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Research Article

Application of Kudryashov Method to Some Equations Used in Physics Science

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Abstract
In this study, Kudryashov Method is used to find the wave solutions of the Gardner equation, fifth order Caudrey-Dodd-Gibbon equation and Sawada-Kotera equation, which are non-linear partial differential equations used as a mathematical model in the physics science field and engineering applications. The exact solutions obtained are compared with the results in the literature and hyperbolic type and soliton solutions are obtained.
Keywords: Caudrey-Dodd-Gibbon equation, Gardner equation, Kudryashov Method, Sawada-Kotera equation

Fizik Biliminde Kullanılan Bazı Denklemlere Kudryashov Metodun Uygulanması

Öz

Bu çalışmada fizik bilim alanında ve mühendislik uygulamalarında matematiksel model olarak kullanılan lineer olmayan kısmi türevli diferansiyel denklemlerden Gardner denklemi, beşinci mertebeden Caudrey-Dodd-Gibbon denklemi ve Sawada-Kotera denkleminin dalga çözümlerini bulmak için Kudryashov Metot kullanılmıştır. Elde edilen tam çözümler literatürde bulunan sonuçlarla karşılaştırılmış ve hiperbolik tip ve soliton çözümler elde edilmiştir.

Anahtar Kelimeler: Caudrey-Dodd-Gibbon denklemi, Gardner denklemi, Kudryashov Metot, Sawada-Kotera denklemi

1. Introduction

Nonlinear partial differential equations appear as mathematical modeling of many problems encountered in the field of science. It is important to know about the analytical solutions of these mathematical models. Because these solutions give information about the character of the problems modeled. The solutions of these nonlinear partial differential equations used in scientific fields such as plasma physics, solid state physics, laser optics, fluid dynamics, chemical kinetics and mathematical biology, shed light on the different disciplines and guide them. In addition, mathematical model of waves such as sound waves, water waves, radio and television waves is also expressed by partial differential equations. When these

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partial differential equations which are considered as mathematical models are solved, a comment can be had about the problem. Therefore, many new methods have been developed to contribute to science. These developed methods reveal the different physical properties of mathematical models. Therefore, these methods have an important place in science. Some of these developed methods are as follows: Homotopy Perturbation Method He (2000), Adomian Decomposition Method Bildik et al (2006), $(G'/G, 1/G)$-expansion method Daghan and Dönmez (2016) and Kudryashov Method (Ryabov et al, 2011; Kabir et al, 2011; Kudryashov, 2012; Mirzazadeh, 2014; Kaplan, 2016).

In this study, exact solutions of the Sawada-Kotera (SK) equation, Caudrey-Dodd-Gibbon (CDG) equation and Gardner equation are obtained by using Kudryashov method. The obtained solutions are compared with the results in the literature and their graphs are demonstrated.

In the first part of this manuscript, the method of Kudryashov is explained and it is shown how this method is used. In the following chapters, the solutions of the Gardner equation, Caudrey-Dodd-Gibbon and Sawada-Kotera equations which are non-linear partial differential equations and which are used in scientific fields such as plasma physics, quantum field theory and fluid dynamics, are obtained with the help of Kudryashov method. In the last section, the results and discussions are given.

2. Kudryashov Method for NPDE

Assume a general structure of a (1+1)-dimensional nonlinear partial differential equation as

$$T(u,\ u_x, u_t, u_{xx}, u_{tt}, u_{xt}, ...) = 0\,, \qquad (1)$$

where T is a polynomial in the unknown function $u = u(x,t)$ and its partial derivatives. The steps of the Kudryashov Method can be summarized as follows:

The first step : To obtain the solutions of Eq.(1), Eq. (1) is turned into the following ordinary differential equation

$$D\left(U, U_\eta, U_{\eta\eta}, U_{\eta\eta\eta}, ...\right) = 0\,, \qquad (2)$$

using the transformation of $u(x,t) = U(\eta)$, $\eta = k_1 x + \nu t$ where k_1 and ν are constants.

