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Mathematical Estimation of Expenditures in The Health Sector in TURKEY with Grey Modeling

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Keywords

Grey Estimation Model,
Grey System Theory,
Forecast Accuracy,
Least Squares Method,

Abstract: Grey theory is an effective theory that deals with systems that lack weak information and / or information. With this theory, effective and very accurate estimates can be created for future times by utilizing a small number of data. The grey modeling method is a sub-branch of the theory of grey systems and the modeling process is carried out with the help of differences equations and differential equations. The least squares approach plays a role in the precise results of the method. Using the GM (1,1) modeling method, which is the basis of grey prediction models with its accuracy and usefulness, health expenditures in Turkey that will be achieved in the coming years were estimated.These estimates are particularly useful for health and economic policies.

Gri Modelleme ile Türkiye Sağlık Sektöründe Harcamaların Matematiksel Tahmini

Anahtar Kelimeler

Gri Tahmin Modeli,
Gri Sistem Teorisi,
Tahmini Doğruluk,
En Küçük Kareler Yöntemi,

Öz: Gri teorisi, zayıf bilgi ve / veya bilgi içermeyen sistemlerle ilgilenen etkili bir teoridir. Bu teoriyle, az sayıda veri kullanılarak gelecek zamanlar için etkili ve çok doğru tahminler oluşturulabilir. Gri modelleme yöntemi, gri sistem teorisinin bir alt dalıdır ve modelleme işlemi, denklemler ve diferansiyel denklemler yardımıyla gerçekleştirilir. En küçük kareler yaklaşımı, yöntemin kesin sonuçlarında rol oynar. Gri tahmin modellerinin temeli olan GM (1,1) modelleme yönteminin doğruluğu ve kullanışlılığı ile Türkiye'de önümüzdeki yıllarda sağlanacak sağlık harcamaları tahmin edilmiştir. Bu tahminler özellikle sağlık ve ekonomi politikaları için faydalıdır.

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1.Introduction

Grey modeling method plays an important role in grey systems theory and was first introduced by Deng (1982).This model is expressed in the form of GM(1,1) and is used to produce high precision estimates based on a small number of data [1].

The grey prediction modeling method has been used successfully in many fields such as industry, science and technology, economy, energy consumption [2-8]. In recent years, improved or modified versions of GM(1.1) have been studied by researchers [9-13]. Grey modeling methods provide effective results in estimating exponential number sequences.

In this study, with the help of GM(1,1) modeling method, expenditures on the health sector in Turkey between 2004-2017 were collected and next expenditures were estimated by 2025. As a result of the predictions made with the same model, the error rate was very low, and this situation encouraged the certainty of subsequent predictions. This article is composed of 4 parts. The Grey modeling GM(1,1) method is given in the section 2. The implementation of the GM(1,1) method on health expenditures derived from the health sector in Turkey was carried out in the third section. In the fourth and final section, results based on the findings are given.

2. Material and Method

2.1 Grey Modelling Theory

Step 1: The initial data is created with an array of $X^{(0)}$ data.

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right). \tag{1}$$

Step 2: $X^{(1)}$ accumulated generating sequence is created.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\right). \tag{2}$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i).$$

Step 3: The first-order average value operator $Z^{(1)}$ is created.

$$Z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}$$

where k = 2, 3, … , n

$$Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\right). \tag{3}$$

Step 4:

The equation

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{4}$$

is called the basic form of the GM (1,1) model.

a and b coefficients of equality are determined by the least squares method. Where k is the coefficient of development and progress of a time point a and b respectively [13]. The equation (4) can be expanded as,

$$\begin{aligned} x^{(0)}(2) + az^{(1)}(2) &= b \\ x^{(0)}(3) + az^{(1)}(3) &= b \\ &\vdots \\ x^{(0)}(n) + az^{(1)}(n) &= b \end{aligned} \tag{5}$$

from the system (5) can be written as follow,

$$Y = B\hat{a} \tag{6}$$

where,

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{pmatrix}$$

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} \quad \hat{a} = \begin{pmatrix} a \\ b \end{pmatrix}$$

The goal is to determine a and b coefficients. The least squares are multiplied by B^T on both sides of the equation $Y = B\hat{a}$ according to the least squares method,

$$B^T Y = B^T B \hat{a}$$

and thence,

$$\hat{a} = (B^T B)^{-1} B^T Y$$

can be found. The matrix multiplication algorithm and the least squares method are used to calculate the parameters of this model.

Proof: Substitute all data values into the grey differential equation

$$x^{(1)}(k+1) = \beta_1 x^{(1)} + \beta_2$$

gives that

$$x^{(1)}(2) = \beta_1 x^{(1)}(1) + \beta_2,$$

$$x^{(1)}(3) = \beta_1 x^{(1)}(2) + \beta_2,$$

$$\dots \quad \dots \quad \dots \quad \dots ,$$

$$x^{(1)}(n) = \beta_1 x^{(1)}(n-1) + \beta_2,$$

That is, $Y = B\hat{a}$, for the evaluated values of β_1 and β_2, substitute $x^{(1)}(k+1)$, (k=1,2,..., n-1) with $\beta_1 x^{(1)}(k) + \beta_2$, which gives the error sequence $\varepsilon = Y - B\hat{a}$.

