MODELING AND OPTIMIZING A VEHICLE NAVIGATION SYSTEM BY G-NETWORK

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ABSTRACT

Increasing the production of vehicles and necessity to use private and public cars have led to heavy traffic that has negative effects in that respect. The aim of intelligent transportation systems (ITS) is improving the quality of transportation, reducing travelling time and reducing fuel consumption via advanced technologies. Clearly, analyzing the routing problems of vehicles and finding optimized routes are among the considerable challenges in intelligent transportation systems.

Vehicle navigation systems are the systems used for leading and routing. Using GPRS communication, these systems provide on-line services for collecting instant traffic information, such as vehicles coordination, speed and type, for enhancing them for efficient routing of vehicles. Furthermore, they can prepare diverse traffic reports regarding time, period, max. and min. speeds, the total driven distance in a desired specific date or time limit.

Many navigation systems have used offline city maps and pre-set maps together with the history of navigation data obtained from GPS. These systems are not suitable due to rapid changes in the traffic conditions.

Since, online systems are preferred. Focusing on online navigation systems and dynamic VRP, we presented a navigation system for the vehicles to receive updated traffic information on reaching each junction, and select the best route with lower traffic to their destination, in case they are permitted to move in it.

In this paper, we used G-Network for modeling the proposed vehicles navigation system. G-Networks are queuing networks with the idea of considering negative customers against positive ones. Negative customers or signals can be considered actual or virtual, operating in different manners in the network. They can destruct positive customers in a queue , cause momentary passing of the customers to another queue, or remove a group of customers from the network.

Vehicles in our proposed model are positive customers and routing decisions are negative customers, here with considered virtual. The queue network is the map of an assumed city. Vehicles may be of different types, such as cars, heavy vehicles and rescue vehicles. Therefore, positive customers in the modeling include different classes. In this graph, each junction and also segments distributed uniformly in each pathway establish the queues of the queuing network. Accordingly, the relevant performance metrics of the network are presented. The given model provide the possibility for us to use gradient descent method for optimization of the routing. Gradient descent is a first order optimization algorithm, used for finding the minimum rate of functions. In optimizing the behavior of the network, it was attempted to minimize the cost function, which includes parameters such as the probability of passing a type of vehicle from a junction and also probability of a routing decision in the junction.

The obtained results from optimization show that the routing problems are improved by considering different criteria including average delay for the vehicles, average delay for routing decisions, average delay for the whole network and average usefulness.

Keywords: Vehicle routing, navigation system, G-Network.

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I. Introduction

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In this paper, we used G-Network for modeling the proposed vehicles navigation system. G-Networks are queuing networks with the idea of considering negative customers against positive ones. Negative customers or signals can be considered actual or virtual, operating in different manners in the network. They can destruct positive customers in a queue [1], cause momentary passing of the customers to another queue [2], or remove a group of customers from the network [3].

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The given model provide the possibility for us to use gradient descent method for optimization of the routing. Gradient descent is a first order optimization algorithm, used for finding the minimum rate of functions. In optimizing the behavior of the network, it was attempted to minimize the cost function, which includes parameters such as the probability of passing a type of vehicle from a junction and also probability of a routing decision in the junction.

The obtained results from optimization show that the routing problems are improved by considering different criteria including average delay for the vehicles, average delay for routing decisions, average delay for the whole network and average usefulness.

To follow, we consider the fulfilled activities in vehicles navigation, VRP and G-Network, in three separate sections. Then, we describe the routing problem in section 1 of III, and deal with modeling the case by G-Network in section 2of III. Section IV explains the calculations of performance criteria, and section V discusses about the optimization of the problem.

II. Related Works

1. Vehicle Routing

VRP research has been underway for over 50 years. Vehicle routing problem was first considered by Dantzig and Ramser (1959). They considered the problem of distributing gas (petrol) by tankers from a source station to a number of fuel stations. With increasing the fuel stations, the number of routes increase, too, making the testing and finding the optimized routes more complicated. Thus, they prop0sed an algorithm that used to find the optimum solution according to proper linear functions [4]. Cooke and Halsey (1966) considered a time-dependent VRP, by classic development of finding the shortest route in a static timing. This method could not be applied for cases with more than a vehicle [5]. There are numerous studies done about VRP, dividing this subject into various classifications mentioned in previous section. Different approaches including the algorithms for finding the shortest route, mathematical models, etc. are given for each classification. The considered classification in this article is "dynamic VRP", the history of which is to follow.

Dynamic VRP refers to Speidel (1976) and Psaraftis (1980), involving the information such as the vehicle location and customer order during the route [6], [7]. Usually, DVRP is used in dynamic operations, by which the customer's order is indicated during the route (online request) and the vehicles should be in "real-time" state. This has various applications in real life, including dynamic transportation management, sales management distributed system, delivery services, repairing or recue services, dial-a-ride emergency service. Ghiani, Guerriero, Laporte & Musmanno, 2003 used DVRP in taxis [8]. Grtz, Klose, Bieding (2009) proposed cell phones for intelligent routing in real time, for communication with the drivers, that reduced the costs followed by customers satisfaction [9]. Block, Gendreau, Ferrucci (2013) have recently presented a method predicting future requests by using previous information [10]. Potvin, Gendreau and Azi provide an ALNS search, showing uncertainty in production scenarios [11]. According to Li, Mirchandani, &Borenstein (2009), Mu, Eglese (2011) and Fu and Lysgaard, dynamic routing is a VRP with "real-time" routing and reprogramming [12], [13].since executing vehicle routing programming is sometimes delayed by unpredicted events such as traffic (Mu, Eglese; 2011) [14]. Dynamic routing is recently been reviewed by Pillac, Gendreau, Guéret, and Medaglia (2012) [15].

