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On The Directional Associated Curves of Timelike Space Curve

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Abstract: In this work, the directional associated curves of timelike space curve in Minkowski 3-space by using q-frame are studied. We investigate quasi normal-binormal direction and donor curves of the timelike curve with q-frame. Finally, some new associated curves are constructed and plotted.

Keywords: Associated curves, Minkowski space, q-frame.

1 Introduction

The theory of curves is the one of the most important subject in differential geometry. The curves are represented in parametrized form and then their geometric properties and various quantities associated with them, such as curvature and arc length expressed via derivatives and integrals using the idea of vector calculus. There are special curves which are classical differential geometric objects. These curves are obtained by assuming a special property on the original regular curve. Some of them are Smarandache curves, curves of constant breadth, Bertrand curves, and Mannheim curves, associated curves, etc. Studying curves can be differed according to frame used for curve [1], [2], [3]. There are many studies on these special curves; for example, Choi and Kim in 2012 introduced the notion of the principal (binormal)-direction curve and principal (binormal)-donor curve of a Frenet curve and gave the relationship of curvature and torsion of its mates in both Euclidean and Minkowski spaces [4]-[5]. Also Macit and Duldul in 2014 worked on the new associated curves in $\mathbf{E^3}$ and $\mathbf{E^4}$ [6]. New associated curves by using the Bishop frame are obtained by some researches in [7], [8], [9] and [10]. In this paper, we give another approach to directional associated curves of timelike space curve with q-frame used in [11], [12], [13] and [14].

The aim of this study in this paper is to define n_q, b_q-direction curves and n_q, b_q-donor curves of timelike curve γ via the q-frame in $\mathbb{E}_1^3$ and give the relationship between q-curvatures and curvature and torsion of its mates in Minkowski space.

2 Preliminaries

Let $\alpha(t)$ be a space curve with a non-vanishing second derivative. The Frenet frame is defined as follows,

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|}, \mathbf{b} = \frac{\alpha' \wedge \alpha''}{\|\alpha' \wedge \alpha''\|}, \mathbf{n} = \mathbf{b} \wedge \mathbf{t}. \tag{1}$$

The curvature κ and the torsion τ are given by

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3}, \tau = \frac{\det(\alpha', \alpha'', \alpha''')}{\|\alpha' \wedge \alpha''\|^2}. \tag{2}$$

The well-known Frenet formulas are given by

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{bmatrix} = v \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix}, \tag{3}$$

where

$$v = \|\alpha'(t)\|. \tag{4}$$

In order to construct the 3D curve offset, Coquillart in [15] introduced the quasi-normal vector of a space curve. The quasi-normal vector is defined for each point of the curve, and lies in the plane perpendicular to the tangent of the curve at this point.

As an alternative to the Frenet frame, a new adapted frame called q-frame in both Euclidean and Minkowski space is defined by Ekici et all in [11] and [13]. Given a space curve $\alpha(t)$ the q-frame consists of three orthonormal vectors, the unit tangent vector $\mathbf{t}$, the quasi-normal vector $\mathbf{n}_q$ and the quasi-binormal vector $\mathbf{b}_q$. The q-frame $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q, \mathbf{k}\}$ is given by

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|}, \mathbf{n}_q = \frac{\mathbf{t} \wedge \mathbf{k}}{\|\mathbf{t} \wedge \mathbf{k}\|}, \mathbf{b}_q = \mathbf{t} \wedge \mathbf{n}_q \tag{5}$$

where $\mathbf{k}$ is the projection vector, which can be chosen $\mathbf{k} = (0, 1, 0)$ or $\mathbf{k} = (1, 0, 0)$ or $\mathbf{k} = (0, 0, 1)$.

A q-frame along a space curve is shown in Figure 1.

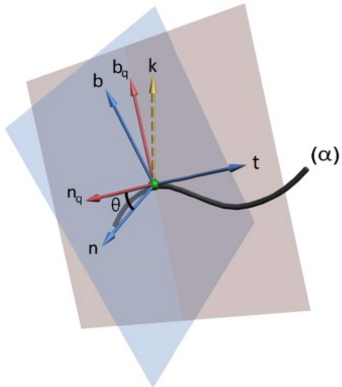

Fig. 1: The q-frame and Frenet frame

Since the derivation formula for the q-frame for the timelike curve in Minkowski space does not depend on projection vector being timelike or spacelike, we work on spacelike projection vector without loss of generality.

