

CAPACITY CHOICE IN A PRICE-SETTING MIXED OLIGOPOLY UNDER DEMAND UNCERTAINTY

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Abstract

We study a mixed oligopoly market in which firms choose the capacities and prices sequentially in an uncertain environment with differentiated products. If the products of the firms are substitutable and realized demand is higher (lower) than expected demand, we find that both firms hold under (excess) capacity. In the case of medium realized demand, private firm chooses under capacity but public firm holds excess capacity. If the products are complements and realized demand is high (low) enough, both firms hold under (excess) capacity. Whenever realized demand is conformable with expectations, private and public firms hold under capacity.

Keywords: Mixed oligopoly, capacity choice problem, uncertain demand, differentiated products

Jel Classifications: D21, D24, D43

DOI: [10.18825/irem.31616](https://doi.org/10.18825/irem.31616)

I. INTRODUCTION

Nowadays, we encounter more and more with a different type of oligopoly called mixed oligopoly where both public and private firms compete. Especially, this type of oligopolies are common in transportation industries, oil industries and telecommunications. The existence of mixed

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oligopolies appears to be very significant in the sectors where privatization efforts just began to take place. In a mixed oligopoly, at least one firm is public firm and such firms aim to maximize public welfare. There are also profit maximizing private firms and public firms have to compete with these public firms within different agendas. Currently, mixed oligopolies become to more and more important and the capacity choice and pricing behavior of firms in mixed oligopolies attract attention of many scholars.

Capacity and quantity choice of firms are investigated in Spence (1977), Dixit (1980), Tirole (1988), and Bau & Singh (1990). However, these studies do not touch upon the issue of mixed oligopolies. On the other hand, Cremer et al. (1989), De Fraja & Delbono (1989), Nett (1993), Anderson et al. (1997), and White (1997) focus on mixed oligopolies. But, these works are also far-off to analyze the capacity choice problem in detail. The issue of capacity choice in mixed oligopoly structures are taken into account in Wen & Sasaki (2001), Nishimori & Ogawa (2004), Lu & Poddar (2005), Ogawa (2006), Lu & Poddar (2006), and Barcen-Ruiz & Garzon (2007). Particularly, Lu & Poddar (2005) studies capacity choice under demand uncertainty in a mixed duopoly market; Lu & Poddar (2006) extend Lu & Poddar (2005) and investigate capacity choice under different time structures in a mixed oligopoly model; and Barcen-Ruiz & Garzon (2007) focus on the capacity choice problem in a mixed oligopoly when firms compete on prices.

In this study, we analyze sequential choice of capacity and price for a mixed oligopoly in an uncertain environment with differentiated products.

II. MODEL

We consider price-setting competition between a private firm, firm 1, and a social welfare maximizing public firm, firm 2, under uncertain demand, and with differentiated products. There are n states of Nature and in state k , each firm faces an inverse demand of the following form:

$$q_{i,k} = a_k - p_{i,k} + bp_{j,k} \quad i, j = 1, 2; i \neq j \text{ and } k = 1, \dots, n \quad (1)$$

where $q_{i,k}$ is quantity produced of firm i , $p_{i,k}$ is the corresponding price and $q_{j,k}$ is the quantity produced of the competitor firm j at state k . Demand parameter $a_k > 0$ takes a different value in each state, $a_k < a_{k+1}$, $k = 1, \dots, (n - 1)$, which leads common demand uncertainty in the mixed duopoly market. The probability of each state is denoted by ρ_k , where $k = 1, \dots, n$; $\sum_{k=1}^n \rho_k = 1$. The degree of product differentiation is measured by $b \in (-1, 1)$, $b \neq 0$,

where positive values of the degree of product differentiation, b , are associated with substitutes, and negative values are associated with complements.

The cost function is assumed to be U-shaped and is given by

$$C_{i,k}(q_{i,k}, x_i) = (q_{i,k} - x_i)^2 \quad i \in (0,1)$$

where x_i represents production capacity of firm i . This form of cost function implies that the long-run average cost is minimized when quantity produced is equal to the firm's production capacity. Moreover, to simplify the analysis, marginal cost is assumed to be zero.

Both firms are risk-neutral. The private firm maximizes its profit, while public firm maximizes social welfare, defined as the sum of firms' profits and consumer surplus. In the first stage, firms simultaneously decide their capacity levels before realization of uncertain demand. In the second stage, demand uncertainty is resolved and firms choose output.

