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What is the O-Corner Interpretation and Does it Save the Traditional Square of Opposition?

O-Köşesi Yorumu Nedir ve Geleneksel Karşıtlık Karesini Kurtarabilir mi?

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ABSTRACT

To salvage traditional logic and traditional square of opposition from the problem of existential import, logicians have been offering solutions for centuries. In this paper, firstly it will be argued that as far as we know, the historically first solution proposed by Abelard in 11th century and by Seuren in 2002 is actually a version of the O-Corner Interpretation of traditional logic, which is generally attributed to the 14th century logician Ockham. Secondly, it will be advocated that two systems of Abelard and of Ockham have the same logical power. Lastly, the main claim will be that Abelard's and Seuren's system shall be favored over Ockham's system.

Keywords: Problem of existential import, the o-corner interpretation, traditional logic, Abelard, Ockham, traditional square of opposition

ÖZET

Geleneksel mantığı ve geleneksel karşıtlık karesini, varlıksal varsayım denen problemden kurtarmak için, mantıkçılar yüzyıllardır çözüm üretmekteler. Bu çalışmada, ilk, bildiğimiz kadarıyla tarihsel ilk çözüm olan ve 11. yüzyılda Abelard ve 2002'de Seuren tarafından önerilen sistemin aslında 14. yüzyıl mantıkçısı olan Ockham'a atfedilen geleneksel mantığın O-köşesi yorumunun bir versiyonu olduğu savunulacaktır. Daha sonra, bu iki sistemin mantıksal güçlerinin eşit olduğu iddia edilecektir. En son olarak da, Abelard ve Seuren'in sisteminin Ockham'ınkine tercih edilmesi gerektiği asıl iddiamız olacaktır.

Anahtar Kelimeler: Varlıksal varsayım problemi, o-köşesi yorumu, geleneksel mantık, Abelard, Ockham, geleneksel karşıtlık karesi

Introduction and Preliminaries

Aristotle, in his well-known logic book *De Interpretatione*, defined certain relations among four categorical statements, such that one is the opposite of the others in a specific sense. The most quoted passage (*De Interpretatione*, 17b17-26) where he outlines these relations is as follows:

> "I call an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally, e.g. 'every man is white' and 'not every man is white', 'no man is white' and 'some man is white'. But I call the universal affirmation and the universal negation contrary opposites, e.g. 'every man is just' and 'no man is just'. So these cannot be true together, but their opposites may both be true with respect to the same thing, e.g. 'not every man is white' and 'some man is white'."[1]

For brevity, the following abbreviations have been traditionally using to refer to these four categorical statements;

A:	Universal Affirmative	Every S is P	*SaP*
E:	Universal Negative	No S is P	*SeP*
I:	Particular Affirmative	Some S is P	*SiP*
O:	Particular Negative	Some S is not P	*SoP*

Aristotle defined *SaP* as the contradictory opposite of *SoP* and *SiP* as that of SeP in the sense that they cannot both be true and cannot both be false together. Thus we define by means of the sentential operator "if and only if (↔)";

$$\neg SaP \leftrightarrow SoP \qquad \text{(Contradictory 1)}$$

$$\neg SiP \leftrightarrow SeP \qquad \text{(Contradictory 2)}$$

In the passage quoted, *SaP* and *SeP* are defined as contrary opposites to the effect that they cannot both be true, but can both be false. Two formalizations in modern notations are available; one with the modal operator "possible (◊)" and one with the sentential operator "and (∧)";

$$\neg\Diamond(SaP \wedge SeP) \wedge \Diamond(\neg SaP \wedge \neg SeP) \qquad \text{(Contrary)}$$

$$\neg(SaP \wedge SeP) \qquad \text{(Contrary)}$$

Depending on the relations Contradictory 1, 2 and Contrary just defined, the following relations can also be deduced. Let us first assume that *SiP* is false: Then, by Contradictory 1, *SeP* must be true. If *SeP* is true, then *SaP* must be false by Contrary. *SaP* and *SoP* are contradictory opposites. Therefore, *SoP* must be true. Thus, *SiP* and *SoP* are subcontrary opposites, in the sense that *SiP* and *SoP* cannot both be false but may both be true. Again, two formalizations may serve. This time, the operator 'or (∨)' shall be employed;

1 Aristotle, *Categories and De Interpretatione*, trans. J.L.Ackrill (Oxford: Clarendon, 1975), 48.

$$\Diamond(SiP \land SoP) \land \neg\Diamond(\neg SiP \land \neg SoP) \qquad \text{(Subcontrary)}$$

$$SiP \lor \mathrm{SoP} \qquad \text{(Subcontrary)}$$

The other relation that can be deduced from the already defined ones is subalternation, which will turn out to be problematic in what follows. Assuming that *SaP* is true, Contrary implies that *SeP* must be false. By Contradiction 2, *SiP* must be true, to the effect that whenever *SaP* is true, *SiP* must be true as well or whenever *SiP* is false, *SaP* must be false too. The same line of reasoning can easily be carried out for the subalternation relation between negative statements as well: if *SeP* is true, *SaP* is false. So its contradictory, *SoP*, must be true;

$$SaP \rightarrow SiP \qquad \text{(Subalternation 1)}$$

$$SeP \rightarrow SoP \qquad \text{(Subalternation 2)}$$

These relations are generally depicted in the following schema called the traditional square of opposition (see Figure 1)[2].