Hence, the S operator is can be defined as,

$$S = \varepsilon^T \varepsilon = (Y - B\hat{a})^T (Y - B\hat{a}) = \sum_{k=1}^{n-1} (x^{(1)}(k+1) - \beta_1 x^{(1)}(k) - \beta_2)^2$$

For minimum values of S, β_1 and β_2 should satisfy following equations,

$$\frac{\partial S}{\partial \beta_1} = -2 \sum_{k=1}^{n-1} \left((x^{(1)}(k+1) - \beta_1 x^{(1)}(k) - \beta_2 \right) x^{(1)}(k)) = 0$$

$$\frac{\partial S}{\partial \beta_2} = -2 \sum_{k=1}^{n-1} \left((x^{(1)}(k+1) - \beta_1 x^{(1)}(k) - \beta_2 \right)$$

$$= 0$$

These equations can be solved easly.Therefore β_1 and β_2 are found as,

$$\beta_1 = \frac{\sum_{k=1}^{n-1} x^{(1)}(k+1) x^{(1)}(k) - \frac{1}{n-1} \sum_{k=1}^{n-1} x^{(1)}(k+1) \sum_{k=1}^{n-1} x^{(1)}(k)}{\sum_{k=1}^{n-1} (x^{(1)}(k))^2 - \frac{1}{n-1} \left(\sum_{k=1}^{n-1} x^{(1)}(k) \right)^2}$$

$$\beta_2 = \frac{1}{n-1} [\sum_{k=1}^{n-1} x^{(1)}(k+1) - \beta_1 \sum_{k=1}^{n-1} x^{(1)}(k)]$$

From $Y = B\hat{a}$, it follows that

$B^T B\hat{a} = B^T Y, \hat{a} = (B^T B)^{-1} B^T Y$

It is clear that,

$$B^T B = \begin{pmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \\ x^{(1)}(n-1) & 1 \end{pmatrix}^T \begin{pmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \\ x^{(1)}(n-1) & 1 \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{n-1} x^{(1)}(k) x^{(1)}(k+1) \\ \sum_{k=1}^{n-1} x^{(1)}(k+1) \end{pmatrix}$$

Therefore,

$$\hat{a} = (B^T B)^{-1} B^T Y = \frac{1}{(n-1)\sum_{k=1}^{n-1}\left(x^{(1)}(k)\right)^2 - \frac{1}{n-1}\left(\sum_{k=1}^{n-1} x^{(1)}(k)\right)^2}$$

$$\text{x} \begin{pmatrix} (n-1)\sum_{k=1}^{n-1} x^{(1)}(k) x^{(1)}(k+1) - \sum_{k=1}^{n-1} x^{(1)}(k+1) \sum_{k=1}^{n-1} x^{(1)}(k) \\ -\sum_{k=1}^{n-1} x^{(1)}(k) \sum_{k=1}^{n-1} x^{(1)}(k) x^{(1)}(k+1) + \sum_{k=1}^{n-1} x^{(1)}(k+1) \sum_{k=1}^{n-1} \left(x^{(1)}(k)\right)^2 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{\sum_{k=1}^{n-1} x^{(1)}(k) x^{(1)}(k+1) - \frac{1}{n-1}\sum_{k=1}^{n-1} x^{(1)}(k+1) \sum_{k=1}^{n-1} x^{(1)}(k)}{\sum_{k=1}^{n-1}\left(x^{(1)}(k)\right)^2 - \frac{1}{n-1}\sum_{k=1}^{n-1}\left(x^{(1)}(k)\right)^2} \\ \frac{1}{n-1}\left(\sum_{k=1}^{n-1} x^{(1)}(k+1) - \beta_1 \sum_{k=1}^{n-1} x^{(1)}(k)\right) \end{pmatrix}$$

$$= [\beta_1, \beta_2]^T$$

Step 5:

The Grey differential equation is defined follow as,

$$\frac{dx^{(1)}(k)}{dk} + ax^{(1)}(k) = b$$

where the grey developmental coefficient a and grey control parameter b are the model parameters to be estimated.

The grey derivative for the first-order grey differential equation with (1) data as the intermediate information is conventionally represented as

$$\frac{dx^{(1)}(k)}{dk} = \lim_{\Delta k \to 0} \frac{x^{(1)}(k + \Delta k) - x^{(1)}(k)}{\Delta k}$$

and

$$\frac{dx^{(1)}(k)}{dk} = \frac{\Delta x^{(1)}(k)}{\Delta k} = x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k+1)$$

when $\Delta k \to 1$ roughly. The background value of $\frac{dx^{(1)}(k)}{dk}$, $x^{(1)}(k)$ is taken as the mean of $x^{(1)}(k)$ *and* $x^{(1)}(k+1)$. The solution to equation

$$\frac{dx^{(1)}(k)}{dk} + ax^{(1)}(k) = b$$

with system parameters determined by least-squares method and initial condition $x^{(1)}(1) = x^{(0)}(1)$ is