2. Vehicle Navigation

There are various ways in ITS, for obtaining traffic information. Some of the promoting methods include processing video images and pictures, IR sensors, magnetic sensors and Piezo electric sensors. Since these methods are not efficient for large scales, some new methods were proposed. In the more advanced methods, GPS and GPRS are used as supplementary tools for collecting traffic information. Dhingral and Gull (2008) focused on vehicle speed, road traffic and load capacity, and by using the history of traffic information provided a model that estimated the number of vehicles in urban areas. The vehicles average speeds were calculated by sensors and the number of vehicles was estimated according to traffic flow theory [16]. Skordili and Trigoni (2008) used "access points" for determining the speeds and the road traffic, and designed navigation routes. The vehicle-vehicle relations were also used in some approaches [17]. Kitani et al. (2008) used "message transition" in inter-vehicle communication for collecting, maintaining and distributing traffic information. They used "bus" as the transiting device for messages, which obtains traffic information from the adjacent vehicles in low-traffic areas to transmit it their adjacent vehicles, periodically [18]. For acquiring real-time traffic information in urban areas, Khosroshahi et al. (2011) proposed a plan that defined a cost function and its parameters such as uncertainty, by using inter-vehicle communication (IVC), the vehicles roadside communication (VRC) and ordinary systems [19]. Lately, Yousefi, Anvari and Abbassi considered online traffic information as the main parameter of routing. In the first stage, ther dealt with gathering traffic information via RSUs and in the second phase, they determined the optimum route for the vehicles movements [20].

3. G-Network

G-Networks were primarily presented by neural networks, in which the signals may be positive or negative [21]. Random neural networks were developed to queuing networks with great speed [1], and the concept of G-Network was introduced as a unique model for neural and queuing networks. A node in a queuing network is equivalent to a neuron in a neural network. Queuing network started with the idea of positive and negative customers, and then, the concept of mobile customer in G-Network was introduced. In this case, when a negative customer reaches the queue, can cause momentary passage of positive customers to another queue and reduce the length of the queue [2]. In [3], negative customers can eliminate the other customers in groups. Moreover, some classes of negative and positive customers were investigated in [22] and [23]. There are other applications of G-Networks, such as using in images texture [24], minimizing the graph coverage [25], optimization of compound cases [26] and modeling defected components in the flow system [27]. A short summary of articles about the G-Network in the years 1989 to 1999 is provided in [28]. Recent publications have indicated G-Network applications on the regular gene networks [29], chemical reaction networks [30] and energy-aware routing in packet networks [31]. Furthermore, the primary neural networks are inspired in some cases for optimizing allocating resources [32] and network routing algorithms [33],[34].

III. Modelling The Routing Problem

1. Describing the problem

In an urban area, there are different types of vehicles, heavy or light vehicles, rescue or police vehicles and etc. Depending on their type, each vehicle should go to a definite route to reach to its destiny. Since, a navigation system must consider types of vehicles.

For devising a navigation system, the city map is transformed to a graph. Each junction is considered as a node or a vertex of this graph.

Moreover, for providing high accuracy and flexibility for the model, we divided the streets into some segments uniformly. Each segment is a vertex of modeled graph too and equipped with a traffic camera, which periodically sends the number of vehicles in that segment to the data center. The routing server is located in the center of the city, updating the traffic in each segment by using the received information from the traffic cameras.

For better understanding, in figure 1, a city with 6 junctions is considered. Figure 2 shows a graph of the considered city.

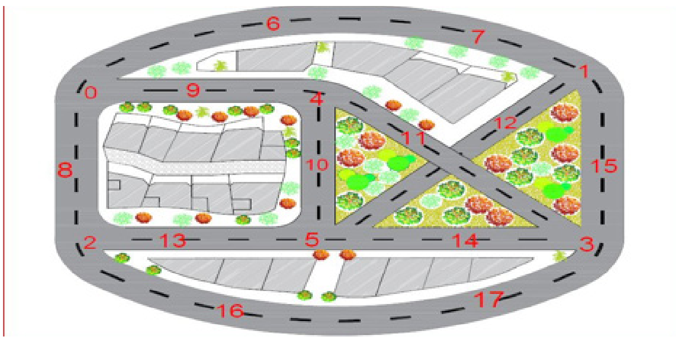

Fig. 1: *City map (routes (3¬–4) and (1–5) are unlevelled)*

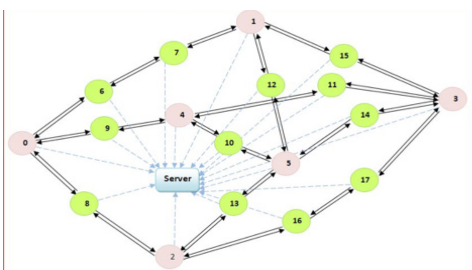

Fig. 2: *Obtained graph from the city map*

Heavy vehicles are only allowed to use the ring road. Other cars are allowed to move inside the city and rescue vehicles and police are allowed to move in all directions.