In [12], the variation equations of the directional q-frame for the timelike space curve when tangent vector (timelike), projection vector $\mathbf{k} = (0, 1, 0)$ (spacelike), quasi-normal vector (spacelike) and quasi-binormal vector (spacelike) are given by

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ k_1 & 0 & k_3 \\ k_2 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n}_q \\ \mathbf{b}_q \end{bmatrix} \tag{6}$$

where the q-curvatures are

$$k_1 = \left\langle \mathbf{t}', \mathbf{n}_q \right\rangle, \ \ k_2 = \left\langle \mathbf{t}', \mathbf{b}_q \right\rangle, \ \ k_3 = \left\langle \mathbf{n}_q', \mathbf{b}_q \right\rangle,$$

In the three dimensional Minkowski space $\mathbb{R}^3_1$, the inner product and the cross product of two vectors $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3) \in \mathbb{R}^3_1$ are defined as

$$< \mathbf{u}, \mathbf{v} >= u_1 v_1 + u_2 v_2 - u_3 v_3 \tag{7}$$

and

$$\mathbf{u} \wedge \mathbf{v} = (u_3 v_2 - u_2 v_3, u_1 v_3 - u_3 v_1, u_1 v_2 - u_2 v_1) \tag{8}$$

where $e_1 \wedge e_2 = e_3$, $e_2 \wedge e_3 = -e_1$, $e_3 \wedge e_1 = -e_2$, respectively [16].

The norm of the vector $\mathbf{u}$ is given by

$$\|\mathbf{u}\| = \sqrt{|\langle u, u\rangle|}. \tag{9}$$

We say that a Lorentzian vector $\mathbf{u}$ is spacelike, lightlike or timelike if $\langle \mathbf{u}, \mathbf{u}\rangle > 0$, $\langle \mathbf{u}, \mathbf{u}\rangle = 0$ and $\mathbf{u} \neq 0$, $\langle \mathbf{u}, \mathbf{u}\rangle < 0$, respectively. In particular, the vector $\mathbf{u} = 0$ is spacelike.

An arbitrary curve $\alpha(s)$ in $\mathbb{R}^3_1$ can locally be spacelike, timelike or null(lightlike), if all its velocity vectors $\alpha'(s)$ are respectively spacelike, timelike or null.

A null curve α is parameterized by pseudo-arc s if $\left\langle \alpha''(s), \alpha''(s) \right\rangle = 1$. On the other hand, a non-null curve α is parameterized by arc-lenght parameter s if $\left\langle \alpha'(s), \alpha'(s) \right\rangle = \pm 1$ [17] and [18].

Then Frenet formulas of timelike curve may be written as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = v \begin{bmatrix} 0 & \kappa & 0 \\ \kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} \tag{10}$$

where $v = \left\| \alpha'(t) \right\|$. The Minkowski curvature and torsion of timelike curve $\alpha(t)$ are obtained by

$$\kappa =< \mathbf{t}', \mathbf{n} >, \ \ \tau =< \mathbf{n}', \mathbf{b} >,$$

respectively [16] and [19].

Let x and y be future painting (or post painting) timelike vectors in E_1^3 , then there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \cosh\theta.$$

This number is called the hyperbolic angle between the vectors x and y [19]. Let x and y be spacelike vectors in E_1^3 that span spacelike vector subspace. Then, there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \cos\theta.$$

This number is called the spacelike angle between the vectors x and y.

Let x be a spacelike and y be a timelike vectors in E_1^3 , then there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \sinh\theta.$$

This number is called the timelike angle between the vectors x and y [19]. The relation between Frenet ($\mathbf{n}$ is timelike) and q-frame ($\mathbf{t}$ is timelike) is given as

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sinh\theta & \cosh\theta \\ 0 & \cosh\theta & \sinh\theta \end{bmatrix} . \begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix}, \tag{11}$$

where the angle is between $\mathbf{n}$ and $\mathbf{n}_q$.

