



# Journal of Turkish Operations Management

## AHP – Binary linear programming approach for multiple criteria real estate investment planning

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### Article info

#### Article History:

Received: 15.11.2019

Revised: 17.12.2019

Accepted: 20.12.2019

#### Keywords:

investment planning;  
integer programming;  
multi-criteria decision making

### Abstract

Capital allocation on real estate investments must be made after careful consideration. An investor can obtain a great amount of revenue with a good investment, however, an investment can result by waste of capital, which is spent on an unprofitable asset. There are different aspects of investments related to economic, legal, location and physical factors, which should be taken into account in assessment of possible investment options. In this study, real estate investment planning problem is considered as a multi-objective knapsack problem. An integrated AHP – Binary Linear Programming model is proposed to determine the best investment plan considering different criteria simultaneously. Within the proposed model, multi-criteria evaluation of investment alternatives is done by using Analytical Hierarchy Process and obtained criteria weights values are written as the objective function coefficient in the knapsack model. A real estate investment planning application, which contains 10 alternatives in Ankara, is presented to test the applicability of the proposed decision model. Obtained results are compared with the results obtained by only considering financial aspects of investment.

## 1 Introduction

Investors try to make a profit by spending their capital on alternatives. While some investors are getting returns by putting their money on investment instruments such as foreign currency or gold, some investors are trying to get returns from real estate assets.

Decisions related to real estate investments are generally based on the property's ability to generate financial benefits (Markhvidaa and Bakera 2018). Because of the high investment costs, real estate investments are long – term investments. Therefore, selection among real estate investment alternatives is extremely important and utilization of analytic techniques should be used before making decisions related to development, purchase, upgrade and redevelopment of real estate.

One of the analytic techniques can be used for real estate investment planning is mathematical programming. Mathematical programming models has a wide range of investment planning applications (Mulvey and Vladimirov 1989). By using these models, investors can evaluate the economic benefits that can be achieved with efficient planning and the decision-making process becomes more systematic.

In this study, real estate investment planning is considered as a linear programming problem. If the problem is analyzed from the perspective of an investor, the best combination of investment alternatives with the highest utility values must be determined subject to budget limitation. To do so, the decision problem is modeled as a knapsack model, in which selection of the alternatives is made by considering their resource consumption and possible returns to the decision maker. Real estate investments have to be considered simultaneously on several aspects related to financial, social and spatial attributes of the alternatives. A pre-evaluation is made by weighted sum of some important factors for each investment alternative in order to ensure the simultaneously consideration requirement. To calculate weights for the pre-evaluation process, Analytical Hierarchy Process is used. The evaluation values of alternatives are taken into account as the objective function coefficients in knapsack model and solution of the model provides the suggested combination

of alternatives. A comparison of suggested alternatives after multi-criteria evaluation of alternatives and by considering only of the financial aspect of each alternative is also presented.

The knapsack model (Lorie and Savage 1955) is widely used by researchers in selection problems. Since real estate investments require high amount of capital and it is expected to obtain the greatest return, it is possible to say utilization of Knapsack model for investment plans would be sensible. Some variations of Knapsack model in the literature can be confronted as bin packing problem, cutting stock problem, etc. Some applications of Knapsack problem are automotive sector project selection (de Souza et al. 2012) and mobbing prevention (Bas 2011). Some other examples of Knapsack problem are given in Table 1, as follows:

**Table 1** Examples of Knapsack Problem

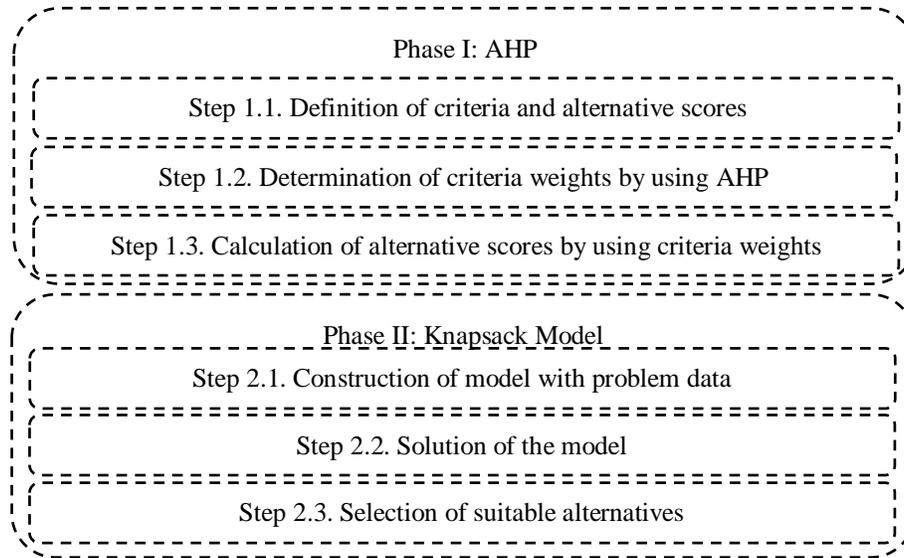
Author	Year	Application	Significance
Yavuz and Captain	2002	Project selection	Multi period model
Alanne	2004	Renovation action selection	Application area
Klamler et al.	2009	Committee selection	Application area
Marinoni et al.	2010	Natural resource management project selection	Multi criteria analysis combination
Chang and Lee	2012	Project selection	DEA combination and Artificial bee colony algorithm solution
Bakirli et al.	2014	Defense project selection	Multi objective multiple knapsack problem
Ic et al.	2017	Order selection in bakery	Fuzzy TOPSIS - Knapsack
Husbands et al.	2017	Transmit Antenna Selection	Application area

