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Classification of the Monolithic Columns Produced in Troad and Mysia Region Ancient Granite Quarries in Northwestern Anatolia via Soft Decision-Making

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Abstract: Ay and Tolun [An Archaeometric Approach on the Distribution of Troadic Granite Columns in the Western Anatolian Coasts. Journal of Archaeology & Art, 156, 2017, 119-130 (In Turkish)] have analysed the distribution of the monolithic columns produced in the ancient granite quarries, located in Troad Region and Mysia Region in Northwestern Anatolia, by archaeometric analyses. Moreover, they have achieved some results by interpreting the prominent data obtained therein. In this study, we propose a novel soft decision-making method, i.e. Monolithic Columns Classification Method (MCCM), constructed via fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) and Prevalence Effect Method (PEM). MCCM provides an outcome by interpreting all the results of the analyses mentioned above. We then apply the method to the problem of monolithic columns classification. Finally, we discuss the need for further research.

Keywords: Ancient Granite Quarries, Classification, *fpfs*-matrices, Monolithic Columns, Soft Decision-Making

1. Introduction

In the Roman Imperial Period, Troad Region and Mysia Region are two essential regions contained ancient granite quarries (Figure 1. a.) (Galetti et al., 1992; Williams-Thorpe and Thorpe, 1993; Williams-Thorpe and Henty, 2000) such as Koçali (Figure 1. b.), Akçakeçili (Figure 1. c.), and Kozak (Figure 1. d.) which known to be produced monolithic granite columns in Anatolia. While Koçali and Akçakeçili ancient granite quarries in Troad Region (Ponti, 1995; Ay, 2017; Ay and Tolun, 2017a, b) are located in Ezine/Çanakkale, Kozak ancient granite quarry in Mysia Region (Williams-Thorpe et al., 2000) is located in Bergama/Izmir.

However, there are not exist a sufficient number of an archaeological document about some subjects such as the exportation of the columns produced in these centres located in Troad and Mysia Region.

For this reason, to locate the source of a column considered in an ancient city, the method commonly used is to compare some archaeological samples taken from this city and some geological samples taken from the granite quarries by using mineralogical-petrographic and geochemical analyses (Williams-Thorpe and Thorpe, 1993; Williams-Thorpe and Henty, 2000; Williams-Thorpe et al., 2000; Potts, 2002; Williams-Thorpe, 2008; Ay, 2017; Ay and Tolun, 2017b).

The mineralogical-petrographic analyses are an examination of the samples in a microscopic environment using their thin sections. These analyses carry out to determine the types, quantities, sizes, and shapes of the minerals forming the rock types, main and secondary components of the samples (Galetti et al., 1992; Williams-Thorpe, 2008; Ay, 2017; Ay and Tolun,

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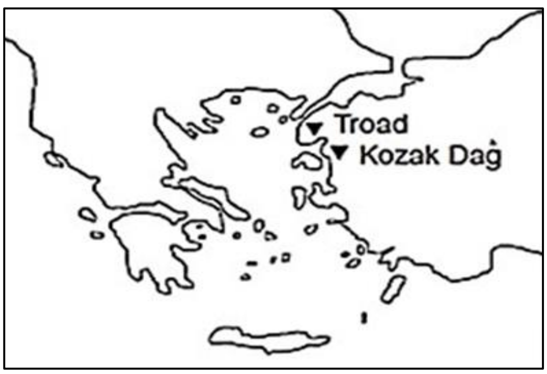
2017b). The geochemical analyses perform in determining the type and number of major elements contained in the samples (Galetti et al., 1992; Potts, 2002; Williams-Thorpe, 2008).

Recently, Ay and Tolun have examined the distribution in Northwestern Anatolia of the monolithic columns produced in the ancient granite quarries, located in Troad Region and Mysia Region, by using archaeometric methods (Ay, 2017; Ay and Tolun, 2017b). For this aim, by using the qualitative mineralogical-petrographic and geochemical analyses, they have compared the geological samples taken from Koçali-Akçakeçili ancient quarries in Troad Region and Kozak ancient quarry in Mysia Region with the archaeological samples taken from Smintheion (Smintheion 1, Smintheion 2), Pergamon Red Hall/Serapeion, Smyrna Agora (Smyrna Agora 1, Smyrna Agora 2), Tlos Stadium, Tlos Theatre, and Side Theatre.

Moreover, Ay and Tolun have divided the samples into two groups as ancient granite quarries and ancient city (Figure 2). They first have compared the results of each group in itself. Afterwards, they have compared separately the archaeological samples with the geological samples and have revealed which archaeological samples are more similar to which geological.

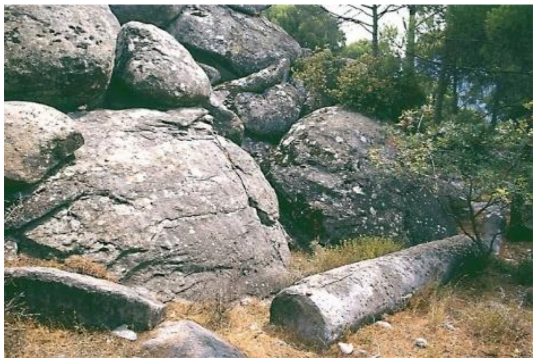
The results show that the granite columns in Smintheion 1, Smintheion 2, Smyrna Agora 2, Tlos Stadium, and Side Theatre may originate from the Koçali-Akçakeçili granite quarries located in Troad Region while the others may originate from Kozak quarry located in Mysia Region.



a.

b.

c.



d.

Figure 1. a. Troad and Kozak ancient quarries in the Roman period (Williams-Thorpe, 2008) b. Akçakeçili quarry c. Koçali quarry d. Kozak quarry (De Vecchi et al., 2000)

The concept of soft sets was introduced by Molodtsov (1999) to cope with uncertainty and have been applied to many areas from analysis to decision-making problems (Maji et al., 2001; Çağman and Enginoğlu, 2010; Çağman et al., 2010; Çağman et al., 2011a; Çağman and Deli, 2012; Deli and Çağman, 2015; Enginoğlu and Demiriz, 2015; Enginoğlu and Dönmez, 2015; Enginoğlu et al., 2015; Karaaslan, 2016; Şenel, 2016; Zorlutuna and Atmaca, 2016; Atmaca, 2017; Bera et al., 2017; Çıtak and Çağman, 2017; Şenel, 2017; Çıtak, 2018; Enginoğlu and Memiş, 2018a, b, c, d; Enginoğlu et al., 2018a, b, c, d; Gulistan et al., 2018; Mahmood et al., 2018; Riaz and Hashmi, 2018; Riaz et al., 2018; Şenel, 2018; Ullah et al., 2018). Recently, some soft decision-making methods constructed by fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) have enabled data processing in many problems containing uncertainty. Being one of these methods, Prevalence Effect Method (PEM) (Enginoğlu and Çağman, In Press) has been applied to a performance-based value assignment to some methods used in noise removal so that the methods can be ordered in terms of performance. We use this method for classification the monolithic columns mentioned in (Ay, 2017; Ay and Tolun, 2017b). The results show that Monolithic Columns Classification Method (MCCM) is successfully model the monolithic columns classification (MCC) problem. Here, *fpfs*-matrices have a row consisting of the significance degrees (membership degrees) of the parameters. These values are usually determined by consulting an expert.