The second step: The solution of Eq. (2) is assumed to be as follows:

$$U(\eta) = \sum_{i=0}^{n} a_i \varphi^i(\eta) \qquad (3)$$

where $a_i\,(\mathrm{i} = 0,1,...)$ are constants that will be determined in a way that $a_n \neq 0$. The following function $\varphi(\eta)$ in Eq. (3) gives the nonlinear ordinary differential equation:

$$\varphi'(\eta) = h_0 + h_1\varphi(\eta) + h_2\varphi^2(\eta) \qquad (4)$$

When $h_0 = 0, h_1 = -1, h_2 = 1$ in Eq.(4), the following nonlinear ordinary differential equation is obtained:

$$\varphi'(\eta) = \varphi^2(\eta) - \varphi(\eta)\,. \qquad (5)$$

Eq. (5) is solved as follows:

$$\varphi(\eta) = \frac{1}{1 \pm Ke^{\eta}}$$

where K is integration constant.

The third step: The positive integer n in Eq. (3) is obtained by taking by the balance between the highest order derivatives and the highest power nonlinear terms in Eq (2).

The fourth step: If Bernoulli differential Eq.(5) and Eq. (3) are substituted into Eq. (2), all coefficients of φ^i $(\mathrm{i}=0,1,...)$ are set to zero in the system of algebraic equations obtained for $a_i\,(\mathrm{i}=0,1,...)$.

The fifth step: The system of algebraic equations obtained in the previous step is solved using the programs such as Mathematica, Maple and the unknown parameters are found. More details can be seen in Refs. (Ryabov et al, 2011; Kabir et al, 2011; Kudryashov, 2012; Mirzazadeh, 2014; Kaplan, 2016).

3. Utilization of the Kudryashov Method to the Gardner Equation

Combined KdV–mKdV equation or the Gardner equation describe Eq. (6) (Zuntao et al, 2004; Wazwaz, 2007; Biswas, 2008; Kamchatnov et al, 2012; Betchewe et al, 2013; Alam and Akbar, 2014; Daghan and Dönmez 2016)

$$u_t + \alpha u u_x + \beta u^2 u_x + q u_{xxx} = 0 \qquad (6)$$

where α, β, q arbitrary constants. The equation given in Eq. (6) can be converted into the following ordinary differential equation by using the transformation of $u(x,t) = U(\eta)$, $\eta = x - pt$,

$$qU''' - pU' + \alpha U'U + \beta U'U^2 = 0. \qquad (7)$$

The integral of Eq.(7) based on the variable η is as follows

$$qU'' - pU + \frac{\alpha}{2}U^2 + \frac{\beta}{3}U^3 + c = 0 \qquad (8)$$

where c is an arbitrary integration constant.

Balancing between therms U″ and U³, we obtain the following form of solution

$$U(\eta) = a_0 + a_1\varphi(\eta) \qquad (9)$$

where $\varphi = \varphi(\eta)$ satisfies Eq. (5) and a_0, a_1 are arbitrary constants. Substituting Eq. (9) into Eq. (8) along with Eq. (5) and then equating all coefficients of the functions φ^k *(k=0,1,…,5)* to zero, we obtain

$$\varphi:\ 6a_1(a_0^2\beta + a_0\alpha + q\ \text{-}p) = 0,$$

$$\varphi^2:\ 3a_1(2a_1a_0\beta + a_1\alpha\ \text{-}6q) = 0,$$

$$\varphi^3:\ 2a_1(a_1^2\beta + 6\beta) = 0.$$

By using Mathematica, the solutions of this system can be obtained as follows

$$a_0 = -\frac{1}{2\beta}\left(\sqrt{\frac{-6q}{\beta}}\beta + \alpha\right),\quad a_1 = \mp\sqrt{\frac{-6q}{\beta}},$$
$$p = \frac{-\alpha^2 - 2\beta q}{4\beta},\quad cb^2 = \frac{\alpha(\alpha^2 + 6\beta q)}{24}. \qquad (10)$$

