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Middle School Mathematics Teachers' Pedagogical Content Knowledge Regarding Student Knowledge about Quadrilaterals

Elif Nur AKKAŞ[1] Elif TÜRNÜKLÜ[2]

ABSTRACT: Pedagogical content knowledge is consisted of two components: student knowledge, teaching strategies. Student knowledge was defined to sub-categories as connecting prior knowledge to new knowledge, noticing students' mistakes, identifying students' difficulties of understanding. The aim of this study is to examine middle school mathematics teachers' pedagogical content knowledge in terms of student knowledge regarding quadrilaterals. Interview method was used. 30 middle school mathematics teachers working at 12 different schools in Turkey participated. Content analysis was used. Results show that teachers teach lessons taking into consideration their students' previous knowledge and new knowledge they do by "reminding quadrilaterals students previously learnt" or "making association between similar quadrilaterals. The teachers pointed out the students' mistakes about quadrilaterals were group under three headings: mistakes regarding defining quadrilaterals, mistakes regarding visual property, classification of quadrilaterals, and family relation within quadrilaterals. The students' difficulties are summarized in two groups: difficulties identified related with trapezoid, difficulties identified related with other quadrilaterals. Different studies can be carried out on different pedagogical content knowledge components to examine the relationship between the results.
Key words: Pedagogical Content Knowledge, Student Knowledge, Quadrilaterals

INTRODUCTION

Teacher education research has been suggested as a new field of study by Shulman (1986) and has created a framework to determine what teachers need and develop effective teaching according to some factors. In this framework, Shulman (1987) has examined pedagogical content knowledge as consisting of two components: student knowledge and teaching strategies. Examining of components of the knowledge to understand students in various different pedagogical content knowledge models shows that An, Wu and Kulm (2004) divided student knowledge into such sub-categories as addressing students' misconceptions, knowing students' thinking, building on students' maths ideas, engaging students in math learning, and promoting students' thinking mathematics. Schoenfeld (1998) discussed student knowledge in sub-categories such as students' knowledge of what they understood, misconceptions in the topic area and students' prior knowledge. Grossman (1990) dealt with the same component in two sub-categories as student comprehension and students' knowledge of misconceptions. In Magnusson, Krajcik and Borko (1999), student knowledge was examined in sub-categories of requirements for learning and areas of students' difficulty within the framework of pedagogical content knowledge adopted from Grossman (1990). Park and Oliver (2008) examined the same topic in relation with sub-categories of students' knowledge about the topic, topics in which students might face difficulty, learning difficulties, motivation, ability, learning style, interest, stages of development and difference of needs.

Kovarik (2008) studied student knowledge in sub-categories of students' mathematical background, misconceptions, associations between their prior knowledge and new knowledge and anticipating in questions. Lastly, in Fennema and Franke (1992), student knowledge was dealt in sub-categories as students' previous and potential difficulties, the way they think and they comprehend a given topic in mathematics. Despite the fact that sub-categories of student knowledge vary within the framework of pedagogical content knowledge put forth by researchers, the component of student knowledge consists of discovering students' existing knowledge- connecting prior knowledge to new knowledge, noticing students' mistakes, identifying students' difficulties of understanding and recognizing individual differences.

[1] Ph.D., Dokuz Eylul University, Buca Education Faculty, elifnurakdogan@hotmail.com
[2] Assoc. Prof. Dr., Dokuz Eylul University, Buca Education Faculty, elif.turnuklu@deu.edu.tr

Various researchers have carried out studies on the pedagogical content knowledge components put forth by Shulman (Park and Oliver 2008, different components have been listed in detail by different researchers). In this examination, Park and Oliver (2008) have determined that many of these researchers generally use two components determined by Shulman and in addition to these components they also create new components called knowledge of assessment of subject matter along with knowledge of curriculum.

When studies carried out on mathematics teachers are examined in terms of pedagogical content knowledge, other studies are found which examine different topics in mathematics in terms of student knowledge (Baştürk, 2009; Bingölbali and Özmantar, 2009). The studies on student knowledge component shed light onto students' difficulties of understanding, misconceptions and teachers' approach to students.

When the studies carried out are examined (Grossman, 1990; Fennema and Franke, 1992; Schoenfeld, 1998; Magnusson et al, 1999; An, Kulm and Wu, 2004; Ball et al, 2008; Park and Oliver, 2008; Kovarik, 2008; Hacıömeroglu, 2009; Baştürk, 2009; Yeşildere and Akkoç, 2010; Bukova-Güzel, 2010), it is seen that the number of studies examining the subject of quadrilaterals in terms of pedagogical content knowledge is fairly low. The aim of this study is to examine middle school mathematics teachers' pedagogical content knowledge in terms of student knowledge regarding quadrilaterals.

Theoretical Framework

In this study, the student knowledge sub-component of pedagogical content knowledge and sub-components comprising student knowledge were used. Magnusson et al. (1999) have presented these sub-components defined for use in science and mathematics education under two sub-headings as learning needs and students' difficulties. In present study, student difficulties were investigated under two headings as students' mistakes and difficulties of understanding, and requirements for learning were handled under heading of the connecting prior knowledge to new knowledge. Connecting prior knowledge to new knowledge are defined as reminding students of previous knowledge, reminding students of conceptually similar situations, associating and reviewing the topic. Bingölbali and Özmantar (2009) point out that the "difficulty" is a comprehensive concept and used by students for expressing difficulties they face in learning mathematics in general terms, and thus it contains mistakes. Difficulty of understanding is covered in student difficulty. In this sense, difficulty of understanding can be defined as students' having difficulty or challenge in understanding a specific topic (Bingölbali and Özmantar, 2009).