As it seen from the literature summary, in some studies the decision problem is considered with evaluating multiple criteria and multi-objective solution methods and/or multi-criteria decision making techniques are utilized to obtain the solution. No studies related to real estate planning decisions is confronted. In this study, real estate investment alternatives are evaluated under multiple criteria consideration. Evaluation criteria are prioritized by using Analytic Hierarchy Process (AHP), which is a commonly used multi-criteria decision making technique. Results of AHP is used in the Knapsack model as objective coefficients and choice between alternatives is made under budget constraint.

The rest of the paper organized as follows: in the second part, explanation of the proposed decision making methodology is given. Next, an application of the proposed approach is demonstrated for real estate investment planning in Ankara in the third part. Results of the application is compared with the results by considering only the financial aspect investment. This analysis is given in the fourth part. Finally, the paper is concluded in the fifth part by summarizing the results and presenting suggestions for extension.

## 2 Definition of the proposed decision model

In this study, multiple criteria real estate investment planning is considered. An investor has a certain budget and he/she has to determine which properties can return the highest profit. Before making this kind of decisions about real estate assets, he/she may face more than one aspect to consider. Real estate investments can be considered as one of such investments. As it is presented in Figure 1, selection decision of an investor, who has to consider a number of criteria simultaneously, is modeled in this study by using AHP integrated Knapsack model. Criteria weights are determined via AHP and alternative investment scores in views of each criterion are aggregated as alternative score by calculating weighted sums. Obtained results of this analysis are written as the objective function coefficient in the Knapsack model (O'Leary 1995). Properties to invest money subject to budget constraint are determined by solving mathematical model. The proposed decision making approach allows considering more than one criteria for an aggregated investment planning.



**Figure 1** General scheme for the proposed approach

#### *Phase 1. AHP*

Analytic Hierarchy Process is a very commonly used multi-criteria decision making method and is introduced by Thomas L. Saaty (Saaty 1980). To solve complex decision problems with AHP, a hierarchical structure of decision problem from goal to the alternatives is firstly formed. Then, pairwise comparisons of both tangible and intangible decision elements at the same hierarchical level are made, by this way the solution of the problem is obtained. It is an easy and flexible method to apply in different applications and applications of AHP is confronted in different applications in a wide range such as environmental sciences, industrial decisions, healthcare systems, etc. Reader may refer to Dağdeviren et al. (2009) and Rouyendegh and Erkan (2012) for detailed explanation of AHP.

In this study, AHP is used for calculation of criteria weights. Importance degree of criteria are must be determined. Because different aspects of investments will not effect selection decision at the same degree and different aspects of investment alternatives must be integrated as a single value by using these weights.

#### *Phase 2. Knapsack Model*

Knapsack problem is one of the most known problems in operations research concept. The aim of the problem is to find a subset of items that yields maximum benefit without exceeding the capacity restrictions. This model is used in different applications with various names such as container loading problem, capital budgeting problem, project selection, etc.

Explanation of used notations and mathematical formulation of Knapsack model is given as follows:

*Sets:*

$i$  : set of items ( $i=1,2,\dots,N$ )

*Parameters:*

$c_i$  : unit benefit of item  $i$

$a_i$  : weight of item  $i$

$B$  : capacity limit of knapsack

*Decision Variable:*

$$x_i = \begin{cases} 1, & \text{if item } i \text{ placed into the knapsack} \\ 0, & \text{otherwise} \end{cases}$$

*Model Formulation:*

*Objective Function:*

$$\max \sum_{i=1}^N c_i x_i$$

*Constraints:*

$$a_i x_i \leq B$$

$$x_i \in \{0, 1\}$$

Solution of the model shows appropriate alternatives subject to capacity limit. Corresponding items to decision variables with 1 value should be placed in the knapsack and with 0 value should not be placed.