Hence, the solution of Eq.(8) is

$$U(\eta)=\frac{-\alpha(1+Ke^{\eta})+\sqrt{\frac{-6\beta}{q}}q(-1+Ke^{\eta})}{2\beta(1+Ke^{\eta})}. \quad (11)$$

In the case of $K=1$ in Eq.(11) is

$$U(\eta)=-\frac{\alpha+\sqrt{-6q\beta}\tanh\left(\frac{\eta}{2}\right)}{2\beta} \quad (12)$$

where $\eta = x - pt$. The graph of the conclusion given in Eq.(12) can be seen in Figure 1.

The hyperbolic function solutions of the Gardner equation are compatible solutions with the literature (Wazwaz, 2007; Daghan and Dönmez 2016).

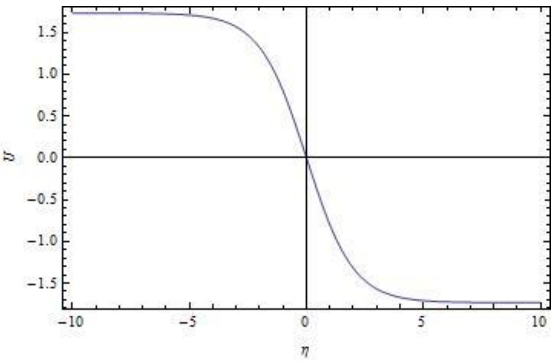

Figure 1. The exact solution according to $\alpha=0,\ \beta=1$ and $q=-2$ for Gardner Equation

4. The Generalized fifth-order KdV Equations

The standard form of the fifth-order KdV equation (Salas, 2008; Wazwaz, 2011);

$$u_t+u_{xxxxx}+\gamma uu_{xxx}+\beta u_x u_{xx}+\alpha u^2 u_x=0$$

where γ, β and α are arbitrary nonzero and real parameters. Some important particular states of the fifth- order Equation are:

Caudrey–Dodd–Gibbon equation (CDG)

$$u_t+u_{xxxxx}+30uu_{xxx}+30u_x u_{xx}+180u^2u_x=0$$

and Sawada-Kotera Equation (SK)

$$u_t+u_x^{(5)}+5(\text{uu}_{xxx}+u_x u_{xx}+u^2u_x)=0$$

for $\gamma=\beta$, $\alpha=\frac{1}{5}\gamma^2$.

4.1 Utilization of the Kudryashov Method to the CDG Equation

CDG equation is

$$u_t+u_{xxxxx}+30uu_{xxx}+30u_x u_{xx}+180u^2u_x=0$$
(13)

(Wazwaz, 2006; Salas et al, 2008; Wazwaz, 2008; Jiang and Bi, 2010; Karaagac, 2019).

The equation given in Eq. (13) can be converted into the following ordinary differential equation by using the transformation of

$$u(x,t)=U(\eta),\ \eta=kx+wt,$$
$$wU'+k^5U^{(5)}+30k^3(U''U)'+60k(U^3)'=0.$$
(14)

Integrating Eq. (14), we obtain

$$wU+k^5U^{(4)}+30k^3U''U+60kU^3+c=0$$
(15)

where c is an arbitrary integration constant.

Balancing between therms $U^{(4)}$ and U³, we obtain the following form of solution

$$U = a_0 + a_1\varphi + a_2\varphi^2 \qquad (16)$$

where $\varphi = \varphi(\eta)$ satisfies (5) and a_0, a_1, a_2 are arbitrary constants. Substituting Eq.(16) into Eq.(15) along with Eq.(5) and then equating all coefficients of the functions $\varphi^i\,(\mathrm{i}=0,1,2,...6)$ to zero, we obtain