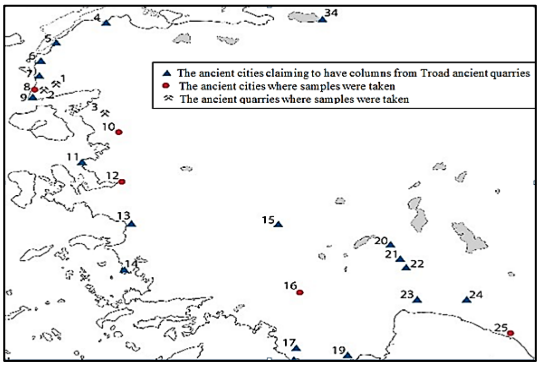


Figure 2. The estimated-distribution of Troad granite columns in Western Anatolia (Ay and Tolun, 2017b)

In this study, we have identified the values, that is, the weights of archaeometric and geochemical parameters, concerning the opinions mentioned in (Ay, 2017; Ay and Tolun, 2017b). Moreover, Ay and Tolun have considered of more effective the geochemical data than the archaeometric data. Therefore, we set a higher value to geochemical data than archaeometric data in the final decision step.

In Section 2 of the present study, we present the concept of *fpfs*-matrices and PEM. In Section 3, we give all the results of the qualitative mineralogical-petrographic and geochemical analyses provided in (Ay, 2017; Ay and Tolun, 2017b). In Section 4, we propose a new method, i.e. MCCM. In section 5, we apply MCCM to the MCC problem. Finally, we discuss the need for further research.

2. Preliminaries

In this section, we first present the concept of fuzzy soft matrices (*fs*-matrices) (Çağman and Enginoğlu, 2012). Throughout this paper, let U be universal set, E be a parameter set, $F(E)$ be the set of all fuzzy sets over E, and $\mu \in F(E)$. Here, a fuzzy set is denoted by $\left\{{}^{\mu(x)}x : x \in E\right\}$.

Definition 2.1. (Çağman et al., 2011b) Let U be a universal set, E be a parameter set, and α be a function from E to $F(U)$. Then, the set $\{(x, \alpha(x)) : x \in E\}$ being the graphic of α is called a fuzzy soft set (*fs*-set) parameterized via E over U (or briefly over U).

In the present paper, the set of all *fs*-sets over U is denoted by $FS_E(U)$. In $FS_E(U)$, since the graphic of α ($graph(\alpha)$) and α generate each other uniquely, the notations are interchangeable. Therefore, as long as it does not cause any confusion, we denote an *fs*-set $graph(\alpha)$ by α.

Example 2.1. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Then,

$$\alpha = \left\{\left(x_1, \{{}^{0.9}u_1, {}^{0.5}u_4\}\right), \left(x_2, \{{}^{0.3}u_2, {}^{0.5}u_3\}\right), \left(x_3, \{{}^{0.7}u_1, {}^{0.8}u_3, {}^{0.6}u_4\}\right), \left(x_4, \{u_3, {}^{0.9}u_5\}\right)\right\}$$

is an *fs*-set over U. Here, for brevity, the notation u_3 is used instead of ${}^{1}u_3$ and also the elements which have zero membership value such as ${}^{0}u_3$ does not show in the sets containing them.

Definition 2.2. (Çağman and Enginoğlu, 2012) Let $\alpha \in FS_E(U)$. Then, $[a_{ij}]$ is called the matrix representation of α (or briefly *fs*-matrix of α) and is defined by

$$[a_{ij}] := \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{1,2,\cdots\}$ and $j \in \{1,2,\cdots\}$, $a_{ij} := \alpha(x_j)(u_i)$, where $\alpha(x_j)(u_i)$ refers to the membership degree of u_i in the fuzzy set $\alpha(x_j)$.Here, if $|U| = m$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

From now on, the set of all fs-matrices parameterized via E over U is denoted by $FS_E[U]$.

Example 2.2. The fs-matrix of α provided in Example 2.1 is as follows:

$$[a_{ij}] = \begin{bmatrix} 0.9 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0.8 & 1 \\ 0.5 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Secondly, we present the concept of *fpfs*-matrices.

Definition 2.3. (Çağman et al., 2010; Enginoğlu, 2012) Let U be a universal set, $\mu \in F(E)$, and α be a function from μ to $F(U)$. Then, the set $\{({}^{\mu(x)}x, \alpha({}^{\mu(x)}x)) : x \in E\}$ being the graphic of α is called a fuzzy parameterized fuzzy soft set (*fpfs*-set) parameterized via E over U (or briefly over U).

In the present paper, the set of all *fpfs*-sets over U is denoted by $FPFS_E(U)$. In $FPFS_E(U)$, since the $graph(\alpha)$ and α generate each other uniquely, the notations are interchangeable. Therefore, as long as it does not cause any confusion, we denote an *fpfs*-set $graph(\alpha)$ by α.

Example 2.3. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Then,

$$\alpha = \{({}^{0.8}x_1, \{{}^{0.9}u_1, {}^{0.5}u_4\}), ({}^{0}x_2, \{{}^{0.3}u_2, {}^{0.5}u_3\}), ({}^{0.1}x_3, \{{}^{0.7}u_1, {}^{0.8}u_3, {}^{0.6}u_4\}), ({}^{1}x_4, \{{}^{1}u_3, {}^{0.9}u_5\})\}$$

is an *fpfs*-set over U.

Definition 2.4. (Enginoğlu, 2012; Enginoğlu and Çağman, In Press) Let $\alpha \in FPFS_E(U)$. Then, $[a_{ij}]$ is called the matrix representation of α (or briefly *fpfs*-matrix of α) and is defined by

$$[a_{ij}] \coloneqq \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0,1,2,\cdots\}$ and $j \in \{1,2,\cdots\}$,

$$a_{ij} \coloneqq \begin{cases} \mu(x_j), & i = 0 \\ \alpha({}^{\mu(x_j)}x_j)(u_i), & i \neq 0 \end{cases}$$

Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

Herein, the set of all *fpfs*-matrices parameterized via E over U is denoted by $FPFS_E[U]$.

Example 2.4. The *fpfs*-matrix of α provided in Example 2.3 is as follows:

$$[a_{ij}] = \begin{bmatrix} 0.8 & 0 & 0.1 & 1 \\ 0.9 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0.8 & 1 \\ 0.5 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Definition 2.5. (Enginoğlu, 2012; Enginoğlu and Çağman, In Press) Let $[a_{ij}] \in FPFS_E[U]$. For all i and j, if $a_{ij} = \lambda$, then $[a_{ij}]$ is called λ-*fpfs*-matrix and is denoted by $[\lambda]$. Here, $[0]$ is called empty *fpfs*-matrix and $[1]$ is called universal *fpfs*-matrix.