Last, the following constraint guaranteeing positive quantities, prices and capacities in equilibrium completes the model:

$$\frac{8(24 + 12b - 18b^2 - 9b^3 + b^4)}{(7b + 8)(b^4 - 18b^2 + 24)} < \frac{a_k}{\sum_{k=1}^n \rho_k a_k}$$

III. EQUILIBRIUM ANALYSIS

We look for subgame perfect Nash equilibrium and start out by considering the second stage of the game. In the second stage, each firm maximizes its objective function taking capacity level as given. The private firm, firm 1, maximizes the following profit function with respect to p_1 .

$$\pi_{1,k} = p_{1,k}q_{1,k} - (q_{1,k} - x_{1,k})^2 \quad k = 1, \dots, n \tag{2}$$

Solving FOC produces the following reaction function of the private firm:

$$p_{1,k} = \frac{3a_k - 2x_1 + 3bp_{2,k}}{4} \tag{3}$$

The objective of the welfare maximizing public firm is to maximize summation of its own profit, the profit of the private firm and the consumer surplus.

$$SW_k = p_{2,k}q_{2,k} - (q_{2,k} - x_{2,k})^2 + p_{1,k}q_{1,k} - (q_{1,k} - x_{1,k})^2 + \frac{a_k(q_{1,k}+q_{2,k})}{1-b} - \frac{(q_{1,k}^2+2bq_{1,k}q_{2,k}+q_{2,k}^2)}{2-2b^2} - \frac{1}{(p_{1,k}q_{1,k} + p_{2,k}q_{2,k})} \quad k = 1, \dots, n \quad (4)$$

Deriving FOC with respect to p_2 , and solving it yields the following best response function of the public firm:

$$p_{2,k} = \frac{2a_k(1-b)-2(x_2-bx_1)+5bp_{1,k}}{2b^2+3} \quad (5)$$

From (3) and (5), we obtain both firms' price choices in terms on exogenous variables and given capacity levels

$$p_{1,k} = \frac{3a_k(3+2b)-6(x_1+bx_2)+2x_1b^2}{12-7b^2} \quad (6)$$

$$p_{2,k} = \frac{a_k(7b+8)-2(4x_2+bx_1)}{12-7b^2} \quad (7)$$

Equations (6) and (7) imply that each firm's price choice is negatively depended to its own production capacity, regardless of whether products are substitutes or complements. Yet, price choice is positively depended to its competitor's installed capacity level if products are complements and positively depended if products are substitutes. Moreover, compared to the private firm's price choice, the public firm's price decision is less responsive to the competitor's installed capacity. This is because the public firm values private firm's profit and consumer surplus besides its own profit.

In the first stage, firms simultaneously decide capacity levels in order to maximize their objective functions. Solving FOCs yields following best response functions:

$$x_1 = \frac{4((2b+3)\sum_{k=1}^n \rho_k a_k - 2bx_2)(3-2b^2)}{72-72b^2+17b^4} \quad (8)$$

$$x_2 = \frac{(b^4-23b^3-34b^2+33b+48)\sum_{k=1}^n \rho_k a_k - 2bx_1(b^4-11b^2+15)}{48-34b^2+b^4} \quad (9)$$

Best response functions indicate that in the case of substitute products if one firm increases its capacity, the competitor reacts by lowering its capacity. Conversely, in the case of complement products, if one firm increases its capacity, the competitor reacts by decreasing its capacity. Another result that stems from (8) and (9) is that compared to the private firm, the public firm is less

responsive to a change in its competitor's installed capacity and this result remain same either products are substitutes or complements. By solving BR functions, we obtain

$$x_1 = \frac{4(3-2b^2) \sum_{k=1}^n \rho_k a_k}{24-18b^2+b^4} \quad (10)$$

$$x_2 = \frac{(b^4-7b^3-18b^2+9b+24) \sum_{k=1}^n \rho_k a_k}{24-18b^2+b^4} \quad (11)$$

Substituting (10) and (11) into price equations (6) and (7), we obtain

$$p_{1,k} = \frac{(6b^5-26b^4-108b^3-18b^2+144b+72) \sum_{k=1}^n \rho_k a_k - 3a_k(2b+3)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6}$$

$$p_{2,k} = \frac{(8b^4-72b^3-144b^2+96b+192) \sum_{k=1}^n \rho_k a_k - a_k(7b+8)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6}$$

Comparison of firm prices indicate that private firm charges higher price than does the public firm, as expected. In addition, the difference between firm prices increases as the product substitutability decreases. The reason behind this result is that since the welfare maximizing-public firm takes consumer surplus into account, it behaves less aggressively when product complementarity increases.

Now, we proceed by deriving equilibrium quantities.

$$q_{1,k} = \frac{(2b^5-46b^4-36b^3+114b^2+48b-72) \sum_{k=1}^n \rho_k a_k - a_k(2b+3)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6} \quad (12)$$

$$q_{2,k} = \frac{(6b^6-26b^5-116b^4+54b^3+288b^2-24b-192) \sum_{k=1}^n \rho_k a_k + a_k(b^2-2b-4)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6}$$

From equations (10) and (12), we can obtain capacity and quantity differences, which lead us with the main result of the paper.

$$x_{1,k} - q_{1,k} = \frac{(-2b^5-10b^4+36b^3+66b^2-48b-72) \sum_{k=1}^n \rho_k a_k + a_k(2b+3)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6} \quad (13)$$

$$x_{2,k} - q_{2,k} = \frac{(b^6-23b^5-24b^4+93b^3+96b^2-84b-96) \sum_{k=1}^n \rho_k a_k - a_k(b^2-2b-4)(b^4-18b^2+24)}{-288+384b^2-138b^4+7b^6}$$

Table 1 represents firms' capacity and quantity (mis)match under different demand realizations when products are substitutes.