$$\varphi^0 : 60a_0^3k + 30a_0w + c = 0,$$

$$\varphi^1 : a_1\left(180a_0^2k + 30a_0k^3 + k^5 + w\right) = 0,$$

$$\varphi^2 : 180a_2a_0^2k + 120a_2a_0k^3 + 16a_2k^5 + a_2w + 180a_1^2a_0k + 30a_1^2k^3 - 90a_1a_0k^3 - 15a_1k^5 = 0,$$

$$\varphi^3 : 10k(36a_2a_1a_0 + 15a_2a_1k^2 - 30a_2a_0k^2 - 13a_2k^4 + 6a_1^3 - 9a_1{}^2k^2 + 6a_1a_0k^2 + 5a_1k^4) = 0,$$

$$\varphi^4 : 30k(6a_2^2a_0 + 4a_2{}^2k^2 + 6a_2a_1^2 - 13a_2a_1k^2 + 6a_2a_0k^2 + 11a_2k^4 + 2a_1k^2 - 2a_1k^4 = 0,$$

$$\varphi^5 : 12k\left(15a_2^2a_1 - 25a_2{}^2k^2 + 20a_2a_1k^2 - 28a_2k^4 + 2a_1k^4\right) = 0,$$

$$\varphi^6 : 60a_2k\left(a_2^2 + 3a_2k^2 + 2k^4\right) = 0.$$

By using Mathematica, the solutions of this system can be obtained as follows

$$a_0 = \frac{-k^2}{6},\ \ a_1 = 2k^2,\ \ a_2 = -2k^2,$$
$$w = \frac{18c - 5k^7}{3k^2},\ c = \frac{k^7}{9} \qquad (17)$$

The solution of Eq. (15) corresponding to (17) is

$$U(\eta) = \frac{-k^2(1 - 10Ke^{\eta} + K^2e^{2\eta})}{6(1 + Ke^{\eta})^2}. \qquad (18)$$

In the case of $K = 1$ in Eq. (18) is

$$U(\eta) = \frac{-k^2(\cosh\eta - 5)}{6(\cosh\eta + 1)} \qquad (19)$$

where $\eta = kx + wt$. The graph of the conclusion given in Eq. (19) can be seen in Figure 2.

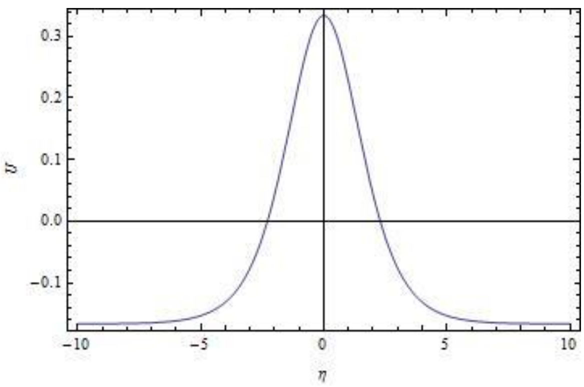

Figure 2. The exact solution according to $K = 1$ and $k = 1$ for CDG Equation

The solutions of the CDG equation is compatible with solutions in the reference Wazwaz (2008).

4.2. Utilization of the Kudryashov Method to the SK Equation

The SK equation is

$$u_t + u_x^{(5)} + 5(\mathrm{uu}_{xxx} + u_x u_{xx} + u^2 u_x) = 0 \quad (20)$$

(Salas et al, 2008; Wazwaz, 2011; Shakeel and Mohyud-Din, 2014).

The equation given in Eq. (20) can be converted into the following ordinary differential equation by using the transformation of

$$u(x,t) = U(\eta),\ \eta = kx + wt,$$
$$wU' + k^5 U^{(5)} + 5k^3 U U^{(3)} + 5k^3 U' U'' + 5kU^2 U' = 0 \quad (21)$$

The integral of Eq.(21) based on the variable η is as follows

$$wU + k^5 U^{(4)} + 5k^3 U'' U + \frac{5k}{3} U^3 + c = 0 \quad (22)$$

where c is an arbitrary integration constant.