Definition 2.6. (Enginoğlu, 2012; Enginoğlu and Çağman, In Press) Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. For all i and j, if $c_{ij} \coloneqq |a_{ij} - b_{ij}|$, then $[c_{ij}]$ is called symmetric difference between $[a_{ij}]$ and $[b_{ij}]$ and is denoted by $[a_{ij}]\tilde{\Delta}[b_{ij}]$.

Finally, we present the soft decision-making method PEM provided in (Enginoğlu and Çağman, In Press). Throughout this paper, $I_n \coloneqq \{1,2,\dots,n\}$ and $I_n^* \coloneqq \{0,1,2,\dots,n\}$.

PEM Algorithm Steps

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{(m+1)\times n}$ such that $i \in I_m^*$ and $j \in I_n$

Step 2. Obtain a matrix $[s_{i1}]$ defined by

$$s_{i1} \coloneqq \sum_{j=1}^{n} \left[\left(\frac{1}{m}\sum_{k=1}^{m} a_{kj}\right)\left(\frac{1}{n}\sum_{t=1}^{n} a_{it}\right) a_{0j} a_{ij}\right], i \in I_m$$

Step 3. Obtain a decision set $\left\{ {}^{\frac{s_{i1}}{\max_k s_{k1}}}u_i \mid u_i \in U \right\}$

3. The Qualitative Mineralogical-Petrographic and Geochemical Analyses Results

In this section, we give tables of the results of the qualitative mineralogical-petrographic and geochemical analyses provided in (Ay, 2017; Ay and Tolun, 2017b). The qualitative mineralogical-petrographic analyses result from Koçali and Akçakeçili are the same, and the geochemical analyses results are close to each other. Since Koçali and Akçakeçili ancient quarries are the same structure, Ay and Tolun have compared eight samples with two sources: Bergama Kozak and Koçali-Akçakeçili in (Ay, 2017; Ay and Tolun, 2017b). Therefore, in the next section, we use the mean results from Koçali and Akçakeçili.

Table 1. Results of the mineralogical-petrographic analyses of alkali feldspars in thin sections

Alkali Feldspars	*Coarse−Medium−Fine*	*Coarse −Medium*	*Medium−Fine*	*Fine*	*Chloritised*	*Sericitised*	*Pertitic*	*Mirmekitic*	*Carbonated*	*Clayed*	*Idiomorphic*	*Hypidiomorphic*	*Xenomorphic*	*Homogeneous*	*Recrystallised*	*Prismatic*	*Flat Prismatic*	*Clean-Surfaced*	*Twinning*
Side Theatre	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Tlos Theatre	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Tlos Stadium	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Smyrna Agora 1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Smyrna Agora 2	1	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0
Pergamon Red Hall	1	0	0	0	0	0	0	1	0	1	0	0	1	1	0	0	0	0	0
Smintheion 1	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
Smintheion 2	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0
Bergama Kozak	0	0	1	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0	0
Koçali	1	0	0	0	0	0	1	1	0	1	0	1	1	1	0	0	0	0	1
Akçakeçili	1	0	0	0	0	0	1	1	0	1	0	1	1	1	0	0	0	0	1

Table 2. Results of the mineralogical-petrographic analyses of amphiboles in thin-sections

Amphiboles	*Coarse−Medium−Fine*	*Coarse −Medium*	*Medium−Fine*	*Fine*	*Chloritised*	*Sericitised*	*Pertitic*	*Mirmekitic*	*Carbonated*	*Clayed*	*Idiomorphic*	*Hypidiomorphic*	*Xenomorphic*	*Homogeneous*	*Recrystallised*	*Prismatic*	*Flat Prismatic*	*Clean-Surfaced*	*Twinning*
Side Theatre	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Tlos Theatre	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Tlos Stadium	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Smyrna Agora 1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Smyrna Agora 2	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0
Pergamon Red Hall	1	0	1	0	1	0	0	0	1	0	0	1	0	1	0	1	0	0	0
Smintheion 1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Smintheion 2	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Bergama Kozak	0	0	1	0	0	0	0	0	1	0	1	1	0	1	0	1	0	0	0
Koçali	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	0
Akçakeçili	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	0

Table 3. Results of the mineralogical-petrographic analyses of biotite in thin-sections

Biotite	*Coarse−Medium−Fine*	*Coarse −Medium*	*Medium−Fine*	*Fine*	*Chloritised*	*Sericitised*	*Pertitic*	*Mirmekitic*	*Carbonated*	*Clayed*	*Idiomorphic*	*Hypidiomorphic*	*Xenomorphic*	*Homogeneous*	*Recrystallised*	*Prismatic*	*Flat Prismatic*	*Clean-Surfaced*	*Twinning*
Side Theatre	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Tlos Theatre	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Tlos Stadium	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Smyrna Agora 1	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Smyrna Agora 2	0	0	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0
Pergamon Red Hall	0	0	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0
Smintheion 1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Smintheion 2	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Bergama Kozak	0	0	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0
Koçali	0	0	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0
Akçakeçili	0	0	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0

Table 4. Results of the mineralogical-petrographic analyses of plagioclase in thin-sections

Plagioclase	*Coarse–Medium–Fine*	*Coarse –Medium*	*Medium–Fine*	*Fine*	*Chloritised*	*Sericitised*	*Pertitic*	*Mirmekitic*	*Carbonated*	*Clayed*	*Idiomorphic*	*Hypidiomorphic*	*Xenomorphic*	*Homogeneous*	*Recrystallised*	*Prismatic*	*Flat Prismatic*	*Clean-Surfaced*	*Twinning*
Side Theatre	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Tlos Theatre	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Tlos Stadium	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Smyrna Agora 1	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Smyrna Agora 2	1	0	0	0	0	1	0	0	1	1	0	1	0	1	0	0	1	0	0
Pergamon Red Hall	0	0	1	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0
Smintheion 1	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1
Smintheion 2	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Bergama Kozak	0	0	1	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0
Koçali	1	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	1
Akçakeçili	1	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	1

Table 5. Results of the mineralogical-petrographic analyses of quartz in thin-sections

Quartz	*Coarse–Medium–Fine*	*Coarse –Medium*	*Medium–Fine*	*Fine*	*Chloritised*	*Sericitised*	*Pertitic*	*Mirmekitic*	*Carbonated*	*Clayed*	*Idiomorphic*	*Hypidiomorphic*	*Xenomorphic*	*Homogeneous*	*Recrystallised*	*Prismatic*	*Flat Prismatic*	*Clean-Surfaced*	*Twinning*
Side Theatre	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Tlos Theatre	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Tlos Stadium	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Smyrna Agora 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Smyrna Agora 2	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Pergamon Red Hall	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
Smintheion 1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0
Smintheion 2	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0
Bergama Kozak	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
Koçali	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0
Akçakeçili	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0