Also the relation between q-curvatures and curvature and torsion are

$$k_1 = \kappa \sinh\theta, \;\; k_2 = \kappa\cosh\theta, \;\; k_3 = -d\theta + \tau. \tag{12}$$

The relation between Frenet ($\mathbf{b}$ is timelike) and q-frame ($\mathbf{t}$ is timelike) is given as

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cosh\theta & \sinh\theta \\ 0 & -\sinh\theta & -\cosh\theta \end{bmatrix} . \begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix}, \tag{13}$$

where the angle is between $\mathbf{b}$ and $\mathbf{n}_q$.

Also the relation between q-curvatures and curvature and torsion are

$$k_1 = \kappa \cosh\theta, \;\; k_2 = \kappa\sinh\theta, \;\; k_3 = -d\theta - \tau. \tag{14}$$

3 Directional Associated Curves of Timelike Space Curve

In this section, we inverstigate $\mathbf{n}_q$ and $\mathbf{b}_q-$ direction and donor curves of the timelike curve with q-frame in $\mathbb{E}_1^3$. For a Frenet frame $\gamma : I \to \mathbb{E}_1^3$, consider a vector field V with q frame as follows:

$$V(s) = u(s)t(s) + v(s)n_q(s) + w(s)b_q(s), \tag{15}$$

where u, v, and w are functions on I satisfying

$$u^2(s) + v^2(s) - w^2(s) = 1. \tag{16}$$

Then, an integral curve $\overline{\gamma}(s)$, that is $V(\overline{\gamma}(s)) = \overline{\gamma}'(s)$, of V defined on I is a unit speed curve in $\mathbb{E}_1^3$.

Let γ be a timelike curve in $\mathbb{E}_1^3$. An integral curve of n_q is called n_q-direction curve of the timelike curve γ via q-frame.

Remark 1. *A n_q-direction curve is an integral curve of the equation* (15) *with* $u(s) = w(s) = 0, v(s) = 1$.

Let γ be a timelike curve in $\mathbb{E}_1^3$. An integral curve of b_q is called b_q-direction curve of the timelike curve γ via q-frame.

Remark 2. *A b_q-direction curve is an integral curve of the equation* (15) *with* $u(s) = v(s) = 0, w(s) = 1$.

3.1 $\mathbf{n}_q-$ direction and donor curves of the timelike curve with q-frame

Theorem 1. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q-curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of γ with the q-curvature $\overline{k}_1, \overline{k}_2, \overline{k}_3$. Then we have*

$$\begin{gathered} \overline{t} = n_q, \;\; \overline{n}_q = -t, \;\; \overline{b}_q = b_q \\ \overline{k}_1 = |k_1| \;\; or \;\; \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}, \;\; \overline{k}_2 = k_3, \;\; \overline{k}_3 = k_2. \end{gathered} \tag{17}$$

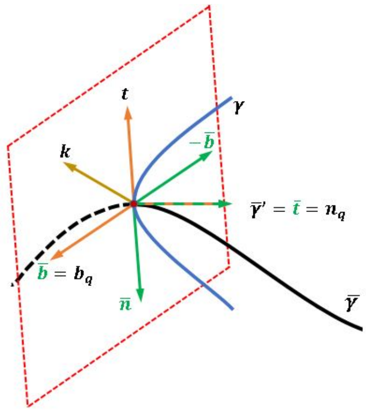

Fig. 2: n_q direction curve

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = n_q. \tag{18}$$

Geometrically, since $\overline{n}_q$ and t lie on the same plane, we can take $\overline{n}_q = -t$. The vectorial product of $\overline{t}$ and $\overline{n}_q$ is as follows:

$$\overline{b}_q = \overline{n}_q \times \overline{t} \tag{19}$$

therefore, $\overline{b}_q = b_q$. Differentiating the expression (18) and then taking its norm, we find

$$\overline{k}_1 = |k_1| \text{ or } \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}. \tag{20}$$

Using definition of $q-$ curvatures and derivation formula of $q-$ frame, one can get $\overline{k}_2 = k_3$, and $\overline{k}_3 = k_2$.