The vehicles having navigation systems request their optimum route query via a GPRS mediator to the routing server, in the beginning of the trip and by reaching a junction. The routing server is connected to the updated database of the traffic in each segment. The updated information is obtained by traffic cameras. The routing server suggests the best next step to the driver for reaching to the destination, using a Dijkestra algorithm on pathways graph. This pathways graph has separate weight in each edge for each type of vehicles. Thus, each vehicle can select the optimum route, with regards to the type and the considered destination, to reach to it.

The routing server has two main functions: 1) Updating the specified database for traffic information; 2) The responding system to the rout selection query.

2. Modeling the problem by G-Network

For modeling proposed system G-Network is applied. G-networks were initially inspired by neural networks and as a comprehensive model for queuing networks which have defined the ideas of "negative costumers" beside ordinary customers called "positive customers" has been presented in [35].

Hence, we introduced a queue network by N={1,2,…,N} with "N" queue or node. Each vertex in the pathways graph which include urban junctions and street segments is considered as a queue. "N+1" is used to refer to the out of the network. A vehicle is sent out of the network when it reach to its destination.

A. Vehicles traffic

Vehicle traffic is modeled by Positive customer. Each vehicle in each type start its trip from a segment (vertex of the pathways graph) and its destination is another segment. Each positive customer can has a classes. Each class is defined by K=(s,d,σ), where s is the origin, d is the destination and σ is the type of vehicle including cars, heavy vehicles, rescue, police vehicles and etc. Note that s,d ЄN. According to its type, each vehicle can go through different routes.

The rate of entrance of the vehicles from outside from class "k" to a node "n" is defined by λ(n,k). This rate is indeed the start of a movement of a vehicle from a segment or junction.

The route of each class of vehicles is determined by the default routing plan, before traffic control. The routing plan is determined by matrix "p", which indicates the probability of transferring a vehicle belonging to class "k" from node "i" to node "j", and the elements are shown by P(i,k,j). The value of each cell of this matrix is "1" in case of transferring node "i" to node "j", and otherwise it is "zero".

B. Traffic control

Negative customers include the decisions of routing server for optimizing the proposed routes, with regards to momentary traffic conditions. Due to these decisions, the default route is taken away for a destination and a new route is proposed. Negative customers can operate on a positive route in a junction, proposing a new route in contrast with the default route, considered by matrix "P".

Negative customers also have different classes in a network, and defined by (a,k), where a∈N and k∈K. hence, each class of negative customers is responsible for changing the routing decision from a specific class of positive customers in a queue, i.e. changing in selecting the route. Negative customer can travel to some different nodes for controlling them.

The rate of entering of the vehicles from outside the network from class (a,k) to node "j" is defined by λ^-(j,(a,k))and the total rate of entering from class (a,k) to node "j" is determined by Λ(j,(a,k)).

Matrix "P^-" indicates the probability of transferring a negative customer belonging to class (a,k) from node "i" to node "j", in order to reach queue "a" that is target queue of this negative customer. The elements are determined by P^-(i,(a,k),j). The values of each cell of this matrix are "0" or "1".

When a negative customer of class (a,k) reaches its target queue defined by "a", it can send a positive customer of class "k" with the probability of Q(i k, j) to the queue "j" immediately and without considering the service rate. Thus, matrix "Q" is the matrix of optimized decisions of the route, that primarily could be equal to P(i, k, j).

C. Details about the model

As mentioned earlier, negative customers are optimized routing decisions for a class of positive customers. These rerouting decisions could be gained using mathematical functions, without simulation activity.

Generally, the following relations should be considered for routing decisions:

$$\sum_{n\in N} P(n,k,n) + P(n,k,N+1) = 1$$

$$\sum_{n\in N} P^-(n,(a,k),n) + P^-(n,(a,k),N+1) = 1$$

$$\sum_{n\in N} Q(n,k,n) = 1$$

It is assumed that the service delivery fashion of routing server is in first-come-first-served basis, while it can do parallel processing, too. In fact, the database structure could be multi-nuclear. Thus, the transfers that are processed simultaneously are often executed by separate threads.

The service rate "μ" is assumed equal in the whole network, and the entering rates from outside in all the queues are considered as Poisson distribution.

D. Model equations

The probability for a queue (a junction or segment of urban) "n" to include at least one vehicle with positive customers "k" is as follows:

$$qn(n,k) = \frac{\Lambda(n,k)}{\mu + \Lambda^-(n,(n,k))} \quad (1)$$

Where the total rate of entry is:

$$\Lambda(n,k) = \lambda(n,k) + \sum_{n\in N} qn(n,k)P(n,k,n)\mu + \sum_{n\in N} qn(n,k)\Lambda^-(n,(n,k))Q(n,k,n)$$

And

$$\Lambda^-(n,(n,k)) = \lambda^-(j,a,k) + \sum_{n\in N} P^-(n,(a,k),n)cn(n,(a,k))\mu$$

The probability for the queue "n" to include at least one negative customer (routing decision) of class (a, K), is as follows:

$$kn(n,(a,k)) = \frac{\lambda^-(n,(a,k)) + \sum_{n\in N} P(n,(a,k),n)}{\mu} \quad (2)$$