Balancing between therms $U^{(4)}$ and U^3, we obtain the following form of solution

$$U = a_0 + a_1\varphi + a_2\varphi^2 \quad (23)$$

where $\varphi = \varphi(\eta)$ satisfies (5) and a_0, a_1, a_2 are arbitrary constants. Substituting Eq.(23) into Eq.(22) along with Eq.(5) for $K = 1$ and then equating all coefficients of the functions $\varphi^i (\mathrm{i} = 0,1,2,...6)$ to zero, we obtain

$$\varphi^6 : \frac{5a_2 k\left(a_2^2 + 18a_2k^2 + 72k^4\right)}{3} = 0,$$

$$\varphi^5 : k(5a_2^2 a_1 - 50a_2^2k^2 + 40a_2a_1k^2 - 336a_2k^4 + 24a_1k^4) = 0,$$

$$\varphi^4 : 5k(a_2^2a_0 + 4a_2^2k^2 + a_2a_1^2 - 13a_2a_1k^2 + 6a_2a_0k^2 + 66a_2k^4 + 2a_1^2k^2 - 12a_1k^4) = 0,$$

$$\varphi^3 : \frac{5k}{3}(6a_2a_1a_0 + 15a_2a_1k^2 - 30a_2a_0k^2 - 78a_2k^4 + a_1^3 - 9a_1^2k^2 + 6a_1a_0k^2 + 30a_1k^4) = 0,$$

$$\varphi^2 : 5a_2a_0^2k + 20a_2a_0k^3 + 16a_2k^5 + a_2w + 5a_0a_1^2k + 5a_0^2k^3 - 15a_1a_0k^3 - 15a_1k^5 = 0,$$

$$\varphi^1 : a_1\left(50a_0^2k + 5a_0k^3 + k^5 + w\right) = 0,$$

$$\varphi^0 : \frac{5a_0^3k + 3a_0w + 3c}{3} = 0$$

By using Mathematica, the solutions of this system can be obtained as follows

$$a_2 = -12k^2,\ a_1 = 12k^2,\ a_0 = -k^2,\ w = -k^5,\ c = \frac{2k^7}{3} \quad (24)$$

The solution of Eq.(22) corresponding to (24) is

$$U(\eta) = \frac{-k^2\left(1 - 10Ke^\eta + K^2e^{2\eta}\right)}{(1+Ke^\eta)^2}. \quad (25)$$

In the case of $K = 1$ in Eq.(25) is

$$U(\eta) = -\frac{k^2\left(-5 + \cosh\eta\right)}{1 + \cosh\eta} \quad (26)$$

where $\eta = kx + wt$. The graph of the conclusion given in Eq. (26) can be seen in Figure 3.

The solutions of the SK equation is compatible with solutions in the reference Shakeel and Mohyud (2015).

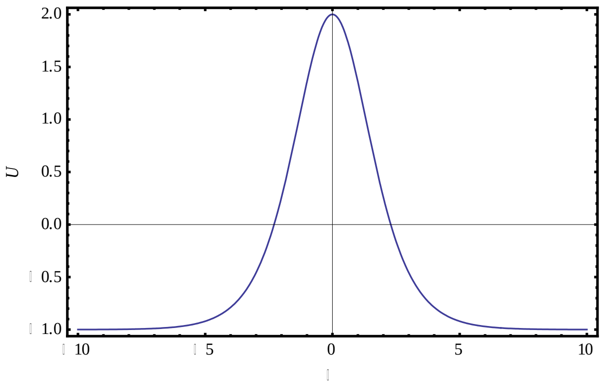

Figure 3. The exact solution according to $K=1$ and $k=1$ for SK Equation

5. Discussions and Conclusion

In this study, the exact solutions of Gardner, CDG and SK equations are obtained with the help of Mathematica by using the Kudryashov Method. Furthermore, given solutions are substituted in the Eq. (8), Eq. (15) and Eq. (22) and this process increased the reliability of the results. These soliton wave solutions are important for analyzing the physical aspects. Kudryashov method can be applied to other nonlinear equations used in physics and engineering.

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