Araştırma Makalesi *BAUN Fen Bil. Enst. Dergisi, 22(1), 345-353, (2020)*
DOI:10.25092/baunfbed.680824 *J. BAUN Inst. Sci. Technol., 22(1), 345-353, (2020)*

On third-order fuzzy differential equations by fuzzy Laplace transform

Hülya GÜLTEKİN ÇİTİL*

Giresun University, Faculty of Arts and Sciences, Department of Mathematics, Giresun, Turkey.

Geliş Tarihi (Received Date): 25.10.2019
Kabul Tarihi (Accepted Date): 19.12.2019

Abstract

In this study, third-order fuzzy differential equations are studied using fuzzy Laplace transform under the approach of Hukuhara differentiability. Examples are solved. Graphics of the solutions are drawn. Conclusions are given.

Keywords: *Fuzzy diffential equation, fuzzy initial value problem, Hukuhara differentiability, fuzzy Laplace transform.*

Fuzzy Laplace dönüşümüyle üçüncü-mertebe fuzzy diferansiyel denklemler üzerine

Öz

Bu çalışmada, Hukuhara diferansiyellenebilirlik yaklaşımı altında fuzzy Laplace dönüşümü kullanarak üçüncü-mertebe fuzzy diferansiyel denklemler çalışıldı. Örnekler çözüldü. Çözümlerin grafikleri çizildi. Sonuçlar verildi.

Anahtar kelimeler: *Fuzzy diferansiyel denklem, fuzzy başlangıç değer problem, Hukuhara diferansiyellenebilirlik, fuzzy Laplace dönüşüm.*

1. Introduction

In recent years, theory of fuzzy differential equations has been rapidly growing. In many papers, solutions of fuzzy differential equation were studied by many different approaches [1-6]. Fuzzy initial or boundary value problems are solved by the fuzzy Laplace transform method. This method is practically important method. Because,

* Hülya GÜLTEKİN ÇİTİL, hülya.citil@giresun.edu.tr, https://orcid.org/0000-0002-3543-033X

problems are solved directly by fuzzy Laplace transform. Thus, many researchers used fuzzy Laplace transform in papers to solve fuzzy differential equations [7-11].

The aim of this work is to research solutions third-order fuzzy differential equations by fuzzy Laplace transform under the Hukuhara differentiability.

This paper is organized as follows:

In section 2, main definitions and theorems are given. In section 3, problems are defined and solved by fuzzy Laplace transform. Then, examples are given. In last section, conclusions are presented.

2. Main definitions and theorems

Definition 2.1. [12] A fuzzy number is a function $u: \mathbb{R} \to [0,1]$ satisfying the following properties:

1. u is normal,
2. u is convex fuzzy set,
3. u is upper semi-continuous on $\mathbb{R}$,
4. $cl\{x \in \mathbb{R} | u(x) > 0\}$ is compact, where cl denotes the closure of a subset.

Let $\mathbb{R}_F$ be the space of fuzzy numbers.

Definition 2.2. [12] Let be $u \in \mathbb{R}_F$. The α-level set of u is $[u]^\alpha = \{x \in \mathbb{R} | u(x) \geq \alpha\}$, $0 < \alpha \leq 1$.

If $\alpha = 0$, the support of u is defined as $[u]^0 = cl\{x \in \mathbb{R} | u(x) > 0\}$.

Definition 2.3. [10] A fuzzy number u in parametric form is a pair $[\underline{u}_\alpha, \overline{u}_\alpha]$ of functions $\underline{u}_\alpha, \overline{u}_\alpha$, $0 \leq \alpha \leq 1$, which satisfty the following requirements:

1. $\underline{u}_\alpha$ is bounded non-decreasing left-continuous in $(0,1]$, right-continuous at $\alpha = 0$.
2. $\overline{u}_\alpha$ is bounded non-increasing left-continuous in $(0,1]$, right-continuous at $\alpha = 0$.
3. $\underline{u}_\alpha \leq \overline{u}_\alpha$, $0 \leq \alpha \leq 1$.