Since we have assumed that the vehicles operate in one node as first-come-first-served, the probability for a queue to be busy is calculated by B(n):

$$B(n)=\sum_{k\in K} qn(n,k)+\sum_{n\in N} cn(n,(a,k)) \tag{3}$$

IV. Performance metrics

qn(n,k) and kn(n,(a,k)) are key values for estimating the criteria. For instance, assuming the unlimited length of the queue, the average queue length in a queue (vehicle traffic in a junction or segment) "n" of class "k" will be:

$$N(n,k)=\frac{qn(n,k)}{1-qn(n,k)} \tag{4}$$

Assuming the unlimited length of queue, the average queue length in queue "n" of negative customer in class (a, k) will be:

$$N(n,(i,k))=\frac{cn(n,(a,k))}{1-cn(n,(a,k))} \tag{5}$$

The average length of queue in queue "n" for all classes:

$$N(n)=\frac{B(n)}{1-B(n)} \tag{6}$$

The probability of positive customer in class "k" or negative customer in "(a, k)" to enter node "n":

$$\pi(n,k)=\frac{\Lambda(n,k)}{\lambda^{+}(k)} \tag{7}$$

$$\pi(n,(a,k))=\frac{\Lambda^{-}(n,(a,k))}{\lambda^{-}(a,k)} \tag{8}$$

Total traffic entry to the network of class "k":

$$\lambda^{+}(k)=\sum_{n\in N}\lambda(n,k) \tag{9}$$

Total traffic entry to the network of class (a, k):

$$\lambda^{-}(i,k)=\sum_{n\in N}\lambda^{-}(n,(a,k)) \tag{10}$$

Total traffic entry to queue "i" of all the positive customer classes:

$$\Lambda^{+}(i)=\sum_{k\in K}\Lambda(i,k)\,,\qquad i\in N \tag{11}$$

Total traffic entry to queue "i" of all the negative customer classes:

$$\Lambda^{-}(i)=\sum_{j\in N,j\neq i}\sum_{k\in K}\Lambda^{-}(i,(j,k)),\, i\in N \tag{12}$$

Thus, the total traffic transferred to a node will be:

$$\Lambda(i)=\Lambda^{+}(i)+\Lambda^{-}(i) \tag{13}$$

The performance of the network can be evaluated using the above formulas. For instance, the end-to-end average delay for customer in class "(k)" which reveal trip delay of the vehicles can be estimated as follows:

$$Tp(k)=\sum_{n\in N}\pi(n,,k)\frac{N(n,k)}{\Lambda(n,k)} \tag{14}$$

Similarly, the average the end-to-end average delay for class (a, k) of negative customers or the routing decisions can be estimated as follows:

$$Tp^{-}(a,k)=\sum_{n\in N}\pi(n,(a,k))\frac{N(n,(a,k))}{\Lambda^{-}(n,(a,k))} \tag{15}$$

Also, the average delay of positive customers (vehicles in the system) could be estimated:

$$\overline{TN}=\sum_{k}\frac{\lambda^{+}(k)}{\Lambda_T^{+}}T(k) \tag{16}$$

Where $\Lambda_T^{+}=\sum_k\lambda^{+}(k)$ indicates the total traffic of positive customers (vehicles) entered into the network, and $T(k)=\sum_k Tp(k)+Tp^{-}(a,k)$.

Finally, average productivity of each queue can be calculated. This metric clarifies how much each queue can be busy for service delivery.

$$\overline{UN}=\frac{1}{N}\sum_{n\in N}B(n) \tag{17}$$

V. Optimizationby Gradient Descent

As mentioned earlier, by arriving at a junction, the vehicles having navigation systems request their optimized route from the routing server, by enquiring for selecting the route. By using the database containing the updated information that include the present traffic in each segment, the routing server responds to the enquiry by the vehicle, offering the optimum route. Hence, it is necessary that an optimizing trend of routing to be done in the continuous routing server, based on the updated information. Optimization procedures are stated as follows.

In this section, the function "f" (cost function) is minimized by optimization of the routing, the required parameters of which for optimization are q_k and k_(a,k) vectors.

$$q\_k=(qn(1,k),qn(2,k),\ldots,qn(n,k)) \quad (18)$$

Where qn(n,k) is the introduction to the probability that a queue "n" includes at least a positive customers (vehicle) in class "k", calculated in equation "1".

$$K_{a,k} = \Big(Kn\big(1,(a,k)\big), Kn\big(2,(a,k)\big), \ldots, Kn\big(n,(a,k)\big)\Big) \quad (19)$$

Where kn(n,(a,k))is the probability for queue "n" to include at least a negative customers in class (a, k), calculated in equation "2".

The cost function will be:

$$f = f(q_{\mathrm{k}}, k_{\mathrm{a,k}}) \qquad \forall\, k \in K, a \in N \quad (20)$$

Where "f" is the sum of q_k and k_(a,k) vectors. The aim in optimization is minimization of cost function.

Optimization can be obtained by proper selection of the route control parameter Q(x,m,y), where x∈ N, m∈ K and y∈ N.