Table 1		
Capacity Choice of Firms under Different Demand Realizations When Products are Substitutes		
Realized Demand	Private Firm	Public Firm
$a_k > T^{Pub}$	Under capacity	Under capacity
$a_k = T^{Pub}$	Under capacity	Capacity is equal to the quantity produced
$T^{Pub} > a_k > T^{Priv}$	Under capacity	Excess capacity
$a_k = T^{Priv}$	Capacity is equal to the quantity produced	Excess capacity
$T^{Priv} < a_k$	Excess capacity	Excess capacity

T^{Priv} and T^{Pub} denote thresholds for the private firm and for the public firm, where

$$T^{Priv} = 2 \left(\frac{b^5 + 5b^4 - 18b^3 - 33b^2 + 24b + 36}{(2b+3)(b^4 - 18b^2 + 24)} \right) \sum_{k=1}^n \rho_k a_k$$

$$T^{Pub} = \left(\frac{b^6 - 23b^5 - 22b^4 + 93b^3 + 96b^2 - 84b - 96}{(b^2 - 2b - 4)(b^4 - 18b^2 + 24)} \right) \sum_{k=1}^n \rho_k a_k$$

Proposition: When products are substitutes both firms hold under capacity if the realized demand is sufficiently high. Conversely, both firms hold excess capacity if the realized is sufficiently low. When the realized demand is medium, the private firm holds under capacity, whereas the public firm holds excess capacity.

When demand is sufficiently high or low than the expected demand, the effect of uncertainty dominates strategic effect and so both firms hold under capacity and excess capacity, respectively. This result is in line with Lu & Poddar (2006). However, if demand realization is in the medium range and is closer to the expected demand the strategic choice dominates the uncertainty effect. In this case, the private holds under capacity in order to increase its price and reduce competition. On the other hand, social welfare maximizing public firm chooses excess capacity in order to increase industry output, weakens competition and increase consumer surplus. The result obtained in this case is the opposite of that Lu & Poddar (2006) derived in quantity competition.

The next table presents results obtained for complement products and the following proposition summarizes findings for this case.

Table 2		
Capacity Choice of Firms under Different Demand Realizations When Products are Complements		
Realized Demand	Private Firm	Public Firm
$a_k > \max\{T^{Pub}, T^{Priv}\}$	Under capacity	Under capacity
$a_k = T^{Pub}$	Under capacity if $T^{Pub} > T^{Priv}$ Excess capacity if $T^{Priv} > T^{Pub}$	Capacity is equal to the quantity produced
$a_k = T^{Priv}$	Capacity is equal to the quantity produced	Under capacity if $T^{Priv} > T^{Pub}$ Excess capacity $T^{Pub} > T^{Priv}$
$\max\{T^{Pub}, T^{Priv}\} > a_k > \min\{T^{Pub}, T^{Priv}\}$	Under capacity if $T^{Pub} > T^{Priv}$ Excess capacity otherwise	Under capacity if $T^{Priv} > T^{Pub}$ Excess capacity otherwise
$\min\{T^{Pub}, T^{Priv}\} > a_k$	Excess capacity	Excess capacity

We should note that when products are substitutes, the difference between thresholds for firms depends on the degree of product substitution. More precisely, $T^{Pub} > T^{Priv}$ if $b < -0,81$ and $T^{Pub} < T^{Priv}$ if $b > -0,81$.

Proposition: When products are complements, both firms hold under capacity if the realized demand is sufficiently high. Conversely, both firms hold excess capacity if the realized is sufficiently low. When the realized demand is medium, both firms hold under capacity.

Similar to the case in substitute products, high or low realization of demand leads both firms to hold under and over capacity, respectively. When demand is medium, the effect of demand uncertainty weakens, and the strategic effect becomes more prominent. (6) and (7) show that price choices of both firms are positively and so quantity produced is negatively depended to the competitor’s installed capacity. Since products are complements, each firm has incentive to hold under capacity in order to increase the opponent’s quantity produced.

Equations (6) and (7) imply that each firm’s price choice is negatively depended to its own production capacity, regardless of whether products are substitutes or complements. Yet, price choice is positively depended to its competitor’s installed capacity level if products are complements and positively depended if products are substitutes. Moreover, compared to the private firm’s price choice, the public firm’s price decision is less responsive to the competitor’s installed capacity.

IV. CONCLUSION

In this study, we analyze sequential choice of capacity and price in a mixed oligopoly context. Different from prior studies, we use an uncertain environment with differentiated products to uncover capacity choice problem. If realized demand is higher (lower) than expected demand, the uncertainty effect becomes dominant and both firms hold under (excess) capacity in the case of substitute products. On the other hand, uncertainty effect loses its dominance when realized demand is medium. In this case, private firm chooses under capacity but public firm aims to increase consumer surplus utilizing over capacity.

When products are complements, our results become slightly different. In the case of complement products, both firms hold under (excess) capacity if the realized demand is high (low) enough. But, the results become quite different compared to the case of substitute products if the realized demand is in harmony with expected demand i.e. it is medium. If such a case is under consideration, both private and public firms hold under capacity. Indeed utilizing under capacity, firms tend to force each other to produce more.

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