Theorem 2. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q−curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of the timelike curve γ with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$. Then we have*

$$\begin{aligned} \overline{t} = n_q, \quad \overline{n} = -t, \quad \overline{b} = b_q \\ \overline{\kappa} = \sqrt{|-k_1^2 + k_3^2|}, \quad \overline{\tau} = -k_2. \end{aligned} \tag{21}$$

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = n_q. \tag{22}$$

Differentiating the expression (22) and then taking its norm, we find

$$\overline{\kappa} = \sqrt{|-k_1^2 + k_3^2|} \tag{23}$$

Differentiation of the expressions (22) gives us

$$\overline{n} = -t. \tag{24}$$

The vectorial product of $\overline{t}$ and $\overline{n}$ is as follows:

$$\overline{b} = \overline{n} \times \overline{t}. \tag{25}$$

Using the expressions (22), (24) in (25) we find that

$$\overline{b} = b_q. \tag{26}$$

Finally, differentiating (26) and using (24) in it, we have

$$\overline{\tau} = -k_2. \tag{27}$$

Corollary 1. *Let γ be a timelike curve in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ. The Frenet frame of $\overline{\gamma}$ is given in terms of the $q-$ frame as follows:*

$$\begin{aligned} &\overline{t}(s) = \overline{n}_q(s), \\ &\overline{n}(s) = -\sinh(\textstyle\int k_2(s)ds)\overline{n}_q(s) + \cosh(\int k_2(s)ds)\overline{b}_q(s), \\ &\overline{b}(s) = \cosh(\textstyle\int k_2(s)ds)\overline{n}_q(s) - \sinh(\int k_2(s)ds)\overline{b}_q(s). \end{aligned} \tag{28}$$

Proof. It is straightforwardly seen by substituting (23) and (27) into (11).

Corollary 2. *If the curve γ is a n_q-donor curve of the curve $\overline{\gamma}$ with the curvatures k_1, k_2, k_3, then the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ of the timelike curve γ are given by*

$$\overline{\tau}=\sqrt{|-k_1^2+k_3^2|},\quad \overline{\kappa}=\pm k_2+\left(\frac{k_3^2}{-k_1^2+k_3^2}\right)\left(\frac{k_1}{k_3}\right)' \tag{29}$$

Proof. Taking the squares of (23) and (27), then subtracting them side by side by using (12) gives us the equation (29).

Corollary 3. *Let γ be a timelike curve with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ with the curvatures k_1, k_2, k_3. Then it satisfies*

$$\frac{k_2}{k_1}=\coth\theta,\quad \frac{\overline{\tau}}{\overline{\kappa}}=\pm\frac{k_2}{\sqrt{-k_1^2+k_3^2}}+\frac{k_3^2}{(-k_1^2+k_3^2)^{\frac{3}{2}}}\left(\frac{k_1}{k_3}\right)' \tag{30}$$

Proof. It is straightforwardly seen by substituting the expressions (23), (27) and (29) into (12).

3.2 $\mathbf{b}_q-$ *direction and donor curves of the timelike curve with q-frame*

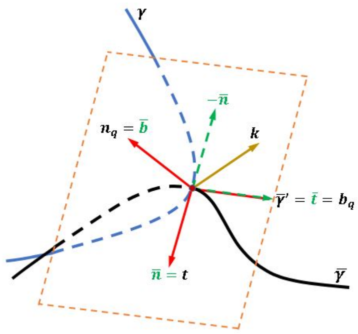

Fig. 3: b_q direction curve

Theorem 3. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q-curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of γ with the q-curvature $\overline{k}_1, \overline{k}_2, \overline{k}_3$. Then we have*

$$\begin{gathered}\overline{t}=n_q,\ \ \overline{n}_q=-t,\ \ \overline{b}_q=b_q\\ \overline{k}_1=|k_1|\ \ or\ \ \overline{k}_1=\sqrt{|2k_3^2-k_1^2|},\ \ \overline{k}_2=k_3,\ \ \overline{k}_3=k_2.\end{gathered} \tag{31}$$