Definition 2.4. [12] If A is a symmetric triangular number with support $[\underline{a}, \overline{a}]$, the α-level set of A is $[A]^\alpha = \left[\underline{a} + \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha, \overline{a} - \left(\frac{\overline{a}-\underline{a}}{2}\right)\alpha\right]$.

Definition 2.5. [6] Let be $u, v \in \mathbb{R}_F$. If there exists $w \in \mathbb{R}_F$ such that $u = v + w$ then w is called the H-difference of u and v and it is denoted $u \ominus v$.

Definition 2.6. [8] Let $f: (a,b) \longrightarrow \mathbb{R}_F$ and $t_0 \in (a,b)$. If there exists $f'(t_0) \in \mathbb{R}_F$ such that for all h > 0 sufficiently small, $\exists f(t_0 + h) \ominus f(t_0)$, $f(t_0) \ominus f(t_0 - h)$ and the limits holds

$$\lim_{h \longrightarrow 0} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \longrightarrow 0} \frac{f(t_0) \ominus f(t_0 - h)}{h} = f'(t_0),$$

f is Hukuhara differentiable at t_0.

Definition 2.7. [8] Let $f:(a,b)\longrightarrow\mathbb{R}_F$ and $t_0\in(a,b)$. If there exists $f'(t_0)\in\mathbb{R}_F$ such that for all $h>0$ sufficiently small, $\exists f(t_0+h)\ominus f(t_0)$, $f(t_0)\ominus f(t_0-h)$ and the limits holds

$$\lim_{h\longrightarrow 0}\frac{f(t_0+h)\ominus f(t_0)}{h}=\lim_{h\longrightarrow 0}\frac{f(t_0)\ominus f(t_0-h)}{h}=f'(t_0),$$

f is (1)-differentiable at t_0. If there exists $f'(t_0)\in\mathbb{R}_F$ such that for all $h>0$ sufficiently small, $\exists f(t_0)\ominus f(t_0+h)$, $f(t_0-h)\ominus f(t_0)$ and the limits holds

$$\lim_{h\longrightarrow 0}\frac{f(t_0)\ominus f(t_0+h)}{-h}=\lim_{h\longrightarrow 0}\frac{f(t_0-h)\ominus f(t_0)}{-h}=f'(t_0),$$

f is (2)-differentiable.

Definition 2.8. [10] The fuzzy Laplace transform of fuzzy function f is

$$F(s)=L\big(f(t)\big)=\int_0^{\infty}e^{-st}f(t)dt=\left[\lim_{\rho\longrightarrow\infty}\int_0^{\rho}e^{-st}\underline{f}(t)dt\,,\lim_{\rho\longrightarrow\infty}\int_0^{\rho}e^{-st}\overline{f}(t)dt\right].$$

$$F(s,\alpha)=L([f(t)]^{\alpha})=\left[L\left(\underline{f}_{\alpha}(t)\right),L\left(\overline{f}_{\alpha}(t)\right)\right],$$

$$L\left(\underline{f}_{\alpha}(t)\right)=\int_0^{\infty}e^{-st}\underline{f}_{\alpha}(t)dt=\lim_{\rho\longrightarrow\infty}\int_0^{\rho}e^{-st}\underline{f}_{\alpha}(t)dt,$$

$$L\left(\overline{f}_{\alpha}(t)\right)=\int_0^{\infty}e^{-st}\overline{f}_{\alpha}(t)dt=\lim_{\rho\longrightarrow\infty}\int_0^{\rho}e^{-st}\overline{f}_{\alpha}(t)dt.$$

Theorem 2.1. [7] Let $f'(t)$ be an integrable fuzzy function and $f(t)$ is primitive of $f'(t)$ on $(0,\infty]$. Then,

$$L\big(f'(t)\big)=sL\big(f(t)\big)\ominus f(0),$$

where f is (1)-differentiable or

$$L\big(f'(t)\big)=\big(-f(0)\big)\ominus\Big(-sL\big(f(t)\big)\Big),$$

where f is (2)-differentiable.