This study focused on all sub-components of student knowledge component of pedagogical content knowledge developed by Magnusson et al (1999). Quadrilaterals in geometry have been selected in this study in the context of student knowledge.

In our study, three important factors in teaching of concepts regarding quadrilaterals were used: visualisation, defining and classification. There are various studies on students' perceptions and concept images on geometry. When studies carried out on the learning of students for quadrilaterals are examined, it was observed that specifically two factors played a role in understanding, perception and comprehension. These are the definition of the concept image and figural concept. Concept image put forth by Tall and Vinner (1981) is not only limited to the concept definition and they have defined concept image as, 'the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes' (p. 152).

Individuals may perceive the same concept in different ways due to their individual epistemological and psychological attributes. Definition of the concept is defined as the whole set of words used to distinguish one concept from the others, whereas concept image is defined as what is conjured up in the mind for that concept either consciously or unconsciously. Concept image includes partially correct definitions and misconceptions.

Hershkowitz (1989, 1990), Tall and Vinner (1981) have interpreted their definition of concept image along with the definition of concept in critical attributes and non-critical attributes putting forth

that critical attributes belong to the concept definition while non-critical attributes belong to the concept image. Geometrical figures have certain visual structures in addition to certain formal definitions. Another important factor that should be considered when giving examples of geometrical figures is that there may be one or more prototypes. Herskowitz (1989) has explained this by stating that all samples have common specific visual features, prototypes namely. Tsamir et al (2008) have accepted prototype samples as the heuristic representative of the concept. In this sense, the prototype factors comprise the key factor. Each concept may have more than one prototype sample. These prototype samples are those that represent some of the features included in the long list of features for the concept. These prototype figures always have an effect on the concept image (Fischbein, 1993; Hershkowitz, 1990).

When all the aforementioned theoretical structures effective in understanding and apprehending quadrilaterals are taken into account, it is important to examine teachers' awareness of sub-components of student knowledge, to determine students' mistakes and difficulty of understanding of the topic and what associations students make between their prior knowledge and new knowledge in classes are like during the process of students' learning quadrilaterals. As a conclusion, mathematics teachers' pedagogical content knowledge regarding student knowledge within the framework created by Magnusson et al (1999) for the quadrilaterals have been examined in this study.

The theoretical framework synthesized above and presented in Figure 1 has been used in this study. Student knowledge was investigated as connecting prior knowledge to new knowledge, students' mistakes to the topic and students' difficulties of the topic (quadrilaterals).

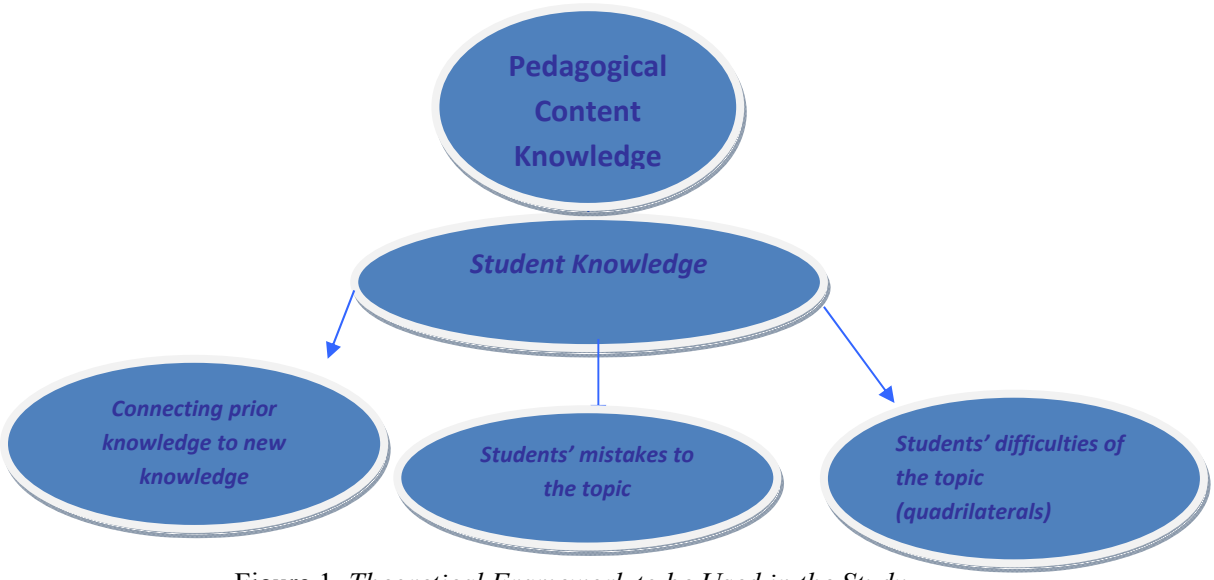

Figure 1. *Theoretical Framework to be Used in the Study*

The purpose of this study is to examine the middle school mathematics teachers' (10-13 year olds' teachers) pedagogical content knowledge in the context of student knowledge during the process of teaching of quadrilaterals. In this study, interview method has been preferred within the context of qualitative research.

METHODS

Participants and Setting

30 middle school mathematics teachers working at 12 different schools in the city of Izmir in Turkey have participated in the study. Teachers were selected on voluntary basis. These teachers have 1-20 years of experience in their fields.

The mathematics teachers who participated in the study have graduated from 4 year mathematics teaching program in faculties of education. The teachers have graduated from these programs after completing the field courses as well as pedagogical knowledge and content knowledge.