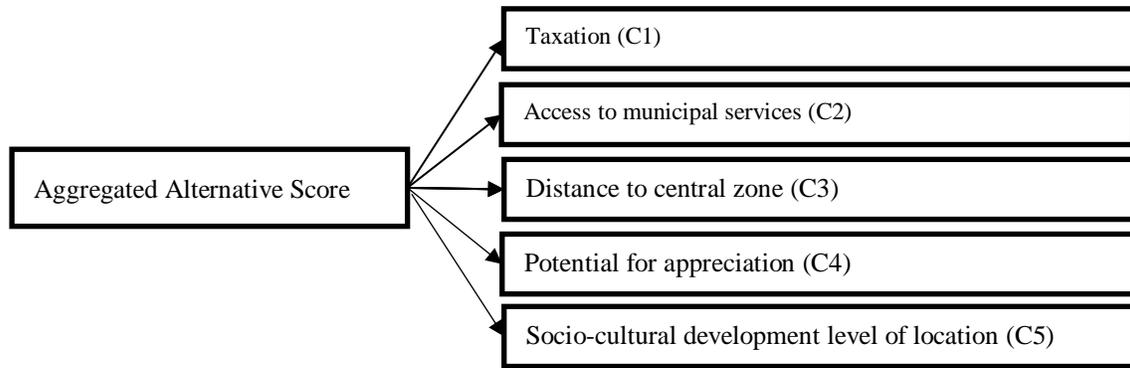
### 3 An application of real estate investment planning in Ankara

To demonstrate the applicability of proposed method, a set of 10 alternatives, which consists real estate assets in various districts of Ankara such as apartment houses, detached house, residence, the land available for construction and the field in the expansion areas of the city, is formed. Application steps are provided as follows:

*Step 1.1:* A set of 10 real estate alternatives are determined according to the decision maker's preferences. Score of each alternative in views of taxation, access to municipal services, distance to central zone, potential for appreciation and socio-cultural development level of location criteria are collected and presented in Table 2. Alternative scores are collected by asking opinion of a real estate office manager in Ankara, who has 30 years of experience in that job. Hierarchical structure of alternative evaluation is presented in Figure 2.

**Table 2** Alternative scores

	C1	C2	C3	C4	C5	Investment Cost
A1	300	52	60	40	30	30
A2	600	72	50	25	60	175
A3	100	25	10	80	15	65
A4	250	20	30	50	10	82
A5	450	45	70	120	20	150
A6	1000	88	90	10	95	350
A7	900	65	60	30	85	400
A8	550	75	75	60	40	250
A9	500	47	60	70	45	65
A10	750	80	80	45	55	190



**Figure 2** Hierarchical structure of alternative evaluation

*Step 1.2:* Criteria values in Table 2 must be aggregated since we need single objective coefficient value for each alternative. To do so, we again asked opinion of the real estate office manager for criteria evaluation.

Assessments made by the real estate office manager on problem criteria are collected to form a pairwise comparison matrix. According to the pairwise comparison matrix, weights of criteria are calculated by using Eigenvector method and calculations are made on Microsoft Excel software. Additionally, consistency ratio of pairwise comparison matrix is also calculated. Pairwise comparison matrix is presented in Table 3 with criteria weights and consistency ratio, as follows:

**Table 3** Pairwise comparison matrix for criteria with respect to alternative score

	C1	C2	C3	C4	C5	Criteria Weight
C1	1	1/3	1/5	1/9	1/3	0.042
C2	3	1	1	1/5	1	0.122
C3	5	1	1	1/5	3	0.180
C4	9	5	5	1	5	0.552
C5	3	1	1/3	1/5	1	0.104
Consistency Ratio						0.043

It is seen from Table 3 that the consistency ratio is less than 0.1, so the pairwise comparisons are consistent. Moreover, the most important criterion on alternative evaluation is potential for appreciation (C4). Distance to central zone (C3) and access to municipal services (C2) follows potential for appreciation. The order of other criteria goes as socio-cultural development level of region (C5) and taxation (C1). It can be summarized from these results that financial return possibility is the most important factor for real estate investments, on the counter taxation, which is the regular expense of properties, is the least important factor.

