SRS}$, $\bar{y}_{R3\_SRS}$, $\bar{y}_{\text{Re}g1\_SRS}$, $\bar{y}_{\text{Re}g2\_SRS}$, $\bar{y}_{R\_\text{MRSS}}$, $\bar{y}_{R1\_MRSS}$, $\bar{y}_{R2\_MRSS}$, $\bar{y}_{R3\_MRSS}$, $\bar{y}_{\text{Re}g1\_MRSS}$ and $\bar{y}_{\text{Re}g2\_MRSS}$ from samples.

4. Computed MSEs and percent relative efficiencies (PREs) of estimators with respect to $\bar{y}_{R\_SRS}$ for $n = 5, 6, 7, 8$ on the basis of 60.000 replications and displayed in Table 1-Table 4. We used following formula to calculate PRE values:

$$PRE_i = \frac{MSE\left(\bar{y}_{R\_SRS}\right)}{MSE\left(\bar{y}_i\right)}*100$$

where i=R_SRS, R1_SRS, R2_SRS, R3_SRS, Reg1_SRS, Reg2_SRS, R_MRSS, R1_MRSS, R2_MRSS, R3_MRSS, Reg1_MRSS, Reg2_MRRS

Table 1: MSE and Efficiency of Estimators for Population I

	$n=5$		$n=6$		$n=7$		$n=8$	
Estimator	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE
$\bar{y}_{R\_SRS}$	13164.0042	100.00	948.5593	100.00	753.8487	100.00	52.6932	100.00
$\bar{y}_{R1\_SRS}$	8.6828	151609.71	4.1258	22991.10	2.9104	25901.97	2.4331	2165.68
$\bar{y}_{R2\_SRS}$	3.3443	393622.86	2.7754	34177.35	2.3691	31819.90	2.0342	2590.32
$\bar{y}_{R3\_SRS}$	26038.5650	50.56	2531.9695	37.46	2295.0400	32.85	172.1833	30.60
$\bar{y}_{\mathrm{Re}g1\_SRS}$	3.0754	428046.87	2.5875	36658.93	2.2239	33897.70	1.9212	2742.74
$\bar{y}_{\mathrm{Re}g2\_SRS}$	2.9395	447824.38	2.4802	38245.61	2.1325	35350.10	1.8431	2858.95
$\bar{y}_{R\_\mathrm{MRSS}}$	3.8511	341828.74	1.5494	61222.79	2.4876	30304.07	1.0921	4824.86
$\bar{y}_{R1\_MRSS}$	3.3535	392550.28	1.3704	69216.08	2.3136	32582.96	1.0117	5208.15
$\bar{y}_{R2\_MRSS}$	3.1856	413229.24	1.3215	71779.46	2.2542	33442.67	0.9921	5311.12
$\bar{y}_{R3\_MRSS}$	115.7138	11376.35	2.1826	43460.93	3.9730	18974.23	1.3606	3872.78
$\bar{y}_{\mathrm{Re}g1\_MRSS}$	3.1153	422555.38	1.2978	73089.57	2.2225	33918.36	0.9807	5372.81
$\bar{y}_{\mathrm{Re}g2\_MRSS}$	**2.9281**	**449576.12**	**1.2190**	**77812.19**	**2.0765**	**36304.05**	**0.9137**	**5766.88**

Table 2: MSE and Efficiency of Estimators for Population II

	$n=5$		$n=6$		$n=7$		$n=8$	
Estimator	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE
$\bar{y}_{R\_SRS}$	211.6173	100.00	6.7393	100.00	7.7910	100.00	216.1899	100.00
$\bar{y}_{R1\_SRS}$	2.1825	9696.16	1.7635	382.16	1.4788	526.85	1.2896	16764.36
$\bar{y}_{R2\_SRS}$	2.1703	9750.69	1.7939	375.68	1.5215	512.07	1.3375	16163.55
$\bar{y}_{R3\_SRS}$	445.1171	47.54	16.1187	41.81	20.0734	38.81	519.0859	41.65
$\bar{y}_{\mathrm{Re}g1\_SRS}$	1.8579	11390.24	1.5479	435.40	1.3277	586.80	1.1668	18528.53
$\bar{y}_{\mathrm{Re}g2\_SRS}$	1.9815	10679.46	1.6448	409.72	1.4072	553.63	1.2367	17480.73
$\bar{y}_{R\_\mathrm{MRSS}}$	2.0655	10245.43	0.8371	805.03	1.4182	549.37	0.6235	34672.35
$\bar{y}_{R1\_MRSS}$	1.9183	11031.28	0.7906	852.48	1.3612	572.35	0.5986	36118.91
$\bar{y}_{R2\_MRSS}$	1.9222	11009.15	0.8097	832.31	1.3687	569.23	0.6098	35455.12
$\bar{y}_{R3\_MRSS}$	2.4221	8736.95	0.9280	726.25	1.5159	513.94	0.6480	33360.22
$\bar{y}_{\mathrm{Re}g1\_MRSS}$	1.8496	11440.94	0.7730	871.86	1.3374	582.54	0.5914	36555.78
$\bar{y}_{\mathrm{Re}g2\_MRSS}$	**1.7780**	**11901.87**	0.7588	**888.11**	1.2653	**615.75**	0.5643	**38307.82**