The topic of quadrilaterals has been examined in this study. Quadrilaterals are included in the 5^{th}-8^{th} class curriculum at middle schools as well as the 5^{th} and 7^{th} class curriculum in Turkey. At these class levels, the topic of quadrilaterals included recognizing, understanding and drawing special quadrilaterals, determining their diagonals along with interior and exterior angles, measurement, forming area relations and solving problems.

Data Collection

Interview method was used for data acquisition. The questions asked to the teachers during interviews were prepared by researchers in accordance with the pedagogical content knowledge student knowledge component. The interview consisted of six questions which were intended to put forth difficulties of learning faced by students, the mistakes they make in relation with the topic and the relations they structure with the topic (see App. 1). These questions were reviewed by three field experts prior to being directed to the teachers and required corrections were made. The questions of the interview which were prepared in such a manner were asked to one teacher and thus the pilot study for the interview questions was carried out. Two mathematics researchers who carried out the study reviewed the questions after the pilot interview thereby deciding on the final form of the questions. The teacher was informed prior to the interview that the interview would be recorded. The interviews lasted an average of 30 minutes.

Data Analysis

Content analysis was used to analyze the data acquired in the study. The basic process carried out in content analysis is to bring together the data that resembles each other within the framework of certain concepts and themes and to arrange and interpret these in a manner that will be understood by the reader (Şimşek and Yıldırım, 2006). Data analysis was carried out in two stages. In the first stage, the voice records were analyzed. In the second stage, two researchers came together to determine the possible codes for the three strategies used in the study. Connecting prior knowledge to new knowledge, students' mistakes to the topic and students' difficulties of the topic are study themes and reminding previously learnt quadrilaterals, associating similar quadrilaterals; mistakes regarding definitions, mistakes regarding visual aspects, mistakes regarding classification- family relation in quadrilaterals; difficulties regarding quadrilaterals are sub- themes. All these processes were carried out separately for each of the 30 teachers after which the analysis results were collected and reported. There was 90% agreement between independent coding of researchers. Since the percentage of agreement should be 70% or above, reliability was enabled in the sense of data analysis (Türnüklü, 2000).

FINDINGS

As a result of data analysis, different codes were determined for three sub-components under student knowledge (connecting prior knowledge to new knowledge, mistakes made about the topic, difficulties of understanding regarding the topic). The codes determined for the sub-categories were examined in the light of the participants' responses.

1. Connecting Prior Knowledge to New Knowledge

During interviews, the teachers pointed out that they teach lessons taking into consideration their students' previous knowledge and they do by "reminding quadrilaterals students previously learnt" or "making association between similar quadrilaterals".

67 % of the teachers (20 out of 30 teachers) stated that they make associations by referring to quadrilaterals (squares and rectangular) previously learnt by students. Assuming that students are more familiar with squares and rectangulars and they are more proficient in those, the teachers underlined

the importance of such reference for better switch to other quadrilaterals. Below are given quotations from some of the participants.

"Students know squares and rectangulars well, so it will be a good introduction to the topic if I remind them of these topics"

"Squares and rectangulars have been in our lives since primary school, even babyhood. So, it is easier for them to remember these and comprehend new quadrilaterals"

"It seems that it is easier for 7th graders to learn quadrilaterals because squares and so on were learnt at 5th grade"

Besides those above, 33 % the teachers stated that they teach quadrilaterals by making connecting their prior knowledge to new knowledge. From participants' responses, "square and equilateral quadrangle", "equilateral quadrangle and rhomboid", "square and rectangular", "rectangular and rhomboid" were grouped as quadrilaterals similar to each other. In this way of grouping, the teachers explained that the quadrilateral which was learnt before could facilitate learning of the new quadrilaterals by students. Expressions of some teachers making such grouping are quoted below.

"Square looks very much alike equilateral quadrangle; so, once students learn square, they can learn equilateral quadrangle easily"

"There are the rules, they look like each other. Rhomboid is like a slanted rectangular. We taught rectangular to students, the rules are similar, so they can comprehend rhomboid easily"

"It is important to present similar quadrilaterals to students due to their prior knowledge; like equilateral quadrangle and rhomboid. A student who knows Equilateral quadrangle also knows rhomboid"

All of the 30 participants noted that students make connections between their prior knowledge and new knowledge in abovementioned manners so that they taught quadrilaterals effectively.

2. Students' Mistakes to Topic

As a result of analysis of the interviews, students' mistakes about quadrilaterals were grouped under three headings. These are mistakes regarding defining quadrilaterals, mistakes regarding visual property, and classification of quadrilaterals and family relations within quadrilaterals.

Definition is important for learning of quadrilaterals. Knowing fundamental elements of quadrilaterals makes learning the topic efficient. In the light of the participants' answers, the mistakes regarding defining quadrilaterals were divided into two. The mistakes in group one are caused by not knowing properties of quadrilaterals and not being able to identifying quadrilaterals properly, while the other mistakes are related with not comprehending the concepts of area-circumference because of memorizing formulas.

During interviews, the teachers identified student mistakes regarding not knowing properties of quadrilaterals and not knowing them exactly as mistakes made due to complete lack of knowledge about basic elements of quadrilaterals (angle-side-vertex-diagonal), mistakes made due to inexact or incomplete knowledge about quadrilaterals. Below are given some examples of such mistakes as defined by teachers.

"..For instance, he does not recognize trapezoid at all, thus he cannot identify it."