Minimize f with the Q(x,m,y) (21)

Since, traffic information are online or real-time and gathered in a gradual fashion, we need a gradual optimization method. In the other hand, previous papers [31,35,36,37] devise a gradient decent optimization for G-Network that considering the matrix inversion is of time complexity O(N3). Since, this paper uses gradient decent optimization for proposed online navigation system which reduces the cost function on a point in the operation, for obtaining optimum *Q(n,k,n)*.

$$Q_{n+1}(i,k,j) = Q_n(i,k,j) - \eta \frac{\partial f}{\partial Q(i,k,j)} \quad (22)$$

Where η> 0 and the gradient is descending. According to chaining rule, the partial derivation of each part of f, proportional to Q(x,y,m,k) is computable as follows:

$$\frac{\partial f}{\partial Q(i,k,j)} = \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial Q(x,m,y)} \quad (23)$$

To calculate $\frac{\partial q_{\mathrm{k}}}{\partial Q(x,m,y)}$, it is required to only consider the cases where

there is a physical connection between the nodes "I" and "j". h(i,j) = 1, or otherwise h(i,j) = 0. When h(i,j) = 0, then the derivations will be partial. According to fig. 2, matrix "h" will be as follows:

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Then, we calculate N*N matrices:

$$A_k = [A_k(n,n)], \qquad A_k(n,n) = \frac{P(n,k,n)}{\mu + \Lambda^{-}(\mathrm{n},(\mathrm{n},\mathrm{k}))} \quad (24)$$

$$D_k = [D_k(n,n)], \quad (25)$$
$$D_k(n,n) = P(n,k,n)\mu \;+ \Lambda^{-}(\mathrm{n},(\mathrm{n},\mathrm{k}))\mathrm{Q}(\mathrm{n},\mathrm{k},\mathrm{n})$$

(26)

$$H_{\mathrm{k}}^{\mathrm{xmy}}(n) = \begin{cases} \Lambda^{-}(x,(x,k))q(x,k) & \mathrm{k{=}m, n{=}y} \\ -h(x,y)\,\Lambda^{-}\big(x,(x,k)\big)q(n,k) & \mathrm{k{=}m, n{\neq}y} \\ 0 & \text{otherwise} \end{cases}$$

Then, the partial derivation can be calculated as follows:

$$\frac{\partial q_{\mathrm{k}}(n)}{\partial Q(x,m,y)} = \frac{\partial q_{\mathrm{k}}(\mathrm{n})}{\partial Q(x,m,y)} \mathrm{D_k A_k} + H_{\mathrm{k}}^{\mathrm{xmy}} \mathrm{A_k} \quad (27)$$
$$= \frac{\partial q_{\mathrm{k}}(\mathrm{n})}{\partial Q(x,m,y)} W_k + \gamma_{\mathrm{k}}^{\mathrm{xmy}}$$

W_k is the matrix and γ_k^{xm} is the vector:

$$W_k(\mathrm{i},\mathrm{j}) = \sum_{\mathrm{n}\in\mathrm{N}} D_k(i,n)\,A_k(n,j) \qquad i,j \in N \quad (28)$$

$$\gamma_{\mathrm{k}}^{\mathrm{xmy}} = \sum_{\mathrm{n}\in\mathrm{N}} H_{\mathrm{k}}^{\mathrm{xmy}}(n) A_k(n,n) \qquad n \in R \quad (29)$$

Thus:

$$\frac{\partial q_{\mathrm{k}}}{\partial Q(x,m,y)} = \gamma_{\mathrm{k}}^{\mathrm{xmy}} (I - W_k)^{-1} \quad (30)$$

Using the results of equation (31) and $\frac{\partial f}{\partial Q(i,k,j)}$, we shall be able to calculate the next step of the algorithm by equation (23).

The procedures of optimization algorithm are as follows:

Step 1: Replacing values for *Q(i,k,j)* and selecting a value for η> 0.

Step 2: Finding $c_{i,k}$

Step 3: Finding q_k

Step 4: Solving equation (31) by using q_k

Step 5: Updating *Q(i,k,j)* regarding the results from previous steps, using equations (23) and (24).

VI. Numerical Results

The model was implemented and the verification of its performance was evaluated in different scenarios. The number of vehicles (positive customer) class was 3 and the number of junctions was 6 in all scenarios. Fig. 2 shows the network graph, in this respect.

Scenario 1: No. of vehicles in this scenario is 42 and the service rate is 2. Table 1 shows the rate of entry of different types of vehicles in the junctions.

Scenario 2: No. of vehicles in this scenario is 42 and the service rate is 3. The rate of entry of different types of vehicles in the junctions is similar to scenario 1.

Scenario 3: No. of vehicles in this scenario is 42 and the service rate is 4. The rate of entry of different types of vehicles in the junctions is similar to scenario 1.

Scenario 4: No. of vehicles in this scenario is 47 and the service rate is2. Table 2 shows the entry rate of different types of vehicles in the junctions.

Scenario 5: No. of vehicles in this scenario is 51 and the service rate is 2. Table 3 shows the entry rate of different types of vehicles in the junctions.

	k_0	k_1	k_2
n_0	4	2	2
n_1	3	2	0
n_2	2	0	2
n_3	0	5	0
n_4	1	5	4
n_5	4	4	2

Table 1: *Entry rate of vehicles in scenarios 1, 2 & 3*

	k_0	k_1	k_2
n_0	3	5	0
n_1	5	5	0
n_2	0	0	4
n_3	1	3	4
n_4	4	0	1
n_5	4	4	4

Table 2: *Entry rate of vehicles in scenario 4*

	k_0	k_1	k_2
n_0	5	5	4
n_1	1	4	2
n_2	2	1	0
n_3	3	1	4
n_4	4	5	0
n_5	1	5	4

Table 3: *Entry rate of vehicles in scenario 5*

The metrics that is calculated previously, Tp(k),Tp-(a,k), TN,UN, stated in rations (14), (15), (16), (17), and (18) are used for evaluation of the model that used for proposed navigation system.