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}'=\overline{t}=n_q. \tag{32}$$

Geometrically, since $\overline{n}_q$ and t lie on the same plane, we can take $\overline{n}_q=-t$. The vectorial product of $\overline{t}$ and $\overline{n}_q$ is as follows:

$$\overline{b}_q=\overline{n}_q\times\overline{t} \tag{33}$$

therefore, $\overline{b}_q=b_q$. Differentiating the expression (32) and then taking its norm, we find

$$\overline{k}_1=|k_1|\ \text{ or }\ \overline{k}_1=\sqrt{|2k_3^2-k_1^2|}. \tag{34}$$

Using definition of $q-$ curvatures and derivation formula of $q-$ frame, one can get

$$\overline{k}_2=k_3\ \text{ and }\ \overline{k}_3=k_2. \tag{35}$$

Theorem 4. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the $q-$curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the b_q-direction curve of the timelike curve γ with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$. Then we have*

$$\begin{gathered}\overline{t}=b_q,\ \ \ \overline{n}=t,\ \ \ \overline{b}=n_q\\ \overline{\kappa}=\sqrt{|-k_2^2+k_3^2|},\ \ \ \overline{\tau}=-k_1.\end{gathered} \tag{36}$$

Proof. By definition of b_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = b_q. \tag{37}$$

Differentiating the expression (37) and then taking its norm, we find

$$\overline{\kappa} = \sqrt{|-k_2^2 + k_3^2|} \tag{38}$$

Differentiation of the expressions (37) with using of (38) gives us

$$\overline{n} = t. \tag{39}$$

The vectorial product of $\overline{t}$ and $\overline{n}$ is as follows:

$$\overline{b} = \overline{n} \times \overline{t}. \tag{40}$$

Using the expressions (37), (39) in (40) we find that

$$\overline{b} = n_q. \tag{41}$$

Finally, differentiating (41) and using definition of curvature, we have

$$\overline{\tau} = k_1 \tag{42}$$

which proves theorem.

Corollary 4. *Let γ be a timelike curve in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the b_q-direction curve of γ. The Frenet frame of $\overline{\gamma}$ is given in terms of the q$-$ frame as follows:*

$$\begin{aligned}
&\overline{t}(s) = \overline{b}_q(s),\\
&\overline{n}(s) = \cosh(\textstyle\int k_1(s)ds)\overline{n}_q(s) + \sinh(\int k_1(s)ds)\overline{b}_q(s),\\
&\overline{b}(s) = -\sinh(\textstyle\int k_1(s)ds)\overline{n}_q(s) - \cosh(\int k_1(s)ds)\overline{b}_q(s).
\end{aligned} \tag{43}$$

Proof. It is straightforwardly seen by substituting (38) and (42) into (13).

Corollary 5. *If the curve γ is a n_q-donor curve of the curve $\overline{\gamma}$ with the curvatures k_1, k_2, k_3, then the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ of the timelike curve γ are given by*

$$\overline{\tau} = \sqrt{|-k_2^2 + k_3^2|}, \quad \overline{\kappa} = \pm k_1 + (\frac{k_3^2}{-k_2^2 + k_3^2})(\frac{k_2}{k_3})' \tag{44}$$

Proof. Taking the squares of (38) and (42)**,** then subtracting them side by side by using (14) gives us the equation (44).

Corollary 6. *Let γ be a timelike curve with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ with the curvatures k_1, k_2, k_3. Then it satisfies*

$$\frac{k_2}{k_1} = \tanh\theta, \quad \frac{\overline{\tau}}{\overline{\kappa}} = \pm\frac{k_1}{\sqrt{-k_2^2 + k_3^2}} + \frac{k_3^2}{(-k_2^2 + k_3^2)^{\frac{3}{2}}}(\frac{k_2}{k_3})' \tag{45}$$

Proof. It is straightforwardly seen by substituting the expressions (38), (42) and (44) into (14).