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{7}$$

is obtained in the form of (k=1,2,...,n-1). The GM(1,1) model is a special modeling approach based on the exponential functions of the solution.Therefore the prediction values can be generated by,

$$\hat{x}^{(0)}(1) = x^{(0)}(1) \tag{8}$$

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1),\ k = 2, 3, \dots, n$$

3. Results

3.1 An Application of The GM (1,1) Model on Health Expenditures in TURKEY

Turkish Statiscal Institute (TUIK) which reports complete, correct and official data. The data includes the amount of total health expenditures between 2004 and 2017 (Million TL). This section presents a systematic predicting methodology which includes data collection, parameter estimation, result analysis and future health expenditures for Turkey.

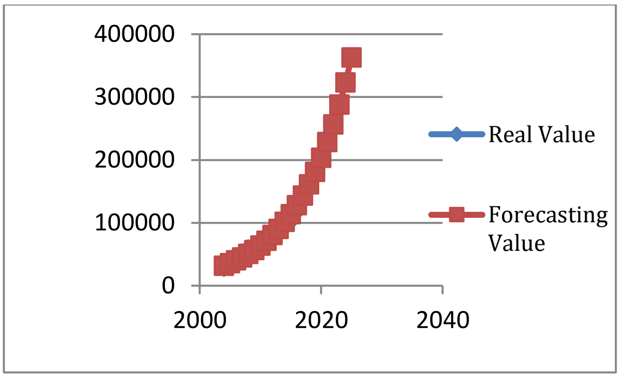
Table 1: Health expenditures and forecasts in health sector in Turkey

Years	Expenditures Amount (Million TL)	GM(1,1)	Error Rate (ε)
2004	30020,8457	31732,0974	5,70
2005	35358,9075	35636,1038	0,78
2006	44068,6811	40020,4209	9,19
2007	50904,3009	44944,1415	11,71
2008	57740,0000	50473,6283	12,58
2009	57910,7320	56683,4091	2,12
2010	61677,5979	63657,1805	3,21
2011	68607,4094	71488,9364	4,20
2012	74188,7119	80284,2348	8,22
2013	84390,0912	90161,6205	6,84
2014	94749,5074	101254,2231	6,87
2015	104567,5392	113711,5507	8,74
2016	119755,7799	127701,5060	6,63
2017	140647,3488	143412,6483	1,97
Average Error Rate			**6,34**
Health Expenditures Estimates (Million TL)			
2018		161056,7356	
2019		180871,5786	
2020		203124,2457	
2021		228114,6630	
2022		256179,6565	
2023		287697,4918	
2024		323092,9727	
2025		362843,1671	

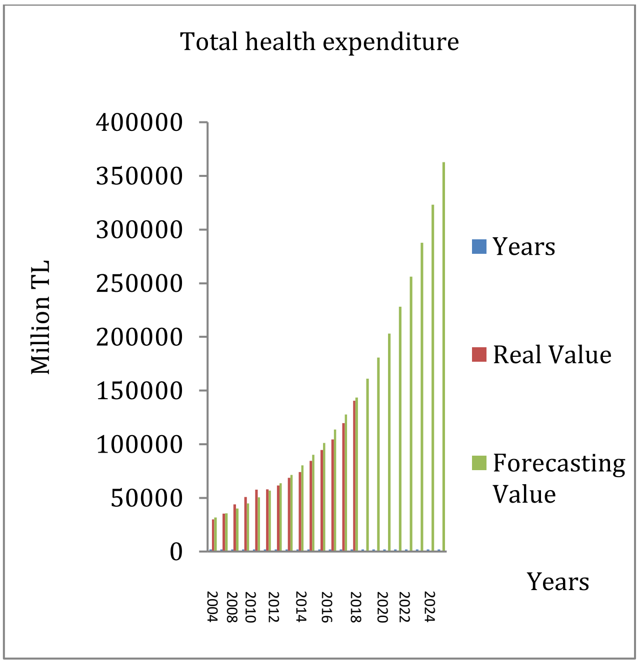
4. Discussion and Conclusions

In the study, expenditures in the health sector in Turkey between 2004-2017 were analyzed and forecasts were made for the following years by using the grey modelling theory. The data at hand were compared with the generated model and the mean error was calculated as 6.34%. In the literature, the results for models with an error rate below 10% are acceptable [9].

The size of a country's health economy represents the development of the country.It is clear that,major investments in the health sector are inevitable to create quality health care.To ensure success in the field of health, the health economy needs to be managed properly.Increased population and following of emerging technologies shows that the inevitable increase in health spending in Turkey.



(a)



(b)

Figure 1: Actual values and forecasting values of health expenditures

Error calculation,

$$\varepsilon(k) = \left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| * 100 \tag{9}$$

and average error,

$$\text{Average Error} = \frac{1}{n}\sum_{k=1}^{n} \varepsilon\,(k) \tag{10}$$

calculated in the form of.

The results show that the increase in health expenditures is mainly due to increased population and other factors.

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