"The students don't distinguish vertex and diagonal, they make wrong definition"

"He can define it only with angles and sides, but he doesn't know the other properties of quadrilaterals"

"If I ask him to define a certain type of quadrilateral, he doesn't list all properties of it, also he sometimes defines them inaccurately"

Also participants told during interviews that students make wrong definitions since they make memorization due to merely sticking to formulas. In addition, they pointed out that the students making such mistakes are not able to comprehend the concepts of area-circumference; thus, they frequently make mistakes with definition. Some examples of teachers' explanations are given below.

"They are trying to use the same formula for rhomboid as quadrilaterals; then, definitions become wrong"

"He knows by heart the formula of area of square as a^2; thus, he recites a^2 for trapezoid, too"

"The students are on such a way that area and circumference are like the same things for them, or for some, area is equal to circumference and circumference is equal to area. Understandably enough, they make wrong definitions due to confusion"

Another type of mistake mentioned during interviews is regarding visual properties of quadrilaterals. This type of mistake is completely focused on visualization of quadrilaterals and measurement mistakes in drawing quadrilaterals. Such mistakes are divided into three groups in the light of the teachers' responses: mistakes made while drawing angles, sides, and diagonals of quadrilaterals. It was found out that the mistake often made in angles is drawing distorted angles. For example, an angle of ninety degrees is not represented as ninety degrees. Their mistakes in drawing of sides and diagonals were also found similar to mistakes made while drawing angles. As an example, it was reported that students draw a rectangular with unequal opposite sides as if each side had a different length. It was inferred from teachers' responses that the most frequent mistake was confusion between "diagonals centering each other" with "orthogonal cutting of diagonals" while drawing diagonals, and drawings were made wrong. Some participants' comments are quoted below to exemplify these mistakes.

"The students try to make drawing without setting a rule; I mean they don't draw angles with a good shape such that they draw an angle of 120 degrees like 30 degrees. There is no certain rule, it's just snapshot."

"I call a student to the board and ask him to draw a square. Yes, he is drawing a square, but it doesn't look like a square at all. When you look at it, you see all sides different from each other."

"There is a big problem with drawing diagonals; he says diagonals centre each other, still he draws a verticality in the middle."

Lastly, students make mistakes with classifying quadrilaterals in reference to family relations. In this context, it was found out that students make mistakes while grouping quadrilaterals or making inappropriate groupings based on their properties since they have no or wrong information about properties of quadrilaterals. The teachers said that students often cannot classify "trapezoid". They added that some of the students recognize individual quadrilaterals with their own properties; however, they cannot establish relationships or they establish wrong relationships between them. Below are given some examples of teachers' responses.

"A student who knows that all sides of a square are equal and its angles are 90 degrees doesn't comprehend that square is a rectangular as well and they belong to the same family"

"The students make a classification as far as they know, but they say trapezoid is not in that group, it is a separate quadrilateral"

It was understood from responses of 30 study participants that not only students but also teachers make some mistakes while classifying quadrilaterals. Nevertheless, those mistakes were not investigated since they are not relevant under this study. Various other studies can be carried out to find how teachers make mistakes while classifying quadrilaterals.

3. Students' Difficulties of Topic

The interviews revealed that teachers notice and explain certain difficulties of comprehension by students depending on the topic learnt. The difficulties inferred from teachers' responses are summarized in two groups: difficulties identified related with trapezoid and difficulties identified related with other quadrilaterals.

According to teachers' responses, students face difficulties in understanding the trapezoid since there is not a clear rule for trapezoid, it is not used much, it has a weird name, it is difficult to calculate area and circumference of it, it has a lot of properties, and it is a kind of quadrilateral encountered late by students, thus students are not familiar with it. Following quotations are given as an example from a participant listing such reasons.

"Students learn the trapezoid very late; therefore, they face difficulty in comprehending it; in my opinion, it should be taught earlier"

"First of all, students mock with what it is called once they hear it, they say it is crooked; so they concentrate on its name and comprehend it differently"

"The students cannot calculate area and circumference of it because it is difficult for them. They simply don't understand it"

"Trapezoid has many properties, that's why they cannot learn it quickly"

The teachers' responses reveal that difficulties regarding the other quadrilaterals than trapezoid are related with rhomboid and equilateral quadrangle. The participants indicate the reason for the biggest challenge with rhomboid as students' (particularly fifth graders) inability to comprehend the concept of parallel. They think the most important reason for difficulties in equilateral quadrangle is that students regard it as a quadrilateral with no clear-cut property such that it is confused with other quadrilaterals- especially square. Examples are given from some of the participants' explanations as following.

"Especially 5th graders fail in comprehending parallelism; therefore, they have the biggest difficulty in comprehending rhomboid"

"There are some challenges with equilateral quadrangle, some mistake it with square, some mistake it with rhomboid; thus, many of those in the class don't comprehend it"

"To explain parallelism, I tell railroad tracks and so one; still, they cannot associate it with rhomboid"

"They take equilateral quadrangle as square; thus, they confuse and don't comprehend it"

According the findings obtained from 30 participants, it can be seen that students do not face challenges in comprehending squares and rectangulars. It can be explained with students' familiarity with square and rectangular for years and their seeing such shapes in their environment often.

DISCUSSION, CONCLUSION AND IMPLICATIONS

In this study, middle school mathematics teachers' pedagogical content knowledge regarding quadrilaterals was investigated in the context of student knowledge component by means of interviews. For the study, the component of student knowledge about quadrilaterals was divided into three sub-components as connecting prior knowledge to new knowledge, students' mistakes the topic, and students' difficulties of the topic. These three sub-components were divided into other sub-categories in the study findings. The size of the study group is not suited for various generalizations. However, since this study was carried out with teachers who teach at different schools and different grades (5th and 7th Grade), it is possible to make some distinctions about teachers' knowledge regarding student knowledge of uadrilaterals.