Table 3: MSE and Efficiency of Estimators for Population III

	$n=5$		$n=6$		$n=7$		$n=8$	
Estimator	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE
$\bar{y}_{R\_SRS}$	27.3732	100.00	24.3081	100.00	1.7696	100.00	1.2123	100.00
$\bar{y}_{R1\_SRS}$	1.2764	2144.49	1.0329	2353.35	0.8629	205.07	0.7527	161.07
$\bar{y}_{R2\_SRS}$	1.3489	2029.24	1.1142	2181.74	0.9431	187.63	0.8285	146.33
$\bar{y}_{R3\_SRS}$	145.0732	18.87	52.8522	45.99	3.1282	56.57	2.0634	58.75
$\bar{y}_{\text{Reg}1\_SRS}$	1.0813	2531.59	0.9002	2700.33	0.7720	229.23	0.6776	178.91
$\bar{y}_{\text{Reg}2\_SRS}$	1.1961	2288.51	0.9925	2449.24	0.8450	209.42	0.7432	163.12
$\bar{y}_{R\_\text{MRSS}}$	1.2418	2204.27	0.4960	4900.34	0.8261	214.21	0.3642	332.85
$\bar{y}_{R1\_MRSS}$	1.1382	2404.96	0.4618	5263.39	0.7906	223.84	0.3462	350.19
$\bar{y}_{R2\_MRSS}$	1.1614	2356.98	0.4856	5005.79	0.8082	218.97	0.3604	336.36
$\bar{y}_{R3\_MRSS}$	1.4280	1916.88	0.5558	4373.20	0.8517	207.78	0.3737	324.38
$\bar{y}_{\text{Reg}1\_MRSS}$	1.1006	2487.15	0.4517	5381.40	0.7789	227.20	0.3426	353.82
$\bar{y}_{\text{Reg}2\_MRSS}$	**1.0248**	**2671.10**	**0.4398**	**5527.28**	**0.7112**	**248.83**	**0.3194**	**379.52**

Table 4: MSE and Efficiency of Estimators for Population IV

	$n=5$		$n=6$		$n=7$		$n=8$	
Estimator	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE	**MSE**	PRE
$\bar{y}_{R\_SRS}$	30.9339	100.00	10.1850	100.00	0.6340	100.00	0.4334	100.00
$\bar{y}_{R1\_SRS}$	0.3656	8461.66	0.2963	3437.76	0.2449	258.86	0.2137	202.82
$\bar{y}_{R2\_SRS}$	0.5267	5872.75	0.4340	2346.62	0.3649	173.73	0.3202	135.35
$\bar{y}_{R3\_SRS}$	73.7932	41.92	27.3506	37.24	1.3909	45.58	0.8715	49.73
$\bar{y}_{\text{Reg}1\_SRS}$	0.2965	10433.07	0.2464	4133.89	0.2111	300.29	0.1854	233.77
$\bar{y}_{\text{Reg}2\_SRS}$	0.4061	7617.18	0.3399	2996.27	0.2880	220.19	0.2556	169.59
$\bar{y}_{R\_\text{MRSS}}$	0.3590	8617.02	0.1481	6879.10	0.2347	270.11	0.1047	413.83
$\bar{y}_{R1\_MRSS}$	0.3132	9876.42	0.1272	8008.96	0.2161	293.41	0.0952	455.33
$\bar{y}_{R2\_MRSS}$	0.3601	8590.04	0.1577	6459.30	0.2407	263.38	0.1137	381.20
$\bar{y}_{R3\_MRSS}$	0.4688	6598.32	0.1785	5706.77	0.2098	302.23	0.1094	396.31
$\bar{y}_{\text{Reg}1\_MRSS}$	0.2964	10435.62	0.1217	8370.56	0.2097	302.28	0.0922	469.95
$\bar{y}_{\text{Reg}2\_MRSS}$	**0.2325**	**13302.55**	**0.1105**	**9213.11**	**0.1534**	**413.41**	**0.0766**	**565.93**

From the simulation results given in Tables 1-4, we can conclude that:

1. When we compare same type estimators in SRS and MRSS sampling designs we can say that MRSS estimators give more efficient results than SRS estimators.
2. Ratio type estimator using two auxiliary variables $\bar{y}_{R3\_SRS}$ is the worst estimator in SRS design for all populations. And $\bar{y}_{R3\_MRRS}$ is the worst estimator in MRSS except the sample size $n=7$ for Population IV.