Theorem 2.2. [11] Suppose that $f(t)$, $f'(t)$ and $f''(t)$ are continuous fuzzy-valued functions on $(0,\infty]$ and of exponential order and $f'''(t)$ is piecewise continuous fuzzy-valued function on $(0,\infty]$ with $f(t)=\left[\underline{f}(t,\alpha),\overline{f}(t,\alpha)\right]$, then

$$L\big(f'''(t)\big)=s^3L\big(f(t)\big)\ominus s^2f(0)\ominus sf'(0)\ominus f''(0).$$

3. Findings and Results

We study solutions of fuzzy initial value problems

$$u''' + \lambda u' = 0, \; x > 0 \tag{1}$$
$$u(0) = [\beta]^\alpha, u'(0) = [\gamma]^\alpha, \;\; u''(0) = [\delta]^\alpha \tag{2}$$

and

$$u''' - \lambda u' = 0, \; x > 0 \tag{3}$$
$$u(0) = [\beta]^\alpha, u'(0) = [\gamma]^\alpha, \;\; u''(0) = [\delta]^\alpha \tag{4}$$

by fuzzy Laplace transform under the Hukuhara differentiability, where $[\beta]^\alpha = \left[\underline{\beta}_\alpha, \overline{\beta}_\alpha\right]$, $[\gamma]^\alpha = \left[\underline{\gamma}_\alpha, \overline{\gamma}_\alpha\right]$, $[\delta]^\alpha = \left[\underline{\delta}_\alpha, \overline{\delta}_\alpha\right]$ are symmetric triangular fuzzy numbers, $\lambda > 0$ and Laplace transform of fuzzy function $u(x)$ is $L\big(u(x)\big) = U(s)$.

I) Consider the fuzzy problem (1)-(2). Using fuzzy Laplace transform to fuzzy differential equation (1),

$$s^3U(s) \ominus s^2u(0) \ominus su'(0) \ominus u''(0) + \lambda\big(sU(s) \ominus u(0)\big) = 0 \tag{5}$$

is obtained. Using Hukuhara difference and fuzzy arithmetic, we have equations

$$s^3\underline{U}_\alpha(s) - s^2\underline{u}_\alpha(0) - s\underline{u}'_\alpha(0) - \underline{u}''_\alpha(0) + \lambda s\underline{U}_\alpha(s) - \lambda\underline{u}_\alpha(0) = 0, \tag{6}$$
$$s^3\overline{U}_\alpha(s) - s^2\overline{u}_\alpha(0) - s\overline{u}'_\alpha(0) - \overline{u}''_\alpha(0) + \lambda s\overline{U}_\alpha(s) - \lambda\overline{u}_\alpha(0) = 0. \tag{7}$$

Using initial conditions (2),

$$\underline{U}_\alpha(s) = \frac{\underline{\beta}_\alpha}{s} + \frac{\underline{\gamma}_\alpha}{s^2+\lambda} + \frac{\underline{\delta}_\alpha}{s(s^2+\lambda)}, \tag{8}$$
$$\overline{U}_\alpha(s) = \frac{\overline{\beta}_\alpha}{s} + \frac{\overline{\gamma}_\alpha}{s^2+\lambda} + \frac{\overline{\delta}_\alpha}{s(s^2+\lambda)}. \tag{9}$$

Taking inverse Laplace transform, lower and upper solutions of fuzzy initial value problem (1)-(2) are obtained as

$$\underline{u}_\alpha(x) = \underline{\beta}_\alpha + \frac{\underline{\gamma}_\alpha}{\sqrt{\lambda}} sin\big(\sqrt{\lambda}x\big) + \frac{\underline{\delta}_\alpha}{\lambda}\Big(1 - cos\big(\sqrt{\lambda}x\big)\Big), \tag{10}$$
$$\underline{u}_\alpha(x) = \overline{\beta}_\alpha + \frac{\overline{\gamma}_\alpha}{\sqrt{\lambda}} sin\big(\sqrt{\lambda}x\big) + \frac{\overline{\delta}_\alpha}{\lambda}\Big(1 - cos\big(\sqrt{\lambda}x\big)\Big). \tag{11}$$

Example 3.1. Consider fuzzy initial value problem

$$u''' + u' = 0, \tag{12}$$
$$u(0) = [0]^\alpha, u'(0) = [1]^\alpha, \;\; u''(0) = [2]^\alpha, \tag{13}$$

where $[0]^\alpha = [-1+\alpha, 1-\alpha]$, $[1]^\alpha = [\alpha, 2-\alpha]$, $[2]^\alpha = [1+\alpha, 3-\alpha]$.