For students' mistakes to the topic, it was found out that students mostly make mistakes due to the lack of exact knowledge about quadrilaterals. In many other studies carried out on comprehending and defining quadrilaterals, it was also noted that students have difficulties in defining quadrilaterals and use arbitrary definitions instead of formal definitions in their attempts to express the image of concepts and geometrical concepts (Akkas and Türnüklü, 2014; Sarfaty and Patkin, 2013; Türnüklü, Alaylı and Akkas, 2013; Hershkowitz, 1989; Burger and Shaughnessy, 1986). As far as teachers' responses reveal in this study, it can be suggested that students make definitions within the framework of their individual perceptions, thus they make mistakes in relation with definitions of quadrilaterals.

Another type of mistake regarding the topic found in this study is related with visual aspects of quadrilaterals. Concerning this type of mistake, it was seen that the students cannot make drawings beyond typical prototypes in their mind, so thus they make mistakes. This finding seems to support other results in the literature. Hershkowitz (1990) identified some mistakes commonly made in his study. The mistakes included overgeneralization of visual properties in reference to the types of quadrilaterals with which students were familiar and application of properties of prototypes to other types of the geometrical figure. As an example, some participants consider rectangular different from squares because of outstanding side structures of these two figures. Likewise, Türnüklü, Alaylı and Akkas (2013) found out in their study that middle school pre-service mathematics teachers could not make draw quadrilaterals other than typical ones and they made mistakes in drawing quadrilaterals since they were not familiar with different drawings. The results above seem to be in parallel with findings of Akkas and Türnüklü (2014), Fujita (2012), Heinze and Ossietzky (2002).

Another type of mistake found in present study was related with classifying of quadrilaterals. In relation with this type of mistake, students could not classify quadrilaterals properly by their respective properties, they could not establish relationships between quadrilaterals in the same family or they made misclassification due to the complete lack of knowledge or lack of exact knowledge of properties of quadrilaterals. The findings yielded by this study seem to be similar to other studies (Akkas and Türnüklü, 2014; Türnüklü, Alaylı and Akkas, 2013; Berkün, 2011; De Villiers, 1994; Monaghan, 2000). This finding implies that teachers were successful in finding students' mistakes.

Our study also found some challenges regarding trapezoid. It was understood in this study that students have difficulty in comprehending this figure. The difficulties of comprehension reported in the study indicate many misconceptions related with trapezoid. There is an abundance of studies accounting for the misconceptions regarding trapezoid (Türnüklü, 2014; Berkün, 2011; Nakahara, 1995). Our findings in this context support these studies. Also Türnüklü (2014) carried out a study on middle school teachers and students' conceptual images regarding trapezoid. He found out that individuals form some misconceptions regarding trapezoid, they exclude trapezoid from family relations drawn in quadrilaterals, denotation of the term trapezoid in Turkish affects their perceptions negatively, and even some participants say the figure is "crooked" or "is subject to no rule". All these findings seem similar due to the fact that the rule applied to trapezoid is not clear, the figure is not used frequently, and it is called "*trapezoid*" meaning crooked, gnarled or lopsided in Turkish. These factors were found to be among reasons for difficulties regarding trapezoid in this particular study. It can be suggested that similar findings in the literature the teachers participating in the study know their students well so they can find out their mistakes and difficulties of comprehension properly.

In this particular study, the component of "student knowledge" was investigated. Study findings helped detail teachers' views about student knowledge about quadrilaterals. The results demonstrating teachers' knowledge regarding student knowledge in teaching of quadrilaterals are displayed in Figure 2.