1.UN Metric

This criterion calculates the average productivity of the nodes. In the proposed routing model, the average productivity increases by performing each optimization step.

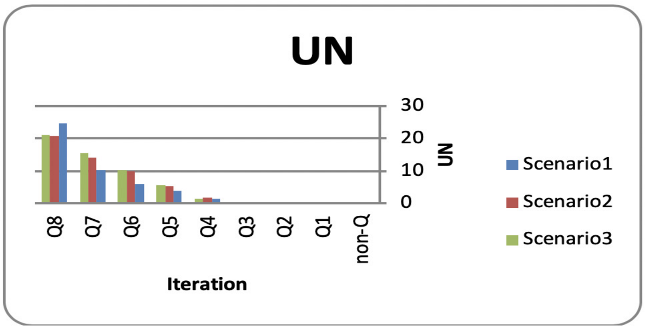

chart.1: *Average usefulness in scenarios 1, 2 & 3*

Chart.1 shows the average usefulness of nodes in scenarios 1, 2 & 3, by using optimization. The number of vehicles and the rates of entry are fixed in these scenarios, but the service rates are increased in each scenario. By increasing the service rate, the rate of usefulness also increases. Moreover, optimization improves the rate of usefulness.

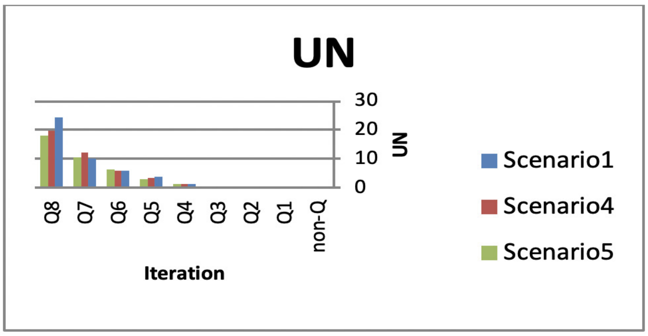

chart.2: *Average usefulness in scenarios 1, 4 & 5*

Chart.2 shows the average usefulness of nodes in scenarios 1, 4&5, by using optimization. The service rates are considered fixed in these scenarios, but the number of vehicles and the rate of entry are variable. Clearly, by increasing the number of vehicles, usefulness id reduced. As it can be observed in fig. 2, usefulness in scenario 1 is higher as compared to scenarios 4 and 5 that involve higher number of vehicles.

2. TN Metric

This criterion indicates the average delay of all the vehicles in the network. The less the delay, the better performance has the network, and routing is done faster.

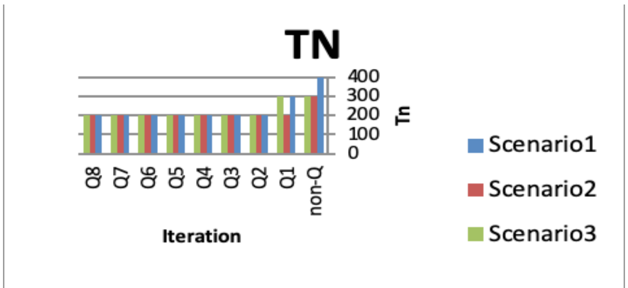

chart.3: Average delay of all the vehicles in the network for scenarios 1, 2 & 3

Chart.3 shows ¯TN in scenarios 1, 2 and 3, which have different rates of entry. It can be observed that delay in scenario 2 with one time of optimization and in scenarios 2 and 3 with two times of optimization has reached to 200ms (required time foe service), and then it was fixed.

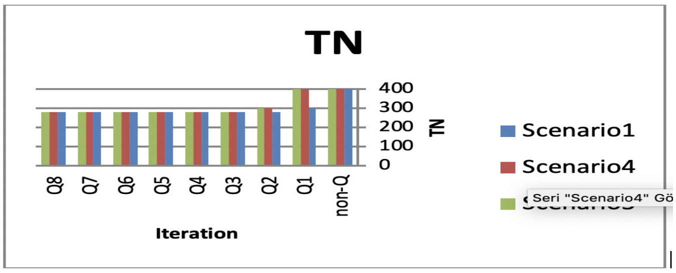

chart.4: Average delay of all the vehicles in the network for scenarios 1, 4 & 5

Chart.4 shows ¯TN in scenarios 1, 4 and 5, which have different rates of entry and different number of vehicles. After two times of optimization for scenario 1 and after 3 times of optimization in scenarios 4 and 5, the delay has reached to 280ms.

3.Tp (k) Metric

Tp(k)is the average end-to-end delay for the vehicles belonging to class "K". Figs. 5-9 show the criteria in each scenario.

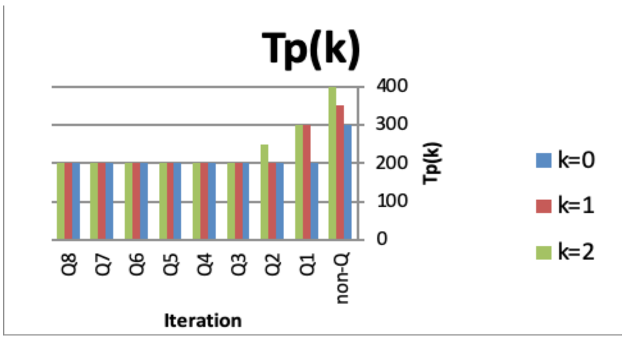

chart. 5: *Average delay for "K" classes in scenario 1*

As observed in chart. 5, delay in class "0", class "1" and class "2" has reached 200ms, after 1, 2, and 3 times of optimization, respectively.