Using the fuzzy Laplace transform, fuzzy solution is

$\underline{u}_\alpha(x) = (-1+\alpha) + \alpha sin(x) + (1+\alpha)\big(1-cos(x)\big),$
(14)
$\underline{u}_\alpha(x) = (1-\alpha) + (2-\alpha)sin(x) + (3-\alpha)\big(1-cos(x)\big),$ (15)
$[u(x)]^\alpha = \big[\underline{u}_\alpha(x), \overline{u}_\alpha(x)\big].$ (16)

If $\frac{\partial \underline{u}_\alpha(x)}{\partial \alpha} \geq 0$, $\frac{\partial \overline{u}_\alpha(x)}{\partial \alpha} \leq 0$, $\underline{u}_\alpha(x) \leq \overline{u}_\alpha(x)$, $[u(x)]^\alpha$ is a valid α −level set. Then, it must be $2 + sin(x) - cos(x) \geq 0$.

According to Figure 1, $[u(x)]^\alpha$ is a valid α −level set. Also, since

$\underline{u}_1(x) = sin(x) + 2\big(1-cos(x)\big) = \overline{u}_1(x),$
(17)
$\underline{u}_1(x) - \underline{u}_\alpha(x) = (1-\alpha)\big(2+sin(x)-cos(x)\big) = \overline{u}_\alpha(x) - \overline{u}_1(x),$ (18)

$[u(x)]^\alpha$ is a symmetric triangular fuzzy function.

According to Figure 2, we can see that $[u(x)]^\alpha$ is a valid fuzzy level set and symmetric fuzzy function.

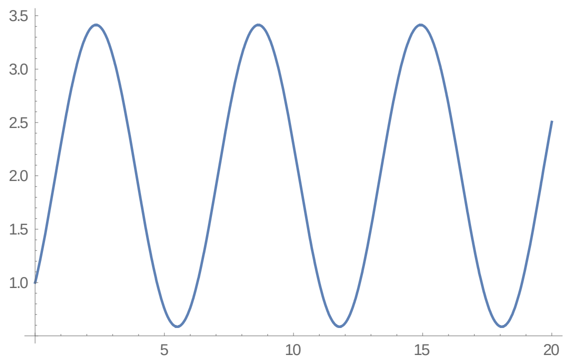

Figure 1. The graphic of the function $f(x) = 2 + sin(x) - cos(x)$.

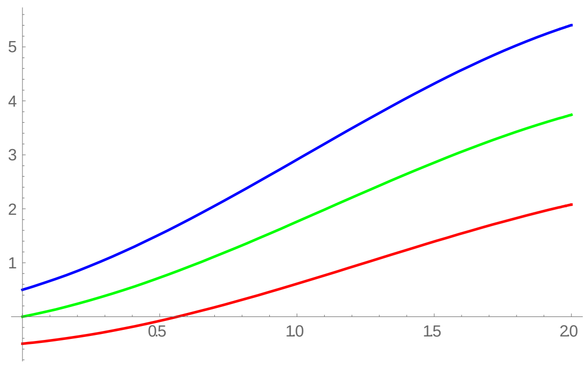

Figure 2. The graphic of $[u(x)]^\alpha$ for $\alpha = 0.5$.
red → $\underline{u}_\alpha(x)$, *blue* → $\overline{u}_\alpha(x)$, *green* → $\underline{u}_1(x) = \underline{u}_1(x)$.