4 Examples

In this section, an example of directional associated curves of timelike space curve with q-frame are constructed and plotted.

Example 1. Consider a timelike curve

$$\gamma(t) = (-\frac{5}{9}\cosh(3t), \frac{4}{3}t, -\frac{5}{9}\sinh(3t)).$$

The Frenet frame vectors and curvatures are calculated by

$$\begin{aligned}
&\mathbf{t} = \big(-\tfrac{5}{3}\sinh(3t), \tfrac{4}{3}, -\tfrac{5}{3}\cosh(3t)\big),\\
&\mathbf{n} = \big(-\cosh(3t), 0, -\sinh(3t)\big),\\
&\mathbf{b} = \big(\tfrac{4}{3}\sinh(3t), -\tfrac{5}{3}, \tfrac{4}{3}\cosh(3t)\big),\\
&\kappa = 5, \quad \tau = 4.
\end{aligned}$$

The q-frame vectors and curvatures are obtained by

$$\begin{aligned}
\mathbf{t} &= \left(-\tfrac{5}{3}\sinh(3t), \tfrac{4}{3}, -\tfrac{5}{3}\cosh(3t)\right),\\
\mathbf{n_q} &= \left(-\cosh(3t), 0, -\sinh(3t)\right),\\
\mathbf{b_q} &= \left(-\tfrac{4}{3}\sinh(3t), \tfrac{5}{3}, -\tfrac{4}{3}\cosh(3t)\right),\\
k_1 &= 5, \quad k_2 = 0, \quad k_3 = -4.
\end{aligned}$$

n_q and b_q- direction curves of γ shown in Figure 4 are written as

$$\begin{aligned}
\overline{\gamma} &= \left(-\tfrac{1}{3}\sinh(3t) + c_1, c_2, -\tfrac{1}{3}\cosh(3t) + c_3\right),\\
\overline{\overline{\gamma}} &= \left(-\tfrac{4}{9}\cosh(3t) + c_4, \tfrac{5}{3}t + c_5, -\tfrac{4}{9}\sinh(3t) + c_6\right),
\end{aligned}$$

respectively.

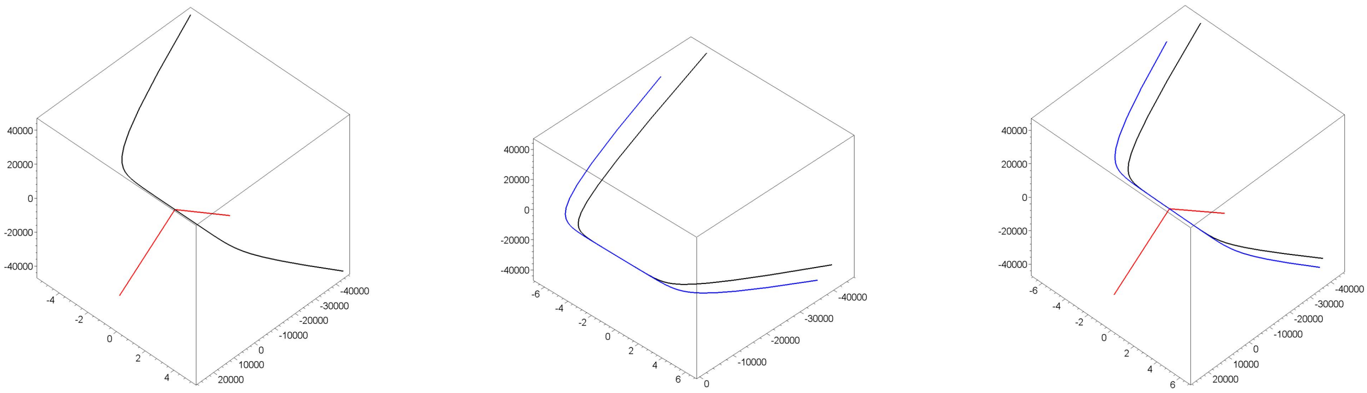

Fig. 4: Timelike curve (black), n_q direction curve (red) and b_q direction curve (blue) for $c_i = 0$.

All the figures in this study were created by using maple programme.

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