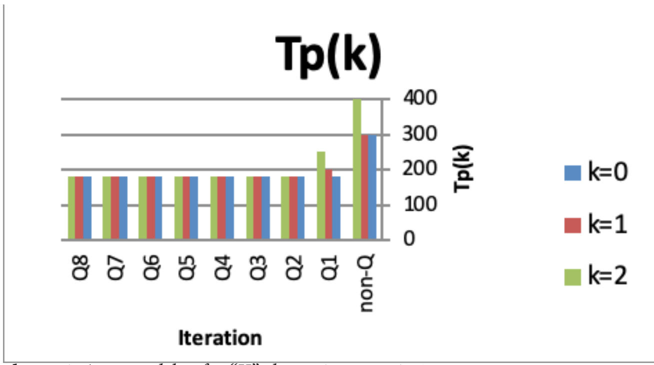

chart. 6: *Average delay for "K" classes in scenario 2*

As observed in chart. 6, in scenario 2, the delay in class "0" and class "1" with one time and the delay in class"2" have reached to their minimum by two times of optimization.

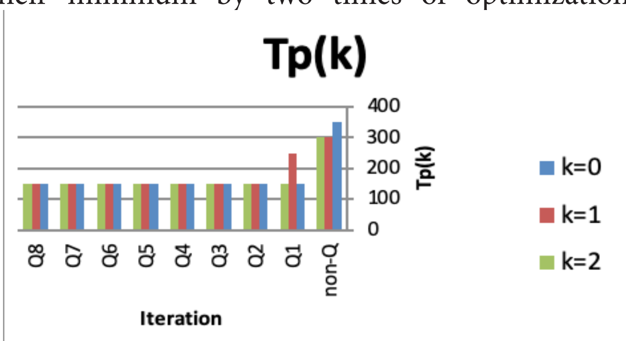

chart. 7: *Average delay of "K" classes in scenario 3*

According to chart. 7, in scenario 3, classes "0" and "2" require one time optimization and class "2" requires 2 times of optimization for their delays to be minimized.

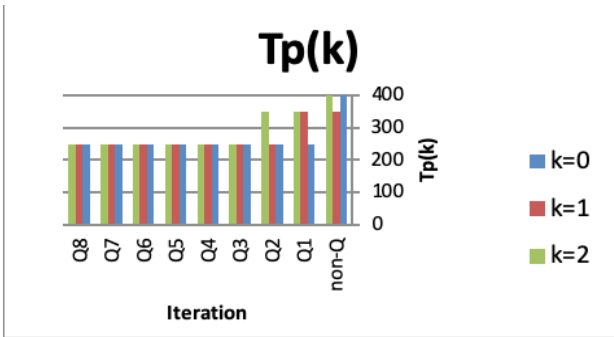

chart. 8: *Average delay of "K" classes in scenario 4*

As observed in chart. 8, classes "0" and "1" require 2, 3 and4 times of optimization, respectively for their delays to be minimized.

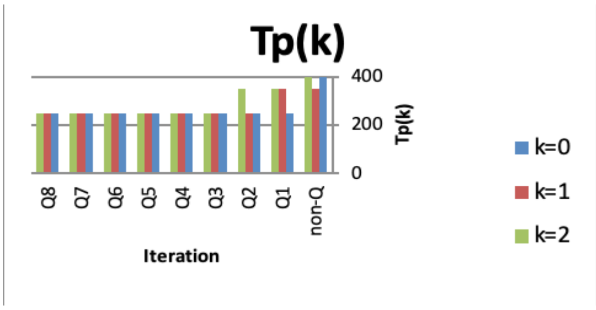

chart. 9: *Average delay of classes "K" in scenario 5*

According to chart. 9, delay in class "0" has reached to minimum by one time of optimization, and delay in class "2" has reached to its minimum by two times of optimization.

4. Tp-(a,k) Metric

Tp-(a,k)is the average end-to-end delay for the routing decisions belonging to class (a,k) that is reduced by optimization, to reach to a fixed value. Chart. 10-14 show the criteria in each scenario.

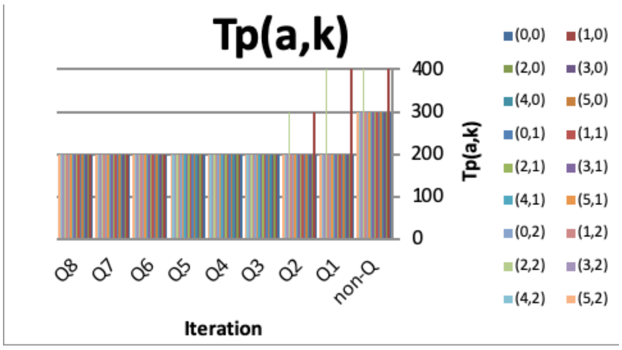

chart. 10: *Average delay of classes (a,k) in scenario 1*

According to chart. 10,class (0,1) and class (2, 2) require three times of optimization and other classes need one time of optimization to get minimum delays.