Table 6. Results of the geochemical analyses of thin-sections

Geochemical Analyses	CaO	Fe_2O_3	MgO	P_2O_5	SiO_2	Al_2O_3	K_2O	MnO	Na_2O	TiO_2
Side Theatre	5.9	5.0	2.0	0.7	59.5	16.3	5.1	0.2	3.4	0.8
Tlos Theatre	5.0	4.5	2.2	0.3	63.5	15.8	3.7	0.1	3.2	0.7
Tlos Stadium	5.3	4.9	1.9	0.6	60.5	16.6	5.1	0.1	3.5	0.7
Smyrna Agora 1	4.4	3.9	1.0	0.1	67.4	15.7	2.8	0.1	3.4	0.5
Smyrna Agora 2	5.7	5.2	2.3	0.5	59.6	16.8	4.1	0.1	3.6	0.9
Pergamon Red Hall	5.5	4.6	2.1	0.3	61.9	16.3	3.4	0.1	3.3	0.7
Smintheion 1	4.6	3.8	1.7	0.4	63.6	15.9	4.4	0.1	3.5	0.6
Smintheion 2	4.8	3.9	1.7	0.4	63.8	15.8	4.3	0.1	3.5	0.6
Bergama Kozak	4.7	4.2	2.0	0.3	64.2	15.9	3.7	0.1	3.4	0.7
Koçali	5.0	4.6	1.9	0.5	61.5	16.4	4.5	0.1	3.6	0.7
Akçakeçili	5.0	4.4	1.9	0.5	61.6	16.1	4.7	0.1	3.5	0.7
Koçali-Akçakeçili	5.0	4.5	1.9	0.5	61.55	16.25	4.6	0.1	3.55	0.7

Table 7. Results of the geochemical analyses of thin-sections (normalised via maximum value in Table 6)

Geochemical Analyses (Normalised)	CaO	Fe_2O_3	MgO	P_2O_5	SiO_2	Al_2O_3	K_2O	MnO	Na_2O	TiO_2
Side Theatre	0.0875	0.0742	0.0297	0.0104	0.8828	0.2418	0.0757	0.0030	0.0504	0.0119
Tlos Theatre	0.0742	0.0668	0.0326	0.0045	0.9421	0.2344	0.0549	0.0015	0.0475	0.0104
Tlos Stadium	0.0786	0.0727	0.0282	0.0089	0.8976	0.2463	0.0757	0.0015	0.0519	0.0104
Smyrna Agora 1	0.0653	0.0579	0.0148	0.0015	1.0000	0.2329	0.0415	0.0015	0.0504	0.0074
Smyrna Agora 2	0.0846	0.0772	0.0341	0.0074	0.8843	0.2493	0.0608	0.0015	0.0534	0.0134
Pergamon Red Hall	0.0816	0.0682	0.0312	0.0045	0.9184	0.2418	0.0504	0.0015	0.0490	0.0104
Smintheion 1	0.0682	0.0564	0.0252	0.0059	0.9436	0.2359	0.0653	0.0015	0.0519	0.0089
Smintheion 2	0.0712	0.0579	0.0252	0.0059	0.9466	0.2344	0.0638	0.0015	0.0519	0.0089
Bergama Kozak	0.0697	0.0623	0.0297	0.0045	0.9525	0.2359	0.0549	0.0015	0.0504	0.0104
Koçali-Akçakeçili	0.0742	0.0668	0.0282	0.0074	0.9132	0.2411	0.0682	0.0015	0.0527	0.0104

4. Research Method

In this section, we first present MCCM and which also uses the abilities of PEM (Enginoğlu and Çağman, In Press).

Algorithm Steps of MCCM

Pre-processing Steps for Archaeometric Data

Step 1. Construct *fs*-matrices $\left[a_{ij}^{z}\right]_{m\times n}$ for archaeological samples, for all $z\in I_w$

Step 2. Construct *fs*-matrices $\left[b_{kj}^{z}\right]_{r\times n}$ for geological samples, for all $z\in I_w$

Step 3. Obtain $\left[c_{ij}^{zk}\right]_{m\times n}$ defined by $c_{ij}^{zk}\coloneqq b_{kj}^{z}$ such that $z\in I_w$ and $k\in I_r$

Step 4. Obtain $\left[d_{ij}^{zk}\right]_{m\times n}$ defined by $\left[d_{ij}^{zk}\right]\coloneqq[1]-\left[c_{ij}^{zk}\right]\tilde{\Delta}\left[a_{ij}^{z}\right]$ such that $z\in I_w$ and $k\in I_r$

Main Steps for Archaeometric Data

Step 1. Construct *fpfs*-matrices $\left[e_{ij}^{zk}\right]_{(m+1)\times n}$ such that $i\in I_m^*$, $j\in I_n$, and $i\neq 0\Rightarrow e_{ij}^{zk}\coloneqq d_{ij}^{zk}$

Step 2. Apply PEM to $\left[e_{ij}^{zk}\right]$ for all $z\in I_w$ and $k\in I_r$. That is, obtain $[f_{ik}^{z}]_{m\times r}$ defined by

$$f_{ik}^{z}\coloneqq\left(\frac{1}{n}\sum_{l=1}^{n}e_{il}^{zk}\right)\sum_{j=1}^{n}\left[\left(\frac{1}{m}\sum_{p=1}^{m}e_{pj}^{zk}\right)e_{0j}^{zk}e_{ij}^{zk}\right]$$

such that $z\in I_w$ and $k\in I_r$

Step 3. Obtain $[g_{ik}]_{m\times r}$ defined by $g_{ik}\coloneqq\frac{1}{w}\sum_{z=1}^{w}f_{ik}^{z}$

Step 4. Obtain $[s_{ik}^{1}]_{m\times r}$ defined by

$$s_{ik}^{1}\coloneqq\begin{cases}\dfrac{g_{ik}}{\max\limits_{t\in I_r}g_{it}}, & \max\limits_{t\in I_r}g_{it}\neq 0\\ g_{ik}, & \max\limits_{t\in I_r}g_{it}=0\end{cases}$$

Pre-processing Steps for Geochemical Data

Step 1. Construct *fs*-matrices $\left[a_{ij}\right]_{m\times n}$ for archaeological samples

Step 2. Construct *fs*-matrices $\left[b_{ij}\right]_{m\times n}$ for geological samples

Step 3. Obtain *fs*-matrices $\left[c_{ij}^{k}\right]_{m\times n}$ defined by $c_{ij}^{k}\coloneqq b_{kj}$ such that $k\in I_r$

Step 4. Obtain *fs*-matrices $\left[d_{ij}^{k}\right]_{m\times n}$ defined by $\left[d_{ij}^{k}\right]\coloneqq[1]-\left[c_{ij}^{k}\right]\tilde{\Delta}\left[a_{ij}\right]$ such that $k\in I_r$