II) Consider the fuzzy problem (3)-(4). Using fuzzy Laplace transform to fuzzy differential equation (3),

$$s^3U(s) \ominus s^2u(0) \ominus su'(0) \ominus u''(0) - \lambda\big(sU(s) \ominus u(0)\big) = 0 \tag{19}$$

is obtained. Using Hukuhara difference and fuzzy arithmetic, we have equations

$$s^3\underline{U}_\alpha(s) - s^2\underline{u}_\alpha(0) - s\underline{u}'_\alpha(0) - \underline{u}''_\alpha(0) - \lambda\overline{U}_\alpha(s) + \lambda\overline{u}_\alpha(0) = 0, \tag{20}$$
$$s^3\overline{U}_\alpha(s) - s^2\overline{u}_\alpha(0) - s\overline{u}'_\alpha(0) - \overline{u}''_\alpha(0) - \lambda\underline{U}_\alpha(s) + \lambda\underline{u}_\alpha(0) = 0. \tag{21}$$

Using initial conditions (4),

$$s^2\underline{U}_\alpha(s) - \lambda\overline{U}_\alpha(s) = s\underline{\beta}_\alpha + \underline{\gamma}_\alpha + \frac{(\underline{\delta}_\alpha - \lambda\overline{\beta}_\alpha)}{s}, \tag{22}$$
$$s^2\overline{U}_\alpha(s) - \lambda\underline{U}_\alpha(s) = s\overline{\beta}_\alpha + \overline{\gamma}_\alpha + \frac{(\overline{\delta}_\alpha - \lambda\underline{\beta}_\alpha)}{s}. \tag{23}$$

Substituting $\overline{U}_\alpha(s)$ in the equation (22) by the equation (23) and making the necessarry operations,

$$\underline{U}_\alpha(s) = \frac{s^3\underline{\beta}_\alpha}{s^4-\lambda^2} + \frac{s^2\underline{\gamma}_\alpha}{s^4-\lambda^2} + \frac{s\underline{\delta}_\alpha}{s^4-\lambda^2} + \frac{\lambda\overline{\gamma}_\alpha}{s^4-\lambda^2} + \frac{\lambda(\overline{\delta}_\alpha - \lambda\underline{\beta}_\alpha)}{s(s^4-\lambda^2)} \tag{24}$$

is obtained. Taking inverse Laplace transform, lower solution of fuzzy initial value problem (3)-(4) is obtained as

$$\begin{aligned}\underline{u}_\alpha(x) = {} & \frac{\underline{\beta}_\alpha}{2}\left(\frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} + cos(\sqrt{\lambda}x)\right) + \frac{\underline{\gamma}_\alpha}{2\sqrt{\lambda}}\left(\frac{e^{\sqrt{\lambda}x}-e^{-\sqrt{\lambda}x}}{2} + sin(\sqrt{\lambda}x)\right)\\ & + \frac{\underline{\delta}_\alpha}{2\sqrt{\lambda}}\left(\frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} - cos(\sqrt{\lambda}x)\right) + \frac{\overline{\gamma}_\alpha}{2\sqrt{\lambda}}\left(\frac{-e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} + cos(\sqrt{\lambda}x)\right)\\ & + \frac{(\overline{\delta}_\alpha - \lambda\underline{\beta}_\alpha)}{\lambda}\left(-1 + \frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{4} + \frac{sin(\sqrt{\lambda}x)}{2}\right).\end{aligned} \tag{25}$$

Similarly, upper solution of fuzzy initial value problem (3)-(4) is obtained as

$$\begin{aligned}\overline{u}_\alpha(x) = {} & \frac{\overline{\beta}_\alpha}{2}\left(\frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} + cos(\sqrt{\lambda}x)\right) + \frac{\overline{\gamma}_\alpha}{2\sqrt{\lambda}}\left(\frac{e^{\sqrt{\lambda}x}-e^{-\sqrt{\lambda}x}}{2} + sin(\sqrt{\lambda}x)\right)\\ & + \frac{\overline{\delta}_\alpha}{2\sqrt{\lambda}}\left(\frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} - cos(\sqrt{\lambda}x)\right) + \frac{\underline{\gamma}_\alpha}{2\sqrt{\lambda}}\left(\frac{-e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{2} + cos(\sqrt{\lambda}x)\right)\\ & + \frac{(\underline{\delta}_\alpha - \lambda\overline{\beta}_\alpha)}{\lambda}\left(-1 + \frac{e^{\sqrt{\lambda}x}+e^{-\sqrt{\lambda}x}}{4} + \frac{sin(\sqrt{\lambda}x)}{2}\right).\end{aligned} \tag{26}$$

Example 3.2. Consider fuzzy initial value problem

$$u''' - u' = 0, \tag{27}$$
$$u(0) = [0]^\alpha, u'(0) = [1]^\alpha, \quad u''(0) = [2]^\alpha, \tag{28}$$

where $[0]^\alpha = [-1+\alpha, 1-\alpha]$, $[1]^\alpha = [\alpha, 2-\alpha]$, $[2]^\alpha = [1+\alpha, 3-\alpha]$.

