



# Some Kinematic Characteristics of Underwater Frog Swimming

## Sualtı Kurbağa Yüzüşünün Bazı Kinematik Özellikleri

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### Abstract

Under liquid swimming for the robots is extremely interesting. In this context one can imagine deep sea beds, oil deposits, acid tanks, etc. It is believed that the next generation of robots will be based on animals rather than humans. If we consider the underwater swimming robots, swimming techniques of frogs are as worthy as fishes. Their underwater motion is trust-drag based. By using the hydrodynamic equations of experimental results of frogs' underwater swimming, we obtain the speed and the distance for such a motion.

**Keywords:** Kinematics, Underwater Swimming, Frogs, Speed, Distance

### Öz

Robotların sıvı altındaki yüzmeleri oldukça ilgi çekicidir. Bu bağlamda derin deniz yatakları, petrol yatakları, asit tankları gibi örnekler düşünülebilir. Yeni nesil robotların insan yerine hayvan bazlı olacağına inanılmaktadır. Sualtı yüzen robotları ele alırsak, yüzüş tekniği açısından kurbağalar balıklar kadar önemlidir. Kurbağaların sualtı yüzüşleri itme-direnç temellidir. Biz böyle bir harekete ait sürat ve mesafe formüllerini, kurbağalara ait deneylerin hidrodinamik denklemlerini kullanarak elde ettik.

**Anahtar Kelimeler:** Kinematik, Sualtı Yüzüşü, Kurbağalar, Sürat, Mesafe

### 1. Introduction

It was striking when some of the swimmers in 1980 Moscow Olympic Games covered near 25 meters by a technique of undulatory swimming at the start. They were better than the others who swam on the surface. Because the swimming at the surface causes five times more drag by generating waves than the same body at a depth of three times its width or body

transversal section (1). There is a significant study about underwater swimming which enlightens energy needs and losses, minimal depth for a better performance, fish tail flaps and high propulsive efficiency, body position analysis for underwater undulatory swimming in [2].

On the other hand aquatic and terrestrial animals have various swimming performances depending on their unlike swimming methods.

Frogs are remarkable swimmers. The relationship between the kinematics and performance of frogs make them worthy for underwater swimming.

The papers published by Gal & Blake [3,4] are key to the studies for frog swimming. In these studies, experiments done by frogs (*Hymenochirus Boettgeri*), establish the relation between trust and drag depending on water density, wetted surface area, drag coefficient and speed. The hydrodynamic mechanism of frog swimming and the hind limb kinematics (in the experimental observations of *Xenopus Leavis*) are given in [5]. There is a comparison of swimming kinematics and hydrodynamics between the purely aquatic (*X. leavis* and *H. boettgeri*) and the semi-aquatic/terrestrial (*R. pipiens* and *B. americanus*) frogs in [6].

**2. Underwater Frog Swimming (U.F.S.)**

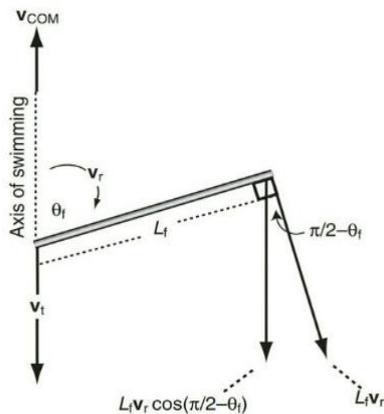
Richards[6] uses the equations,

$$\begin{aligned} d_{t,hip} &= L_{fem} \cos(\pi - \theta_{hip}) \\ d_{t,knee} &= L_{tib} \cos(\theta_{hip} - \theta_{knee}) \\ d_{t,ankle} &= L_{tars} \cos(\Phi) \end{aligned} \quad (1)$$

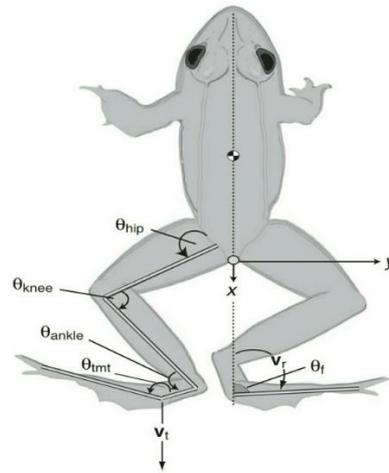
where  $\Phi = \pi - \theta_{hip} + \theta_{knee} - \theta_{ankle}$  and

$$d_t = d_{t,hip} + d_{t,knee} + d_{t,ankle}$$

to compute foot speed components directly from joint angles. In these computations the snout-vent axis is taken as the x-axis where the medio-lateral is the y-axis (figure 1).