Main Steps for Geochemical Data

Step 1. Construct *fpfs*-matrices $\left[e_{ij}^{k}\right]_{(m+1)\times n}$ such that $i\in I_m^*$, $j\in I_n$, and $i\neq 0\Rightarrow e_{ij}^{k}\coloneqq d_{ij}^{k}$

Step 2. Apply PEM to $\left[e_{ij}^{k}\right]$ for all $k\in I_r$. That is, obtain $[f_{ik}]_{m\times r}$ defined by

$$f_{ik}\coloneqq\left(\frac{1}{n}\sum_{l=1}^{n}e_{il}^{k}\right)\sum_{j=1}^{n}\left[\left(\frac{1}{m}\sum_{p=1}^{m}e_{pj}^{k}\right)e_{0j}^{k}e_{ij}^{k}\right]$$

such that $i\in I_m$ and $k\in I_r$

Step 3. Obtain $[s_{ik}^{2}]_{m\times r}$ defined by

$$s_{ik}^{2}\coloneqq\begin{cases}\dfrac{f_{ik}}{\max\limits_{t\in I_r}f_{it}}, & \max\limits_{t\in I_r}f_{it}\neq 0\\ f_{ik}, & \max\limits_{t\in I_r}f_{it}=0\end{cases}$$

Output Steps

Step 1. Obtain the decision matrix $[s_{ik}]_{m\times r}$ such that $s_{ik}=0.25s_{ik}^{1}+0.75s_{ik}^{2}$

Step 2. Obtain the decision sets $D_k\coloneqq\{u_i\,|s_{ik}=\max\limits_{p}s_{ip}\}$ such that $k,p\in I_r$

Secondly, we illustrate MCCM for $z=k=2$, that is, for the Amphibols data given in the previous. Faithfully to the Ay and Tolun's opinions, we set

[0.01, 0.01, 0.01, 0.01, 1, 1, 1, 1, 1, 1, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01]

and

[0.6, 0.6, 0.01, 0.7, 1, 0.8, 1, 0.01, 0.7, 0.01]

to the weights of the archaeometric and geochemical parameters, respectively. Moreover, Ay and Tolun (2017b) consider more effective the geochemical data than archaeometric data. Therefore, we set 0.25 and 0.75 values to these data as weights, respectively, in the final decision stage.

Pre-process Steps for Archaeometric Data

Step 1. and Step 2.

Let $U = \{u_i : i \in I_8\}$, $V = \{v_k : k \in I_2\}$, and $E_1 = \{x_j : j \in I_{19}\}$ such that $u_1 =$*Side Theatre*, $u_2 =$*Tlos Theatre*, $u_3 =$*Tlos Stadium*, $u_4 =$*Smyrna Agora 1*, $u_5 =$*Smyrna Agora 2*, $u_6 =$*Pergamon Red Hall / Serapeion*, $u_7 =$*Smintheion 1*, $u_8 =$*Smintheion 2*, $v_1 =$*Bergama Kozak*, $v_2 =$*Koçali-Akçakeçili*, $x_1 =$ *Coarse-Medium-Fine*, $x_2 =$ *Coarse - Medium*, $x_3 =$ *Medium-Fine*, $x_4 =$ *Fine*, $x_5 =$ *Chloritised*, $x_6 =$ *Sericitised*, $x_7 =$ *Pertitic*, $x_8 =$ *Mirmekitic*, $x_9 =$ *Carbonated*, $x_{10} =$ *Clayed*, $x_{11} =$*Idiomorphic (Self-Shaped)*, $x_{12} =$*Hypidiomorphic (Semi-Self-Shaped)*, $x_{13} =$*Xenomorphic (Self-Shapeless)*, $x_{14} =$ *Homogeneous*, $x_{15} =$*Recrystallised (Wavy)*, $x_{16} =$*Prismatic*, $x_{17} =$*Flat Prismatic*, $x_{18} =$*Clean Surfaced, and* $x_{19} =$*Twinning.*

Therefore, the *fs*-sets of the Amphiboles data are as follows:

$$\alpha = \{ (x_1, \{u_5, u_6\}), (x_3, \{u_3, u_5, u_6, u_8\}), (x_4, \{u_1, u_2, u_4, u_7\}), (x_5, \{u_6\}), (x_9, \{u_6\}), (x_{11}, \{u_1, u_2, u_3\}), (x_{12}, U), (x_{14}, \{u_5, u_6\}), (x_{16}, \{u_5, u_6\})\}$$

$$\beta = \{(x_3, V), (x_9, V), (x_{11}, \{v_1\}), (x_{12}, V), (x_{14}, V), (x_{16}, V)\}$$

where $\emptyset$ denotes empty fuzzy set. Here, for brevity, the notation u_3 has been used instead of ${}^{1}u_3$. Also, the elements such ${}^{0}u_3$ and $(x_4, \emptyset)$ have not been shown in the sets containing them.

The *fs*-matrices corresponded to the *fs*-sets α and β, respectively, are as follows:

$$[a_{ij}^2] = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0&0&0&1&0&1&0&1&0&0&0 \\ 1&0&1&0&1&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[b_{ij}^2] = \begin{bmatrix} 0&0&1&0&0&0&0&0&1&0&1&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \end{bmatrix}$$

Step 3.

$$[c_{ij}^{22}] = \begin{bmatrix} 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0&1&0&1&0&1&0&0&0 \end{bmatrix}$$

Step 4.

$$\left[d_{ij}^{22}\right] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Main Steps for Archaeometric Data

Step 1.

$$\left[e_{ij}^{22}\right] = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 & 1 & 1 & 1 & 1 & 1 & 1 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Step 2.

$$[f_{ik}^{2}] = \begin{bmatrix} 3.6520 & 3.3886 \\ 3.6520 & 3.3886 \\ 4.1821 & 3.9178 \\ 3.3886 & 3.6538 \\ 4.1768 & 4.4435 \\ 3.5453 & 3.7724 \\ 3.3886 & 3.6538 \\ 3.9178 & 4.1842 \end{bmatrix}$$

Step 3.

$$[g_{ik}] = \begin{bmatrix} 3.5157 & 3.2720 \\ 3.6240 & 3.3803 \\ 3.5548 & 3.4438 \\ 3.5167 & 3.3788 \\ 4.2189 & 4.1625 \\ 4.4318 & 4.1732 \\ 3.6016 & 3.7574 \\ 3.5155 & 3.8252 \end{bmatrix}$$

Step 4.

$$[s_{ik}^{1}] = \begin{bmatrix} 0.7933 & 0.7383 \\ 0.8177 & 0.7627 \\ 0.8021 & 0.7771 \\ 0.7935 & 0.7624 \\ 0.9519 & 0.9392 \\ 1.0000 & 0.9416 \\ 0.8127 & 0.8478 \\ 0.7932 & 0.8631 \end{bmatrix}$$

Pre-process Steps for Geochemical Data

Step 1. and Step 2.