Using the fuzzy Laplace transform, fuzzy solution is

$$\underline{u}_\alpha(x) = \frac{(-1+\alpha)}{2}\left(\frac{e^x+e^{-x}}{2} + cos(x)\right) + \frac{\alpha}{2}\left(\frac{e^x-e^{-x}}{2} + sin(x)\right) + \frac{(1+\alpha)}{2}\left(\frac{e^x+e^{-x}}{2} - cos(x)\right) + \frac{(2-\alpha)}{2}\left(\frac{-e^x+e^{-x}}{2} + cos(x)\right) + (4-2\alpha)\left(-1 + \frac{e^x+e^{-x}}{4} + \frac{sin(x)}{2}\right), \quad (29)$$

$$\overline{u}_\alpha(x) = \frac{(1-\alpha)}{2}\left(\frac{e^x+e^{-x}}{2} + cos(x)\right) + \frac{(2-\alpha)}{2}\left(\frac{e^x-e^{-x}}{2} + sin(x)\right) + \frac{(3-\alpha)}{2}\left(\frac{e^x+e^{-x}}{2} - cos(x)\right) + \frac{\alpha}{2}\left(\frac{-e^x+e^{-x}}{2} + cos(x)\right) + 2\alpha\left(-1 + \frac{e^x+e^{-x}}{4} + \frac{sin(x)}{2}\right), \quad (30)$$

$$[u(x)]^\alpha = [\underline{u}_\alpha(x), \overline{u}_\alpha(x)]. \quad (31)$$

If $\frac{\partial \underline{u}_\alpha(x)}{\partial \alpha} \geq 0$, $\frac{\partial \overline{u}_\alpha(x)}{\partial \alpha} \leq 0$, $\underline{u}_\alpha(x) \leq \overline{u}_\alpha(x)$, $[u(x)]^\alpha$ is a valid α −level set. Then, it must be $4 + e^x - e^{-x} - (sin(x) + cos(x)) \geq 0$.

According to Figure 3, $[u(x)]^\alpha$ is a valid α −level set. Also, since

$$\underline{u}_1(x) = e^x + e^{-x} + \frac{3}{2}(\sin(x) - \cos(x)) - 2 = \overline{u}_1(x),$$
(32)

$$\underline{u}_1(x) - \underline{u}_\alpha(x) = (1-\alpha)\left(\frac{e^x - e^{-x} + cos(x) - sin(x)}{2} + 2\right) = \overline{u}_\alpha(x) - \overline{u}_1(x), \quad (33)$$

$[u(x)]^\alpha$ is a symmetric triangular fuzzy function.

According to Figure 4, we can see that $[u(x)]^\alpha$ is a valid fuzzy level set and symmetric fuzzy function.

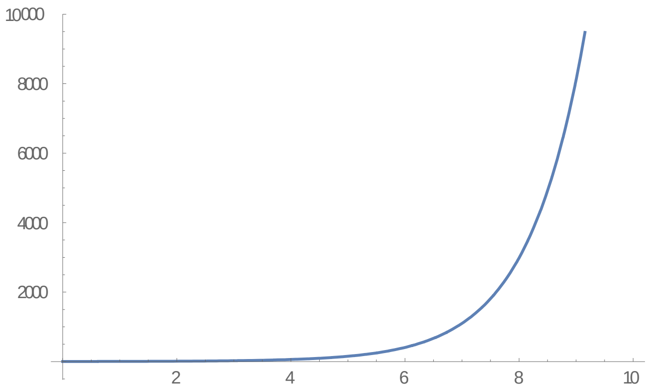

Figure 3. The graphic of the function $g(x) = 4 + e^x - e^{-x} - (sin(x) + cos(x))$.