3. From the Table 1-Table 4, we can say that suggested class of estimator performs better than Al-Omari (2012) estimator. We can conclude that regression type estimator gives always more efficient results than Al-Omari (2012) estimator.
4. If we use any function of u in our estimator we can not reduced the MSE than MSE of regression estimator. The usual ratio, product and power estimators are special case of class of estimator.
5. To use ratio type estimators in SRS or MRSS, we assume that there are positive correlations between study and auxiliary variables. Otherwise to get efficiency we need to use product type estimators.

References:

[1] A. I., Al-Omari, 2012, Ratio estimation of the population mean using auxiliary information in simple random sampling and median ranked set sampling, *Statistics and Probability Letters*, 82, 1883-1890.
[2] M.F., Al-Saleh, A.I., Al-Omari, 2002, Multistage ranked set sampling, *Journal of Statistical Planning and Inference*, 102, 273-286.
[3] A.A., Jemain, A.I., Al-Omari, 2006, Multistage median ranked set samples for estimating the population mean, *Pakistan Journal of Statistics*, 22, 195-207.
[4] A.A., Jemain, A.I., Al-Omari, K., Ibrahim, 2007, Multistage extreme ranked set sampling for estimating the population mean, *Journal of Statistical Theory and Application*, 6, 4, 456-471.
[5] O., Ozturk, M., Jafari Jozani, 2014, Inclusion probabilities in partially rank ordered set sampling, *Computational Statistics and Data Analysis*, 69, 122-132.
[6] S., Amiri, R., Modarres, D.S., Bhoj, 2015, Ranked set sampling with random subsamples, *Journal of Statistical Computation and Simulation*, 85, 5, 935-946.
[7] A.I., Al-Omari, A.D., Al-Nasser, 2012, On the population median estimation using robust extreme ranked set sampling, *Monte Carlo Methods and Applications*, 18, 2, 109-118.
[8] A.I., Al-Omari, M.Z., Raqab, 2013, Estimation of the population mean and median using truncation-based ranked set samples, *Journal of Statistical Computation and Simulation*, 83, 8, 1453-1471.
[9] A., Haq, J., Brown, E., Moltchanova, A.I., Al-Omari, 2013, Partial ranked set sampling design, *Environmetrics*, 24, 3, 201-207.
[10] A., Haq, J., Brown, E., Moltchanova, A.I., Al-Omari, 2014, Mixed ranked set sampling design, *Journal of Applied Statistics*, 41, 10, 2141-2156.
[11] D.S., Bhoj, D., Kushary, 2014, Ranked set sampling with unequal samples and unequal replications, *Pakistan Journal of Statistics*, 30, 3, 351-362.
[12] D.S., Bhoj, D., Kushary, 2016, Ranked set sampling with unequal samples for skew distributions, *Journal of Statistical Computation and Simulation*, 86**,** 4, 676-681.
[13] M.H., Samuh, A., Qtait, 2015, Estimation for the parameters of the exponentiated exponential distribution using a median ranked set sampling, *Journal of Modern Applied Statistical Methods*, 14, 1, 215-237.
[14] A., Omar, K., Ibrahim, 2013, Estimation of the shape and scale parameters of the pareto distribution using extreme ranked set sampling, *Pakistan Journal of Statistics*, 29, 1, 33-47.
[15] S., Gupta, J., Shabbir, 2015, Estimation of finite population mean in stratified random sampling with two auxiliary variables under double sampling design, *Communications in Statistics - Theory and Methods*, 44, 13, 2798-2808.
[16] J., Shabbir, S., Gupta, 2015, A note on generalized exponential type estimator for population variance in survey sampling, *Revista Colombiana de Estadistica*, 38, 2, 385-397.
[17] J., Shabbir, A., Haq, S., Gupta, 2014, A new difference-cum-exponential type estimator of finite population mean in simple random sampling, *Revista Colombiana de Estadistica*, 37, 1, 197-209.
[18] N., Koyuncu, 2015, Ratio estimation of the population mean in extreme ranked set and double robust extreme ranked set sampling, *International Journal of Agricultural and Statistical Sciences*, 11, 1, 21-28.
[19] N., Koyuncu, 2016, New difference-cum-ratio and exponential type estimators in median ranked set sampling, *Hacettepe Journal of Mathematics and Statistics*, 45, 1, 207-225.
[20] S., Singh, 2003, *Advanced Sampling Theory With Applications: How Michael "Selected" Amy*, Springer Science & Business Media.
[21] S.K., Srivastava, S.K., Jhajj, 1980, A class of estimators using auxiliary information for estimating finite population variance, *Sankhya*, C, 42, 87-96.
[22] N., Koyuncu, C., Kadilar, 2010, On improvement in estimating population mean in stratified random sampling, *Journal of Applied Statistics*, 37, 6, 999-1013.
[23] S.K., Srivastava, 1980, A class of estimator using auxiliary information in sample surveys, *Canadian Journal of Statistics*, 8, 253-254.