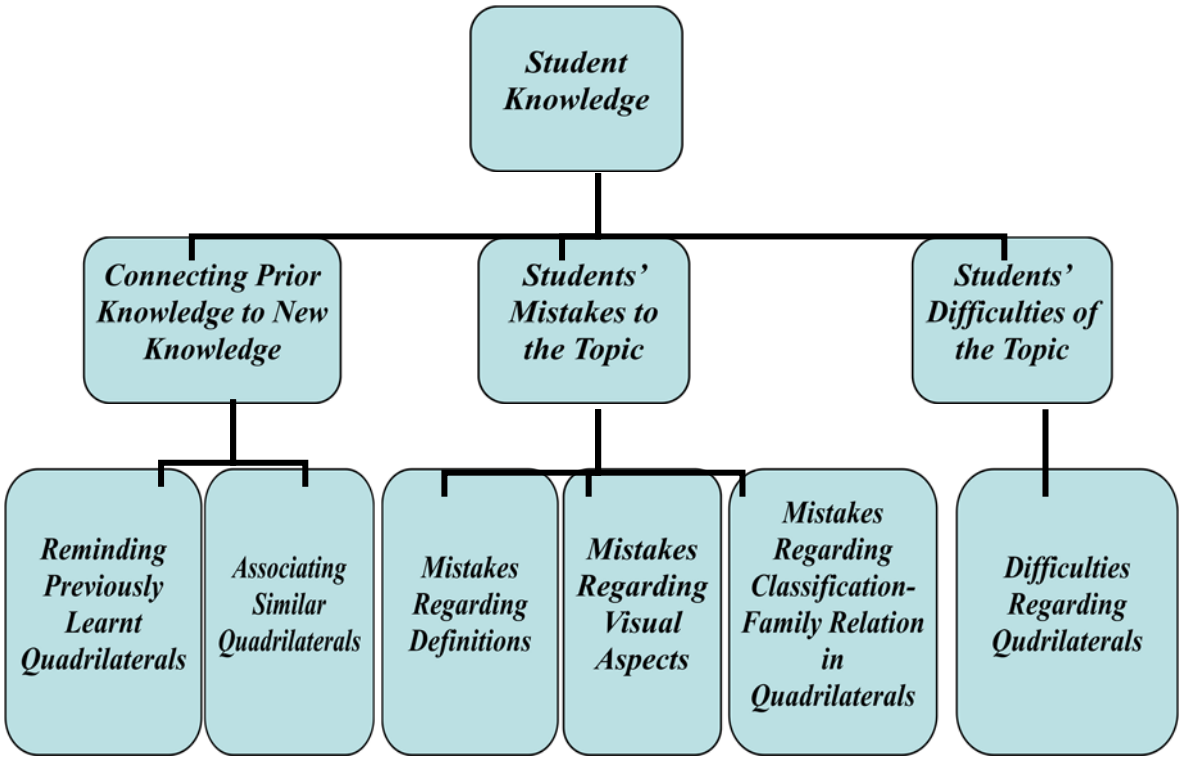

Figure. 2. *Student Knowledge regarding Quadrilaterals*

The study has been carried out with 30 middle school mathematics teachers. It is suggested to repeat it with a larger sample in order to reach further generalizable information about teachers' knowledge regarding their students' knowledge during teaching of quadrilaterals. Moreover, making observations on those teachers in classes might provide deeper and more satisfactory data in order to find and compare data obtained in interviews with processes realized during classes. In this way, it could be investigated whether teachers reporting mistakes and difficulties of comprehension help eliminate and prevent such mistakes and difficulties beyond finding them during classes. In addition, in the end of the study, all of the participant teachers pointed out that students could associate their prior knowledge and new knowledge and they could make particular emphasis on quadrilaterals they had seen before. It would be useful to do classroom research with these teachers in order to find out the extent at which these findings remain unchanged during observation.

Pedagogical content knowledge was investigated within the context of "student knowledge" component in this study. However, further research on various components could make contribution to the literature by comparing findings.

REFERENCES

Akkas, E., Türnüklü, E. (2014). Middle school mathematics teachers' pedagogical content knowledge regarding teaching strategies on quadrilaterals. *Educational Research and Reviews*, 9(7), 183-191.

An, S., Kulm, G., Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U. S., *Journal of Mathematics Teacher Education*, 7, pp. 145- 172.

Ball, D., Thames, M., Phelps, G. (2008). Content knowledge for teaching what makes it special?, *Journal of Teacher Education*, Vol. 59, No. 5, 389- 407.

Baştürk, S. (2009). Mutlak değer kavramı örneğinde öğretmen adaylarının öğrenci hatalarına yaklaşımları. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, 3(1), 174-194.

Berkün, M. (2011). *İlköğretim 5 ve 7. Sınıf Öğrencilerinin Çokgenler Üzerindeki İmgeleri ve Sınıflandırma Stratejileri*. Yayınlanmamış Yüksek Lisans Tezi, D.E.Ü.

Bingölbali, E., Özmantar, M.F (2009). İlköğretimde Karşılaşılan Matematiksel Zorluklar ve Çözüm Önerileri, Pegem Akademi, 1. Baskı.

Bukova- Güzel, E. (2010). An investigation of pre-service mathematics teachers' pedagogical content knowledge, using solid objects, *Scientific Research and Essays*, 5(14), 1872-1880.

Burger, W., Shaughnessy, J. (1986). Characterizing the van hiele levels of development in geometry, *Journal for Research in Mathematics Education*, Vol. 17, No. 1, 31- 48.

De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. For the learning of mathematics, 14, 11-18.

Fennema, E. and Franke, M. (1992) Teachers' knowledge and its impact in: D. A. Grouws (Ed) Handbook of Research on Mathematics Teaching and Learning (New York: Macmillan Publishing).

Fichbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics* , *24 (2),* 139-162.

Fujita, T. (2012). Learners' Level of Understanding of Inclusion Relations of Quadrilaterals and Prototype Phenomenon. *The Journal of Mathematical Behavior, 31*: 60-72.

Grossman, P. L. (1990). The making of a teacher: Teacher knowledge and teacher education. New York: Teachers College Press.

Hacıömeroğlu, G. (2009). Examining a preservice secondary teacher's growth: implications for teaching. *Journal of Theory and Practice in Education*, 5(2), 261- 273.

Heinze, A. ve Ossietzky, C. (2002). "…Because a Square is not a Rectangle" Students' Knowledge of Simple Geometrical Concepts When Starting to Learn Proof. In A. Cockburn ve E. Nardi (Eds.): *Proceedings of The 26th Conference of the International Group for the Psychology of Mathematics Education*, V.3, 81-88.

Hershkowitz, R. (1989). Visualization in geometry- two sides of the coin. *Focus on Learning Problems in Mathematics,* 11(1), 61-76.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. P.Nesher ve J. Kilpatrick (Eds.): *Mathematics and Cognition*, Cambridge University Press: Cambridge.

Kovarik, K. (2008). *Mathematics educators' and teachers' perceptions of pedagogical content knowledge*, Dissertation at the Columbia University, 2008.

Nakahara, T. (1995). Children's construction process of the concepts of basic quadrilaterals in Japan. In A.Oliver & K. Newstead (Eds.), Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education. 3: 27–34.