*Step 1.3:* Weighted sum method is used to aggregate alternative scores. To do so, obtained weight values in Step 1.2. are used. An example of aggregated score calculation is presented as follows:

$$\begin{aligned}
 A_4 &= 0.042 * 250 + 0.122 * 20 + 0.180 * 30 + 0.552 * 50 + 0.104 * 10 \\
 &= 10.5 + 2.44 + 5.4 + 27.6 + 1.04 = 46.98
 \end{aligned}$$

Aggregated scores of all alternatives are presented in Table 4 as follows:

**Table 4** Aggregated alternative scores

Alternative	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Total Score	54.944	63.024	54.77	46.98	105.31	84.336	81.93	83.03	80.854	86.22

*Step 2.1:* Aggregated alternative scores in Table 4 are written as objective function coefficients and investment costs of alternatives in Table 2 are written as constraint coefficients. Budget limit of the investor is assumed to be 1000. According to these data, Knapsack model of the problem is constructed.

*Step 2.2:* Solution of the Knapsack model is obtained by using LINDO software. Solution results are presented as follows:

Objective Value : 528.1520

Decision Variable Values :  $X_1=X_2= X_3= X_5= X_8=X_9= X_{10}=1$  and  $X_4=X_6=X_7=0$

*Step 2.3:* According to the solution results of the model, investment alternatives A1, A2, A3, A5, A8, A9 and A10 are suggested to invest. On the other hand, investor is suggested to avoid investing on A4, A6 and A7 based on the costs and aggregated scores of alternatives.

Solution results indicate that under the budget limit, total benefit of selected alternatives is maximized as 528.1520. To achieve this benefit level, selected alternatives are determined as A1, A2, A3, A5, A8, A9 and A10. As we see in Table 2, A6 is the best alternative in views of C2, C3 and C5. Moreover, it has the third best aggregated alternative score value. But its investment cost is very high and it is not selected. That shows a multiple criteria consideration is meaningful instead of single criterion.

#### 4 Comparison of results of the proposed model with only financial evaluation

In this part of the study, an analysis of single criterion consideration and multiple criteria consideration is compared. For single criterion consideration potential of appreciation (C4) which is the most important criterion is taken into consideration. Aim of the analysis is the comparison of selection by considering only potential of appreciation (C4) with by considering all five criteria.

In the single criterion analysis, alternative values based on C4 in Table 2 are considered as objective function coefficients in the model. Under budget limitation consideration solution results are obtained by LINDO. Obtained results by using LINDO software are given as follows:

Objective Value : 465

Decision Variable Values :  $X_1= X_3= X_4= X_5= X_8=X_9= X_{10}=1$  and  $X_2=X_6=X_7=0$

This results show that selected alternatives for financial evaluation is A1, A3, A4, A5, A8, A9 and A10 with a financial return value of 465. The alternatives, which have the worst three values of C4 are not selected in this evaluation. Selected alternatives in single criterion evaluation and multiple criteria evaluation is given in Table 5 as follows:

**Table 5** Selected alternatives in two cases

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Multiple criteria evaluation	+	+	+		+			+	+	+
Single criterion evaluation	+		+	+	+			+	+	+

The difference between two cases is the selection of A2 in multiple criteria evaluation, whereas A4 is selected in single criterion evaluation. A4 has a good potential for appreciation and when we only consider C4, selection of A4 is sensible. A2 is much better than A4 in views of the other four criteria and it makes A2's aggregated score higher than A4. For this reason, A2 is selected in multiple criteria evaluation.

This comparison indicates possible occurrence of changes in selected alternatives between evaluation with single criterion and multiple criteria. Decision criteria and weight of these criteria can also cause changes in this results. It can be said according to the results that investors have to consider carefully about selection criteria and importance degree of these criteria in real estate investments. Because of the high investment costs, they must avoid making a wrong decision.

#### 5 Conclusion

Real estate investments can be considered as strategic level decisions. Effects of these decisions last for a long time and they require an important amount of capital. A wrong decision would waste resources and such consequence is

not desired by investors. For this reason, these kind of decisions need analytical evaluations before choosing. Due to the requirement of truly management of capital, the effects of decision should be analyzed by considering all possible factors simultaneously.

This study proposes an integrated investment planning model combining multi-criteria decision-making with Knapsack model in order to support selection decisions among real estate investment alternatives. The applicability of the decision model is tested with an application of selection among 10 real estate investment alternatives in Ankara. Comparison of results with only financial aspect of decision shows that multiple criteria consideration of investment alternatives seems sensible.

This study can be extended in the future by considering different criteria and/or sub-criteria. Interdependency among criteria may be considered and for this case decision models can be constructed by combining Analytic Network Process instead of Analytic Hierarchy Process. In applications with much more number of criteria and sub-criteria, hybrid multi-criteria approaches to evaluate alternatives can be considered in the model. In such applications with a complex structure, multi-criteria decision making methods such as TOPSIS, VIKOR, or MOORA can make it easier to evaluate the alternatives.

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