**Figure 1:** Vectorial and angular components of a frog's right foot



**Figure 2.** Angles and direction of joint extension

Here  $\theta_{hip}$ ,  $\theta_{knee}$  and  $\theta_{ankle}$  are joint angles and  $d_{t,hip}$ ,  $d_{t,knee}$  and  $d_{t,ankle}$  are hip, knee and ankle components of foot translational displacement ( $d_t$ ) with respect to the hip joint.  $L_{fem}$ ,  $L_{tib}$  and  $L_{tars}$  are lengths of the femur, tibio-fibula and proximal tarsal hind limb segments (figure 1 and figure 2).

The time (t) derivatives of equations (1) yield the speed components  $v_{t,hip}$ ,  $v_{t,knee}$  and  $v_{t,ankle}$  of translational speed  $v_t$ . In the observations of [6] lateral translational speed,  $v_l$ , acting on the total trust is negligible. Right foot padding causes a rotational trust.

The method verified above is a way to compute the speed of U.F.S. But we prefer to compute speed from the hydrodynamics of such a swimming.

**3. Hydrodynamics of U.F.S.**

A nonzero acceleration causes a net force

$$F_{net} = F - D \quad (2)$$

where F is the trust, that is the total forward force and D is the drag, that is the resistive force. Drag is obtained as,

$$D = \frac{1}{2} \rho S_w C_D v^2 \quad (3)$$

where  $\rho$  is the fluid density,  $S_w$  is the wetted surface area of the frog,  $C_D$  is the drag coefficient, and  $v$  is the speed of the frog. Here the drag coefficient can be taken as

$$C_D = 3,64Re^{-0,378} \quad (4)$$

This is the drag coefficient of H. boettgeri computed in the drop-tank experiments of (Gal & Blake ,1987), and  $Re$  is the Reynolds number based on the snout-vent length,

$$Re = 10^6 \times \text{speed(m/s)} \times \text{lenght(m)} \quad (5)$$

calculated by Alexander (1971).  $S_W$  in  $m^2$  is the surface area of a frog measured by geometric surface area determination (Gal & Blake,1987),

$$S_W = 0,188\lambda^{1,52} \quad (6)$$

where  $\lambda$  is the snout-vent length in meters. Equations (4-6) are given for detailed results but here in after we use only equations (2) and (3) for our kinematic computations.

**4. Results**

We compute the speed  $v$  from the equation (2) and by using  $v$  we get the distance formula of this motion.

Newton's laws of motion reveals;

$$F_{net} = m.a = m. \frac{dv}{dt}$$

(where  $m$  is the mass of a frog and  $a$  is the acceleration of the motion). Hence

$$m. \frac{dv}{dt} = F - \frac{1}{2} \rho S_W C_d v^2 \quad (7)$$

Neglecting the effect of  $v$  on  $C_d$  we can take

$$\frac{1}{2} \rho S_W C_d = \alpha \quad (8)$$

(where  $\alpha$  is a constant for a frog under consideration)

$$m. \frac{dv}{dt} = F - \alpha v^2 \quad (9)$$

The method of seperation of variables gives

$$\frac{dv}{F - \alpha v^2} = \frac{dt}{m} \quad (10)$$

Integrating both sides of (10) yields

$$\int \frac{1}{F - \alpha v^2} dv = \int \frac{1}{m} dt \quad (11)$$

The trigonometric substitution  $v = \sqrt{\frac{F}{\alpha}} \sin \theta$  in

(11) implies  $dv = \sqrt{\frac{F}{\alpha}} \cos \theta d\theta$ . Then we have

$$\int \frac{\sqrt{\frac{F}{\alpha}} \cos \theta}{F - F \sin^2 \theta} d\theta = \int \frac{1}{m} dt \quad (12)$$

Hence  $\frac{1}{\sqrt{\alpha F}} \int \sec \theta d\theta = \int \frac{1}{m} dt$  and

$$\frac{1}{\sqrt{\alpha F}} \ln | \sec \theta + \tan \theta | + c = \frac{t}{m} \quad (13)$$

where  $c$  is the integral constant.