Let $U = \{u_i : i \in I_8\}$, $V = \{v_k : k \in I_2\}$, and $E_2 = \{y_l : l \in I_{10}\}$ such that u_1 =*Side Theatre*, u_2 =*Tlos Theatre*, u_3 =*Tlos Stadium*, u_4 =*Smyrna Agora 1*, u_5 =*Smyrna Agora 2*, u_6 =*Pergamon Red Hall / Serapeion*, u_7 =*Smintheion 1*, u_8 =*Smintheion 2*, v_1 =*Bergama Kozak*, v_2 =*Koçali-Akçakeçili*, $y_1 = CaO$, $y_2 = Fe_2O_3$, $y_3 = MgO$, $y_4 = P_2O_5$, $y_5 = SiO_2$, $y_6 = AlO_3$, $y_7 = K_2O$, $y_8 = MnO$, $y_9 = Na_2O$, *and* $y_{10} = TiO_2$. Therefore, the *fs*-sets of the Amphiboles data are as follows:

$$\begin{aligned}\gamma = \; & \{(y_1, \{^{0.0875}u_1, {}^{0.0742}u_2, {}^{0.0786}u_3, {}^{0.0653}u_4, {}^{0.0846}u_5, {}^{0.0846}u_6, {}^{0.0682}u_7, {}^{0.0712}u_8\}), \\ & (y_2, \{^{0.0742}u_1, {}^{0.0668}u_2, {}^{0.0727}u_3, {}^{0.0579}u_4, {}^{0.0772}u_5, {}^{0.0682}u_6, {}^{0.0564}u_7, {}^{0.0579}u_8\}), \\ & (y_3, \{^{0.0297}u_1, {}^{0.0326}u_2, {}^{0.0282}u_3, {}^{0.0148}u_4, {}^{0.0341}u_5, {}^{0.0312}u_6, {}^{0.0252}u_7, {}^{0.0252}u_8\}), \\ & (y_4, \{^{0.0104}u_1, {}^{0.0045}u_2, {}^{0.0089}u_3, {}^{0.0015}u_4, {}^{0.0074}u_5, {}^{0.0045}u_6, {}^{0.0059}u_7, {}^{0.0059}u_8\}), \\ & (y_5, \{^{0.8828}u_1, {}^{0.9421}u_2, {}^{0.8976}u_3, {}^{1,0000}u_4, {}^{0.8843}u_5, {}^{0.9184}u_6, {}^{0.9436}u_7, {}^{0.9466}u_8\}), \\ & (y_6, \{^{0.2418}u_1, {}^{0.2344}u_2, {}^{0.2463}u_3, {}^{0.2329}u_4, {}^{0.2493}u_5, {}^{0.2418}u_6, {}^{0.2359}u_7, {}^{0.2344}u_8\}), \\ & (y_7, \{^{0.0757}u_1, {}^{0.0549}u_2, {}^{0.0757}u_3, {}^{0.0415}u_4, {}^{0.0608}u_5, {}^{0.0504}u_6, {}^{0.0653}u_7, {}^{0.0638}u_8\}), \\ & (y_8, \{^{0.0030}u_1, {}^{0.0015}u_2, {}^{0.0015}u_3, {}^{0.0015}u_4, {}^{0.0015}u_5, {}^{0.0015}u_6, {}^{0.0015}u_7, {}^{0.0015}u_8\}), \\ & (y_9, \{^{0.0504}u_1, {}^{0.0475}u_2, {}^{0.0519}u_3, {}^{0.0504}u_4, {}^{0.0534}u_5, {}^{0.0490}u_6, {}^{0.0519}u_7, {}^{0.0519}u_8\}), \\ & (y_{10}, \{^{0.0119}u_1, {}^{0.0104}u_2, {}^{0.0104}u_3, {}^{0.0074}u_4, {}^{0.0134}u_5, {}^{0.0104}u_6, {}^{0.0089}u_7, {}^{0.0089}u_8\})\}\end{aligned}$$

$$\begin{aligned}\delta = \; & \{(y_1, \{^{0.0697}v_1, {}^{0.0742}v_2\}), (y_2, \{^{0.0623}v_1, {}^{0.0668}v_2\}), (y_3, \{^{0.0297}v_1, {}^{0.0282}v_2\}), \\ & (y_4, \{^{0.0045}v_1, {}^{0.0074}v_2\}), (y_5, \{^{0.9525}v_1, {}^{0.9132}v_2\}), (y_6, \{^{0.2359}v_1, {}^{0.2411}v_2\}), \\ & (y_7, \{^{0.0549}v_1, {}^{0.0682}v_2\}), (y_8, \{^{0.0015}v_1, {}^{0.0015}v_2\}), (y_9, \{^{0.0504}v_1, {}^{0.0527}v_2\}), \\ & (y_{10}, \{^{0.0104}v_1, {}^{0.0104}v_2\})\}\end{aligned}$$

The *fs*-matrices corresponded to the *fs*-sets γ and δ, respectively, are as follows:

$$[a_{ij}] = \begin{bmatrix} 0.0875 & 0.0742 & 0.0297 & 0.0104 & 0.8828 & 0.2418 & 0.0757 & 0.0030 & 0.0504 & 0.0119 \\ 0.0742 & 0.0668 & 0.0326 & 0.0045 & 0.9421 & 0.2344 & 0.0549 & 0.0015 & 0.0475 & 0.0104 \\ 0.0786 & 0.0727 & 0.0282 & 0.0089 & 0.8976 & 0.2463 & 0.0757 & 0.0015 & 0.0519 & 0.0104 \\ 0.0653 & 0.0579 & 0.0148 & 0.0015 & 1.0000 & 0.2329 & 0.0415 & 0.0015 & 0.0504 & 0.0074 \\ 0.0846 & 0.0772 & 0.0341 & 0.0074 & 0.8843 & 0.2493 & 0.0608 & 0.0015 & 0.0534 & 0.0134 \\ 0.0816 & 0.0682 & 0.0312 & 0.0045 & 0.9184 & 0.2418 & 0.0504 & 0.0015 & 0.0490 & 0.0104 \\ 0.0682 & 0.0564 & 0.0252 & 0.0059 & 0.9436 & 0.2359 & 0.0653 & 0.0015 & 0.0519 & 0.0089 \\ 0.0712 & 0.0579 & 0.0252 & 0.0059 & 0.9466 & 0.2344 & 0.0638 & 0.0015 & 0.0519 & 0.0089 \end{bmatrix}$$

$$[b_{ij}] = \begin{bmatrix} 0.0697 & 0.0623 & 0.0297 & 0.0045 & 0.9525 & 0.2359 & 0.0549 & 0.0015 & 0.0504 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \end{bmatrix}$$

Step 3.

$$[c_{ij}^2] = \begin{bmatrix} 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \\ 0.0742 & 0.0668 & 0.0282 & 0.0074 & 0.9132 & 0.2411 & 0.0682 & 0.0015 & 0.0527 & 0.0104 \end{bmatrix}$$

Step 4.