Magnusson, S., Krajcik, J. and Borko, H. (1999). Nature, sources and development of pedagogical content knowledge for science teaching. In J. GessNewsome and N.G. Lederman (Eds.), *Examining Pedagogical Content Knowledge.* (95–132). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Monaghan, F. (2000). What Difference Does It Make? Children's Views of the Differences Between Some Quadrilaterals. *Educational Studies in Mathematics*, *42*(2),179-196.

Park, S., Oliver, J. (2008) Revisiting the conceptualisation of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand Teachers as Professionals. *Research Science Education, 38*,261- 284.
Tall, D., and Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics,* 12(2), 151-16.
Türnüklü, A. (2000). Eğitim bilim araştırmalarında etkin olarak kullanılabilecek nitel bir araştırma tekniği: Görüşme. *Kuram ve Uygulamada Eğitim Yönetimi*,*24*,543-559.
Türnüklü, E. (2014). Concept Images of Trapezoid: Some cases from Turkey. *Educational Journal, 3*(3), 179- 185.
Türnüklü, E., Alaylı, F., Akkaş, E. (2013). Investigation of prospective primary mathematics teachers' perceptions and images for quadrilaterals. *Educational Sciences: Theory & Practice, 13*(2), 1225-1232.
Tsamir, P., Tirosh, D., and Levenson, E. (2008). Intuitive nonexamples: The case of triangles. *Educational Studies in Mathematics, 69*(2), 81- 95.
Sarfaty, Y., and Patkin, D. (2013). The ability of second grades to identify solids in different positions and to justify their answer. *Pythagoras, 34*(1), 212 - 222.
Schoenfeld, A., H. (1998) Toward a theory of teaching-in- context. *Issues in Education,* 4(1), 1-94.
Shulman, L. (1986) Those who understand: knowledge growth in teaching. *Educational Researcher*, *15* (2), 4- 14.
Shulman, L. (1987) Knowledge and teaching: foundations of the new reform. 57, 1- 22.
Şimşek, H., Yıldırım, A.(2006). *Nitel Araştırma Yöntemleri*. Ankara: Seçkin Yayıncılık.
Yeşildere, S., Akkoç, H. (2010). Examining pre-service mathematics teachers' pedagogical content knowledge of number patterns with regard to topic- specific strategies. *Journal of Ondokuz Mayıs University Faculty of Education, 29*(1), 125-149.

APPENDIX 1. INTERVIEW QUESTIONS

1. How do you give opportunities to your students?
2. Which quadrilaterals are much more difficult to students?
3. What kind of mistakes to students make for quadrilaterals?
4. How do you work to try to prevent students' mistakes?
5. Are there students' misconceptions on the topic?
6. What do you do to prevent the misconceptions?

Ortaokul Matematik Öğretmenlerinin Dörtgenler Konusunda Pedagojik Alan Bilgilerinin Öğrenci Bilgisi Bileşeninde İncelenmesi

Elif Nur AKKAŞ[1] Elif TÜRNÜKLÜ[2]

ÖZ. Pedagojik alan bilgisini farklı bileşenlerde inceleyen çalışmalarda ortak iki bileşenin varlığı dikkat çekmektedir. Bu bileşenler; öğrenci bilgisi ve öğretim stratejileri bilgisi bileşeni olarak tanımlanmaktadır. Öğrenci bilgisi bileşeni öğrencilerin ön bilgileri- yeni bilgileri arasında kurulan bağlantılar, konuya yönelik öğrenci hataları ve öğrencilerin konuya özgü yaşadıkları anlama güçlükleri alt bileşenlerinden oluşmaktadır. Araştırmanın amacı ortaokul matematik öğretmenlerinin dörtgenler konusunda pedagojik alan bilgilerinin öğrenci bilgisi bileşeninde incelenmesidir. Nitel araştırma yöntemi, görüşme metodu kullanılmıştır. Katılımcılar, Türkiye'de bir ilde 12 farklı ortaokulda çalışan 30 matematik öğretmenidir. İçerik analizi kullanılmıştır. Sonuçlara göre; öğretmelerin öğrencilerin ön bilgileriyle, yeni öğrendikleri arasında bağlantı kurduklarını, bunu, " önceden öğrenilen dörtgenler" ya da "benzer dörtgenleri ilişkilendirerek" kurdukları belirlenmiştir. Öğretmenler dörtgenlere yönelik öğrenci hatalarını dörtgenleri tanımlama, dörtgenleri görselleştirme ve dörtgenleri sınıflandırma- aile ilişkisi kurma hataları olmak üzere üç başlıkta gruplamışlardır. Öğrencilerin konuya özgü yaşadıkları anlama güçlükleri ise yamuğa ilişkin anlama güçlükleri ve diğer dörtgenlere ilişkin anlama güçlükleri olmak üzere iki grupta incelenmiştir. Farklı çalışmalarla, öğretmenlerin dörtgenler konusundaki farklı öğrenci bilgileri ortaya çıkarılıp, bu çalışmanın sonucu zenginleştirilebilir.
Anahtar Kelimeler: Pedagojik Alan Bilgisi, Öğrenci Bilgisi, Dörtgenler