Replacing  $\theta$  by  $\arcsin \frac{v}{\sqrt{\frac{F}{\alpha}}}$  we obtain

$$\frac{1}{\sqrt{\alpha F}} \ln \left( \frac{\sqrt{\frac{F}{\alpha} + v}}{\sqrt{\frac{F}{\alpha} - \alpha v^2}} \right) + c = \frac{t}{m} \quad (14)$$

Rearranging the equation gives

$$\frac{\sqrt{\frac{F}{\alpha} + v}}{\sqrt{\frac{F}{\alpha} - \alpha v^2}} = e^{\sqrt{\alpha F}(\frac{t}{m} - c)} \quad (15)$$

Taking the square of both sides we obtain,

$$\alpha \left( 1 + e^{2\sqrt{\alpha F}(\frac{t}{m} - c)} \right) v^2 + 2\sqrt{\alpha F} v + F \left( 1 - e^{2\sqrt{\alpha F}(\frac{t}{m} - c)} \right) = 0 \quad (16)$$

which has the roots

$$v_{1,2} = \frac{-\sqrt{\alpha F} \pm \sqrt{\alpha F} \sqrt{e^{\frac{4t\sqrt{\alpha F}}{m}} - 1}}{\alpha(1 + e^{\frac{2t\sqrt{\alpha F}}{m}})} \quad (17)$$

Then the speed in the direction of motion is

$$v = \sqrt{\frac{F}{\alpha}} \left( \frac{e^{2\sqrt{\alpha F}(\frac{t}{m} - c)} - 1}{e^{2\sqrt{\alpha F}(\frac{t}{m} - c)} + 1} \right) \quad (18)$$

Now let  $s = e^{2\sqrt{\alpha F}(\frac{t}{m} - c)}$  to integrate the equation of the speed with respect to  $t$  to obtain the distance travelled at U.F.S.

Then  $ds = \frac{2\sqrt{\alpha F}}{m} s dt$  and  $dt = \frac{1}{s} \frac{m}{2\sqrt{\alpha F}} ds$ .

Thus

$$\int v(t) dt = \frac{m}{2\alpha} \int \frac{s-1}{s(s+1)} ds \quad (19)$$

Integrating by simple fractions method and substituting  $s = e^{2\sqrt{\alpha F}(\frac{t}{m} - c)}$  we obtain the distance

$$\frac{m}{2\alpha} \ln \left( \frac{(e^{2\sqrt{\alpha F}(\frac{t}{m} - c)} + 1)^2}{e^{2\sqrt{\alpha F}(\frac{t}{m} - c)}} \right) + c_1 \quad (20)$$

where  $c_1$  is the integral constant.

### Discussion and Conclusion

For slow swimming of *Xenopus laevis* frogs in the experiments of Richards [6]  $v$  is between 0 and  $0.25 \text{ ms}^{-1}$ , and for fast swimming  $v$  is between 0 and  $0.4 \text{ ms}^{-1}$ , where  $0 < t < 0.065$  and  $0 < t < 0.075$  seconds, respectively. Hence one can eliminate integral constants  $c$  and  $c_1$  above by using these boundaries and derive the force-mass relations of the motion for a given species.

In this paper we use the biological experimental results obtained by Gal & Blake [3,4]. These experiments yield hydrodynamic equation (7). By using the Hydrodynamic equation (7) for a frog underwater, we obtain the speed formula (18) and the distance formula (20).

These results and equations (1) of Richards [6] will be used in the future studies of endemic Anatolian swimming frogs.

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