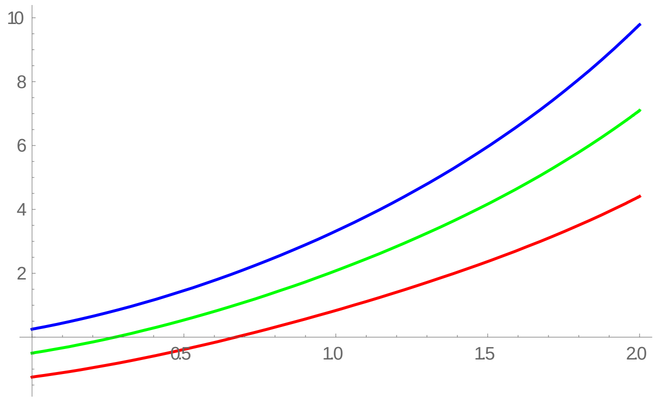

Figure 4. The graphic of $[u(x)]^{\alpha}$ for $\alpha = 0.5$.
$red \rightarrow \underline{u}_{\alpha}(x)$, $blue \rightarrow \overline{u}_{\alpha}(x)$, $green \rightarrow \underline{u}_{1}(x) = \underline{u}_{1}(x)$.

4. Conclusions

In this paper, fuzzy initial value problems for third-order fuzzy diferential equation with positive and negative constant coefficients are studied. Solutions are found by fuzzy Laplace transform. Examples are solved. Graphics of solutions are drawn. It is seen that solutions are valid fuzzy functions. Also, it is shown that when initial values are symmetric triangular fuzzy numbers, solutions are symmetric triangular fuzzy functions.

References

[1] Buckley J. J., Feuring T., Fuzzy differential equations, **Fuzzy Sets and Systems,** 110, 1, 43–54, (2000).

[2] Gasilov N, Amrahov Ş. E., Fatullayev A. G., A geometric approach to solve fuzzy linear systems of differential equations, **Applied Mathematics and Information Sciences,** 5, 3, 484-499, (2011).

[3] Gültekin Çitil H., The relationship between the solutions according to the noniterative method and the generalized differentiability of the fuzzy boundary value problem, **Malaya Journal of Matematik,** 6, 4, 781-787, (2018).

[4] Gültekin Çitil H., Altınışık N., The Examination of Eigenvalues and Eigenfunctions of the Sturm-Liouville Fuzzy Problem According to Boundary Conditions, **International Journal of Mathematical Combinatorics**, 1, 51-60, (2018).

[5] Hüllermeier E., An approach to modelling and simulation of uncertain dynamical systems, **International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems,** 5, 2, 117-137, (1997).

[6] Khastan A., Nieto J. J., A boundary value problem for second order fuzzy differential equations, **Nonlinear Analysis,** 72, 9-10, 3583-3593, (2010).

[7] Allahviranloo T., Barkhordari Ahmadi M., Fuzzy Laplace transforms, **Soft Computing,** 14, 3, 235-243, (2010).

[8] Patel K. R., Desai N. B., Solution of fuzzy initial value problems by fuzzy Laplace transform, **Kalpa Publications in Computing**, 2, 25-37, (2017).

[9] Patel K. R., Desai N. B., Solution of variable coefficient fuzzy differential equations by fuzzy Laplace transform, **International Journal on Recent and Innovation Trends in Computing and Communication,** 5, 6, 927-942, (2017).

[10] Salahshour S., Allahviranloo T., Applications of fuzzy Laplace transforms, **Soft Computing**, 17, 1, 145-158, (2013).

[11] Muhammad Ali H. F., Haydar A. K., On fuzzy Laplace transforms for fuzzy differential equations of the third order, **Journal of Kerbala University,** 11, 3, 251-256, (2013).

[12] Liu H.-K., Comparison results of two-point fuzzy boundary value problems, **International Journal of Computational and Mathematical Sciences**, 5, 1, 1-7, (2011).