Amaç ve Önem: Literatürde farklı araştırmacılar, Shulman'ın ortaya attığı pedagojik alan bilgisi bileşenleri üzerinde çalışmalar yapmışlardır (Park ve Oliver 2008). Bu çalışmalarda öğrenci bilgisi ve öğretim stratejileri bileşenin ortak olduğu görülmektedir. Öğrenci bilgisi bileşenine yönelik yapılan araştırmalar araştırılan konuya dair öğrencilerin yaşadıkları anlama güçlükleri, sahip oldukları kavram yanılgıları ve öğretmenlerin öğrencileri yaklaşımlarının neler olduğuna dair bilgiler vermektedir. Yapılan çalışmalar incelendiğinde (Grossman, 1990; Fennema ve Franke, 1992; Schoenfeld, 1998; Magnusson vd., 1999; An, Kulm ve Wu, 2004; Ball vd., 2008; Park ve Oliver, 2008; Kovarik, 2008; Hacıömeroglu, 2009; Baştürk, 2009; Yeşildere ve Akkoç, 2010; Bukova-Güzel, 2010) literatürde dörtgenler konusunu pedagojik alan bilgisi bileşenleri yönünden inceleyen çalışma sayısı oldukça azdır. Literatüre bu yönde katkı sağlamak için bu araştırmada matematik öğretmenlerinin dörtgenler konusunu işleyişlerinde, pedagojik alan bilgisinin öğrenci bilgisi bileşenini incelemek amaçlanmıştır.

Yöntem: Araştırmaya Türkiye'de bir ilde 12 farklı okulda görev yapmakta olan 30 ortaokul matematik öğretmeni katılmıştır. Araştırmada dörtgenler konusu incelenmiştir. Görüşme yöntemi kullanılmıştır. Görüşmelerde öğretmenlere sorulan sorular, araştırmacılar tarafından pedagojik alan bilgisi öğrenci bilgisi bileşenine yönelik hazırlanmıştır. Görüşme soruları altı sorudan oluşmakta olup, araştırma çerçevesine uygun hazırlanmıştır. Görüşmeler ortalama 30 dakika sürmüştür. İçerik analizi kullanılmıştır. Verilerin analizi iki aşamada gerçekleştirilmiştir. Birinci aşamada ses kaydı olarak elde edilen verilerin çözümlemesi yapılmıştır. İkinci aşamada, iki araştırmacı bir araya gelerek, araştırmada kullanılan çerçevede olası kodları belirlemiştir. Tüm bu işlemler 30 öğretmen için ayrı ayrı gerçekleştirilmiş ve analiz sonuçları bir araya getirilerek raporlaştırılmıştır.

Bulgular: Görüşmelerde, öğretmenler derslerde öğrencilerinin önceden öğrendikleri bilgileri de dikkate alarak ders işlediklerini, bunu iki farklı şekilde yaptıklarını belirtmişlerdir. Öğretmenlerin %67'si, öğrencilerin önceden öğrendikleri dörtgenleri hatırlatarak bağlantı kurduklarını belirtmişlerdir. Öğretmenlerin %33'ü benzer dörtgenleri birbirleriyle ilişkilendirerek, ön bilgilerle yeni bilgiler arasında bağlantı kurduklarını belirtmişlerdir. Öğretmenler dörtgenler konusuna yönelik öğrenci hatalarını; dörtgenleri tanımlamaya yönelik hatalar, görselliğe yönelik hatalar ve dörtgenlerde sınıflamaya yönelik hatalar olarak üç başlıkta gruplamışlardır. Dörtgenleri tanımlama hataları dörtgen özelliklerini bilmemekten ve formüle bağlı kalıp ezber kaynaklı hatalar olarak iki grupta incelenmiştir.

Görselleştirme hataları ise dörtgende açı çizim hatası, dörtgende kenar çizim hatası, dörtgende köşegen çizim hatası olarak üç grupta incelenmiştir. Öğrencilerin dörtgen konusunda yaptıkları son hata ise dörtgenlerde sınıflandırma, aile ilişkilerine yönelik belirlenen hatalar olarak belirlenmiştir. Bu hata tipinde, öğrencilerde, dörtgen özelliklerini iyi bilmemekten ötürü ortaya çıkan, dörtgenleri özelliklerine göre birbirleriyle gruplayamama durumlarında yaptıkları hatalar tespit edilmiştir. Öğrencilerde belirlenen anlama güçlükleri; yamuğa ait belirlenen güçlükler, diğer dörtgenlere ait güçlükler olarak tespit edilmiştir.

Tartışma, Sonuç ve Öneriler: Dörtgenleri tanımlama üzerine yapılan pek çok çalışmada, bireylerin kavrama dair oluşturdukları imge ile geometrik kavramları tanımlarken, kişisel tanımları kullandıkları tespit edilmiştir. Bu bağlamda, bu araştırmada öğretmenlerin ifadelerine göre, öğrencilerin tanımlama yaparken, kendi algılarında tanımlama yaptıkları, bu nedenle dörtgen tanımlarında hatalar ortaya çıktığı söylenebilir. Görselliğe yönelik hatada, öğrenciler şekillerin tipik çizimlerinin dışına çıkamadıkları, alıştıkları tipik çizimden farklı çizimler yapamadıkları belirlenmiştir. Bu sonuç, literatürle uyum göstermektedir. Bu araştırma yamukla ilgili sıkıntılara da işaret etmektedir. Araştırma da öğrencilerin yamuğu anlamakta güçlük yaşadıkları sonucuna ulaşılmıştır. Ortaya çıkan anlama güçlükleri de yamukta yaşanan birçok yanılgının varlığını ortaya koymaktadır. Çalışma daha genellenebilir sonuçlara ulaşmak için daha büyük öğretmen grubuyla yapılabilir. Farklı pedagojik alan bilgisi bileşenleri üzerinde incelemeler yapılıp, sonuçlar arasındaki ilişki incelenebilir.