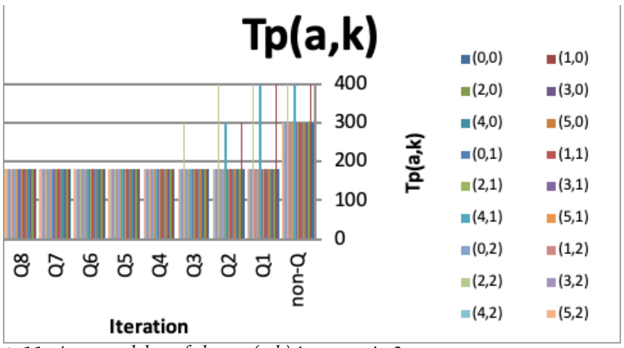

chart. 11: *Average delay of classes (a,k) in scenario 2*

According to chart. 11, class (0,1) and class (1, 4) require three times of optimization and class (2, 2) need 4 times of optimization, while the other classes need one time of optimization for their delays to be minimized.

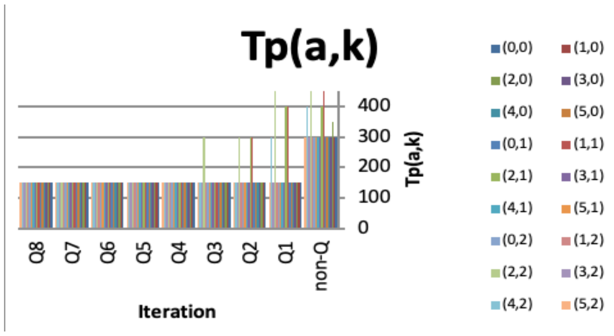

chart. 12: *Average delay of classes (a,k) in scenario3*

According to chart. 12, class (2, 2) require four times of optimization and classes (1, 1) and (1, 2) require three times of optimization, class (2,4) need two times of optimization and the other classes need one time of optimization for their delays to be minimum.

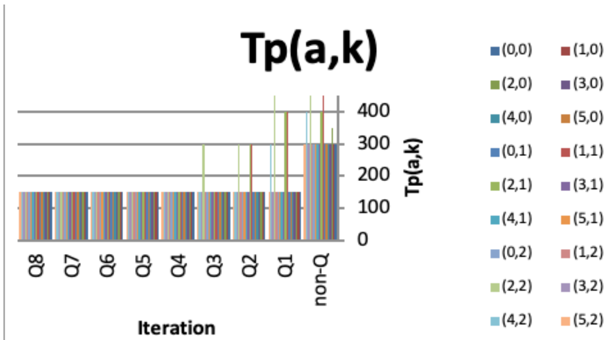

chart. 13: *Average delay of classes (a,k) in scenario4*

According to chart. 13, class (1, 3) needs five times of optimization, class (0, 2) require four times of optimization and classes (2, 2) and (2, 3) and (2, 4) require three times of optimization, class (0,0) need two times of optimization for their delays to get to minimum. Other classes need one time of optimization for their delays to be minimum.

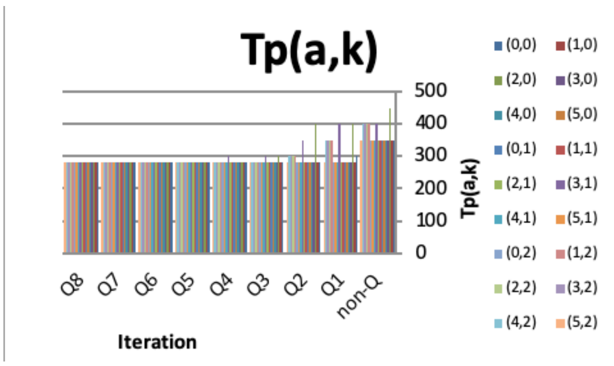

chart. 14: *Average delay of classes (a,k) in scenario5*

According to chart. 14, class (2, 1) require six times of optimization and classes (0, 1) and (1, 3) and (1, 4) require five times of optimization, and the other classes need one time of optimization for their delays to be minimum.

VII. Conclusion

The aim of this article is providing a model for routing of the vehicles having definite destinations, trying to find low-traffic and fast routes.

Vehicle navigation systems are the systems used for routing and conducting the vehicles, mainly using offline and pre-set city maps together with the history of obtained navigation data from GPS. Since traffic is always changing, these systems are not suitable, and online systems are preferred. The proposed model focuses on online navigation systems and dynamic routing for the vehicles to find their routes, dynamically.

G-Network is used for modeling the vehicles routing problems, and regarding their definite routes to reach to their destinations, 3 types of vehicles are considered that are equipped with navigation systems. By arriving at a junction, the vehicles request the optimum route from the routing server, and the routing server receives the updated traffic information from traffic cameras, to suggest the best route in order for the vehicle to get to the required destination.

We used gradient descent method for reducing the traffic in each segment, and optimized the subject. In optimization of the network behavior, the cost function is minimized. The cost function indicates the probability of existence of different types of vehicles in each junction.

The performance of this network was evaluated by the criteria regarding different scenarios. The evaluations showed that by increasing the service rates, the usefulness of the network is improved and the delays are reduced. On the other hand, by increasing the number of vehicles, the usefulness is reduced a little and the delay is increased a bit.

The obtained results from optimization shows that the network usefulness is increased after optimization, and the delay in it is reduced to its possible minimum extent.

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