$$[d_{ij}^2] = \begin{bmatrix} 0.9867 & 0.9926 & 0.9985 & 0.9970 & 0.9696 & 0.9993 & 0.9925 & 0.9985 & 0.9977 & 0.9985 \\ 1 & 1 & 0.9956 & 0.9971 & 0.9711 & 0.9933 & 0.9867 & 1 & 0.9948 & 1 \\ 0.9956 & 0.9941 & 1 & 0.9985 & 0.9844 & 0.9948 & 0.9925 & 1 & 0.9992 & 1 \\ 0.9911 & 0.9911 & 0.9866 & 0.9941 & 0.9132 & 0.9918 & 0.9733 & 1 & 0.9977 & 0.9970 \\ 0.9896 & 0.9896 & 0.9941 & 1 & 0.9711 & 0.9918 & 0.9926 & 1 & 0.9993 & 0.9970 \\ 0.9926 & 0.9986 & 0.9970 & 0.9971 & 0.9948 & 0.9993 & 0.9822 & 1 & 0.9963 & 1 \\ 0.9940 & 0.9896 & 0.9970 & 0.9985 & 0.9696 & 0.9948 & 0.9971 & 1 & 0.9992 & 0.9985 \\ 0.9970 & 0.9911 & 0.9970 & 0.9985 & 0.9666 & 0.9933 & 0.9956 & 1 & 0.9992 & 0.9985 \end{bmatrix}$$

Main Steps for Geochemical Data

Step 1.

$$[e_{ij}^2] = \begin{bmatrix} 0.6 & 0.6 & 0.01 & 0.7 & 1 & 0.8 & 1 & 0.01 & 0.7 & 0.01 \\ 0.9867 & 0.9926 & 0.9985 & 0.9970 & 0.9696 & 0.9993 & 0.9925 & 0.9985 & 0.9977 & 0.9985 \\ 1 & 1 & 0.9956 & 0.9971 & 0.9711 & 0.9933 & 0.9867 & 1 & 0.9948 & 1 \\ 0.9956 & 0.9941 & 1 & 0.9985 & 0.9844 & 0.9948 & 0.9925 & 1 & 0.9992 & 1 \\ 0.9911 & 0.9911 & 0.9866 & 0.9941 & 0.9132 & 0.9918 & 0.9733 & 1 & 0.9977 & 0.9970 \\ 0.9896 & 0.9896 & 0.9941 & 1 & 0.9711 & 0.9918 & 0.9926 & 1 & 0.9993 & 0.9970 \\ 0.9926 & 0.9986 & 0.9970 & 0.9971 & 0.9948 & 0.9993 & 0.9822 & 1 & 0.9963 & 1 \\ 0.9940 & 0.9896 & 0.9970 & 0.9985 & 0.9696 & 0.9948 & 0.9971 & 1 & 0.9992 & 0.9985 \\ 0.9970 & 0.9911 & 0.9970 & 0.9985 & 0.9666 & 0.9933 & 0.9956 & 1 & 0.9992 & 0.9985 \end{bmatrix}$$

Step 2.

$$[f_{ik}] = \begin{bmatrix} 5.1804 & 5.2810 \\ 5.3326 & 5.2865 \\ 5.2087 & 5.3150 \\ 5.2470 & 5.1519 \\ 5.1927 & 5.2767 \\ 5.2773 & 5.3157 \\ 5.3211 & 5.2906 \\ 5.3276 & 5.2869 \end{bmatrix}$$

Step 3.

$$[s_{ik}^2] = \begin{bmatrix} 0.9715 & 0.9903 \\ 1.0000 & 0.9913 \\ 0.9768 & 0.9967 \\ 0.9840 & 0.9661 \\ 0.9738 & 0.9895 \\ 0.9896 & 0.9968 \\ 0.9978 & 0.9921 \\ 0.9991 & 0.9914 \end{bmatrix}$$

Output Steps

Step 1.

$$[s_{ik}] = \begin{bmatrix} 0.9269 & 0.9273 \\ 0.9544 & 0.9342 \\ 0.9331 & 0.9418 \\ 0.9363 & 0.9152 \\ 0.9683 & 0.9769 \\ 0.9922 & 0.9830 \\ 0.9516 & 0.9560 \\ 0.9476 & 0.9594 \end{bmatrix}$$

Step 2.

Side Theatre	*Koçali-Akçakeçili*
Tlos Theatre	*Bergama Kozak*
Tlos Stadium	*Koçali-Akçakeçili*
Smyrna Agora 1	*Bergama Kozak*
Smyrna Agora 2	*Koçali-Akçakeçili*
Pergamon Red Hall	*Bergama Kozak*
Smintheion 1	*Koçali-Akçakeçili*
Smintheion 2	*Koçali-Akçakeçili*

5. Conclusion

We, in this paper, proposed a novel method MCCM to model an MCC problem. We then applied MCCM to the data provided in (Ay, 2017; Ay and Tolun, 2017b). The results affirmed those obtained by archaeometric analyses. Since this method is the first, it could not be compared with other methods for now. Soon, however, if another soft decision-making method that differs from PEM is applied to this problem, then a comparison of these methods can be given.

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References

Atmaca, S. (2017). Relationship between fuzzy soft topological spaces and (X, τ_e) parameter spaces. Cumhuriyet Science Journal, 38(4), 77-85.

Ay, M. (2017). Distribution of the granite columns from the Troadic quarries in Western Anatolia an archaeometric approach, Master's Thesis, Çanakkale Onsekiz Mart University, Çanakkale, Turkey (In Turkish).

Ay, M., Tolun, V. (2017a). Ancient granite quarries in Troad: New findings. The Turkish Yearbook of Çanakkale Studies, 15(23), 265-295 (In Turkish).

Ay, M., Tolun, V. (2017b). An archaeometric approach on the distribution of Troadic granite columns in the Western Anatolian coasts. Journal of Archaeology & Art, 156, 119-130 (In Turkish).

Bera, S., Roy, S. K., Karaaslan, F., Çağman, N. (2017). Soft congruence relation over lattice. Hacettepe Journal of Mathematics and Statistics, 46(6), 1035-1042.

Çağman, N., Çıtak, F., Enginoğlu, S. (2010). Fuzzy parameterized fuzzy soft set theory and its applications. Turkish Journal of Fuzzy Systems, 1(1), 21-35.

Çağman, N., Çıtak, F., Enginoğlu, S. (2011a). FP-soft set theory and its applications. Annals of Fuzzy Mathematics and Informatics, 2(2), 219-226.

Çağman, N., Deli, İ. (2012). Means of FP-soft sets and their applications. Hacettepe Journal of Mathematics and Statistics, 41(5), 615-625.

Çağman, N., Enginoğlu, S. (2010). Soft set theory and uni-int decision making. European Journal of Operational Research, 207, 848-855.

Çağman, N., Enginoğlu, S. (2012). Fuzzy soft matrix theory and its application in decision making. Iranian Journal of Fuzzy Systems, 9(1), 109-119.

Çağman, N., Enginoğlu, S., Çıtak, F. (2011b). Fuzzy soft set theory and its applications. Iranian Journal of Fuzzy Systems, 8(3), 137-147.

Çıtak, F. (2018). Soft k-uni ideals of semirings and its algebraic applications. Journal of the Institute of Science and Technology, 8(4), 281-294.

Çıtak, F., Çağman, N. (2017). Soft k-int-ideals of semirings and its algebraic structures. Annals of Fuzzy Mathematics and Informatics, 13(4), 531-538.

De Vecchi, G., Lazzarini, L., Lünel, T., Mignucci, A., Visonà, D. (2000). The genesis and characterisation of marmor misium from Kozak (Turkey) a granit used antiquity. Journal of Cultural Heritage, 1, 145-153.

Deli, İ., Çağman, N. (2015). Relations on FP-soft sets applied to decision making problems. Journal of New Theory, (3), 98-107.

Enginoğlu, S. (2012). Soft matrices, PhD Dissertation, Gaziosmanpaşa University, Tokat, Turkey, (In Turkish).

Enginoğlu, S., Çağman, N. Fuzzy parameterized fuzzy soft matrices and their application in decision-making. TWMS Journal of Applied and Engineering Mathematics, (In Press).

Enginoğlu, S., Çağman, N., Karataş, S., Aydın, T. (2015). On soft topology. El-Cezerî Journal of Science and Engineering, 2(3), 23-38.

Enginoğlu, S., Demiriz, S. (2015). A comparison with the convergent, Cesàro convergent and Riesz convergent sequences of fuzzy numbers. In: The 4th International Fuzzy Systems Symposium, 5-6 November 2015 İstanbul, Turkey, pp. 413-416.

Enginoğlu, S., Dönmez, H. (2015). An application on decision making problem by using intuitionistic fuzzy parameterized intuitionistic fuzzy soft expert sets. In: The 4th International Fuzzy Systems Symposium, 5-6 November 2015 İstanbul, Turkey, pp. 413-415.

Enginoğlu, S., Memiş, S. (2018a). Comment on fuzzy soft sets [The Journal of Fuzzy Mathematics, 9(3), 2001, 589-602]. International Journal of Latest Engineering Research and Applications, 3(9), 1-9.

Enginoğlu, S., Memiş, S. (2018b). A review on an application of fuzzy soft set in multicriteria decision making problem. In: Proceedings of The International Conference on Mathematical Studies and Applications, pp. 173-178.

Enginoğlu, S., Memiş, S. (2018c). A configuration of some soft decision-making algorithms via *fpfs*-matrices. Cumhuriyet Science Journal, 39(4), 871-881.

Enginoğlu, S., Memiş, S. (2018d). A review on some soft decision-making methods. In: Proceedings of The International Conference on Mathematical Studies and Applications, pp. 437-442.

Enginoğlu, S., Memiş, S., Arslan, B. (2018a). A fast and simple soft decision-making algorithm: EMA18an. In: Proceedings of The International Conference on Mathematical Studies and Applications, pp. 426-438.

Enginoğlu, S., Memiş, S., Arslan, B. (2018b). Comment (2) on soft set theory and uni-int decision making [European Journal of Operational Research, (2010) 207, 848-855]. Journal New Theory, (25), 84-102.

Enginoğlu, S., Memiş, S., Öngel, T. (2018c). A fast and simple soft decision-making algorithm: EMO18o. In: Proceedings of The International Conference on Mathematical Studies and Applications, pp. 179-187.

Enginoğlu, S., Memiş, S., Öngel, T. (2018d). Comment on soft set theory and uni-int decision-making [European Journal of Operational Research, (2010) 207, 848-855]. Journal of New Results in Science, 7(3), 28-43.

Galetti, G., Lazzarini, L., Maggetti, M. (1992). A first characterization of the most important granites used in antiquity. In Ancient Stones: Quarrying, Trade and Provenance. Acta Archaeologica Lovaniensia Monographia, pp. 167-178.

Gulistan, M., Feng, F., Khan, M., Sezgin, A. (2018). Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. Mathematics, 6(293), 20 pages.

Karaaslan, F. (2016). Soft classes and soft rough classes with applications in decision making. Mathematical Problems in Engineering, Article ID 1584528, 11 pages.

Mahmood, T., Rehman, Z. U., Sezgin, A. (2018). Lattice ordered soft near rings. Korean Journal of Mathematics, 26(3), 503-517.

Maji, P. K., Biswas, R., Roy, A. R. (2001). Fuzzy soft sets. The Journal of Fuzzy Mathematics, 9(3), 589-602.

Molodtsov, D. (1999). Soft set theory-first results. Computers and Mathematics with Applications, 37(4-5), 19-31.

Ponti, G. (1995). Granite quarries in the troad. a preliminary survey. In Studia Troica, pp. 291-320.

Potts, P. J. (2002). Geochemical and magnetic provenancing of Roman granite columns from Andalucia. Oxford Journal of Archaeology, 21(2), 167-194.

Riaz, M., Hashmi, M. R. (2018). Fuzzy parameterized fuzzy soft topology with applications. Annals of Fuzzy Mathematics and Informatics, 13(5), 593-613.

Riaz, M., Hashmi, M. R., Farooq, A. (2018). Fuzzy parameterized fuzzy soft metric spaces. Journal of Mathematical Analysis, 9(2), 25-36.

Şenel, G. (2016). A new approach to Hausdorff space theory via the soft sets. Mathematical Problems in Engineering, Article ID 2196743, 6 pages.

Şenel, G., 2017. The parameterization reduction of soft point and its applications with soft matrix. International Journal of Computer Applications, 164(1), 1-6.

Şenel, G. (2018). Analyzing the locus of soft spheres: Illustrative cases and drawings. European Journal of Pure and Applied Mathematics, 11(4), 946-957.

Ullah, A., Karaaslan, F., Ahmad, I. (2018). Soft uni-Abel-Grassmann's groups. European Journal of Pure and Applied Mathematics, 11(2), 517-536.

Williams-Thorpe, O. (2008). A thousand and one columns: Observations on the roman granite trade in the Mediterranean area. Oxford Journal of Archaeology, 27(1), 73-89.

Williams-Thorpe, O., Henty, M. M. (2000). The sources of Roman granite columns in Israel. Levant, 32, 155-170.

Williams-Thorpe, O., Thorpe, R. S. (1993). Magnetic susceptibility used in non-destructive provenancing of Roman granite columns. Archaeometry, 35(2), 185-195.

Williams-Thorpe, O., Webb, P. C., Thorpe, R. S. (2000). Non-destructive portable gamma ray spectrometry used in provenancing Roman granitoid columns from Leptis Magna, North Africa. Archaeometry, 42(1), 77-99.

Zorlutuna, İ., Atmaca, S. (2016). Fuzzy parametrized fuzzy soft topology. New Trends in Mathematical Sciences, 4(1), 142-152.