Additionally, in Prior Analytics 1.2, 25a1-252[3], Aristotle defined the immediate inference called Conversion, by which one can simply interchange the subject term and predicate term of a statement of the form E and I and the statement remains true. Thus, conversion validates the following inferences;

$$SeP \rightarrow PeS \qquad \text{(Conversion 1)}$$

$$SiP \rightarrow PiS \qquad \text{(Conversion 2)}$$

$$SaP \rightarrow PiS \qquad \text{(Conversion per accidens)}$$

2 The vowels characterizing the categorical statements (**A**, **E**, **I** and **O**) in the traditional square of opposition are the invention of medieval logicians, not to be found in Aristotle's original works.

3 Cf: "In universal statement the negative premise is necessarily convertible in its terms: e.g., if no pleasure is good, neither will anything good be pleasure; but the affirmative, though necessarily convertible, is so not as a universal but as a particular statement: e.g., if every pleasure is good, some good must also be pleasure. In particular statements the affirmative premise must be convertible as particular, for if some pleasure is good, some good will also be pleasure." See; Aristotle, *Aristotle: Categories. On interpretation. Prior analytics.* Trans. Cooke, H.P., Tredennick, H., (London: Harvard University Press, 1938), 203.

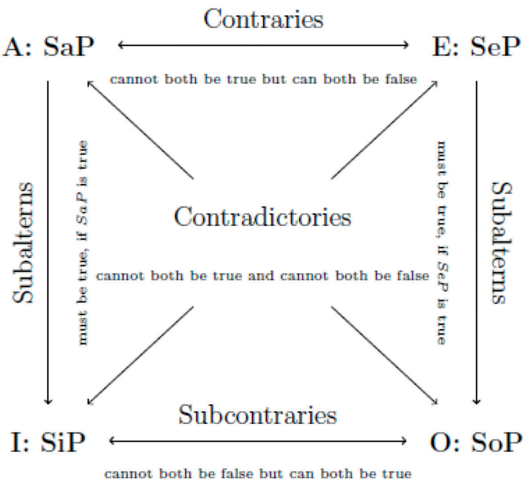

Figure 1. *Traditional Square of Opposition*

To the best of my knowledge, no one involved in the discussion disagrees that the traditional square of opposition as depicted above and the immediate inference of conversion stem from Aristotle. I have already cited the relevant passages where Aristotle plainly elucidates Conversion and all the other relations. Yet, whether Aristotle is to be credited for the following immediate inferences is far from being uncontroversial. Since those can be found in any medieval, as well as, modern textbook[4], I shall define the rest of immediate inferences without attributing them to either Aristotle or any other logician.

The inference of Contraposition allows one to interchange the subject term and the predicate term in A and O form statements if one replaces both terms with their complementary terms[5]. The complementary term is generally achieved in English by putting the prefix "non"[6] in front of the term, i.e., the complementary term of, say, "man" is "non-man". To formalize it, $\overline{P}$ is the complementary of P. Thus, contraposition validates the following inferences:

$$SaP \rightarrow \overline{P}a\,\overline{S} \qquad \text{(Contraposition 1)}$$

$$SoP \rightarrow \overline{P}o\,\overline{S} \qquad \text{(Contraposition 2)}$$

$$SeP \rightarrow \overline{P}o\,\overline{S}^{7} \qquad \text{(Contraposition per accidens)}$$

4 See, for example, Keynes' 19th century classical textbook: John Neville Keynes, *Studies and Exercises in Formal Logic*, (London: Macmillan, 1906).

5 See Thom's book for the discussion about relation between negative terms and Aristotelian syllogism: Paul Thom, *The syllogism*, (München: Philosophie Verlag, 1981).

6 Sometimes, the prefixes "un" or "in" do the job as well.

7 It is "per accidens" because the inference of subalternation is also used to infer this: firstly, from "Every P is Q" to "Some P is Q" by subalternation, then, from "Some P is Q" to "Some Q is P" by Conversion 2.

Obversion is the inference that validates the entailment from an affirmative statement to its negative counterpart (or vice versa) if the predicate term is replaced with its complementary term.

$$SaP \rightarrow Se\overline{P} \qquad \text{(Obversion 1)}$$

$$SeP \rightarrow Sa\overline{P} \qquad \text{(Obversion 2)}$$

$$SiP \rightarrow So\overline{P} \qquad \text{(Obversion 3)}$$

$$SoP \rightarrow Si\overline{P} \qquad \text{(Obversion 4)}$$

Inversion is less common than the immediate inferences[8]. It states that for A and E form statements, their subalterns are allowed to be inferred, even when both the subject terms and predicate terms are replaced with their complement terms:

$$SaP \rightarrow \overline{S}i\overline{P} \qquad \text{(Inversion 1)}$$

$$SeP \rightarrow \overline{S}o\overline{P} \qquad \text{(Inversion 2)}$$

Since the discussion of to what extent these rules can be attributed to Aristotle has not been settled down yet, I shall use the generic name "Traditional logic", instead of "Aristotelian logic", to refer the traditional square of opposition as stated above together with the collection of four immediate inferences.

In the eye of the modern logician, traditional logic is inconsistent. It leads one to derive falsehood from truth. The focus of the modern criticisms is the relation of Subalternation, although it extends to cover all immediate inferences and the relations in traditional logic, except for Contradictories. The inconsistency comes in sight when the logician deals with empty terms, the terms whose extension is empty ($[[S]] = \varnothing$)[9]. Inconsistencies can be derived in a couple of ways:

1) The statement "Every chimera is monster" is written in modern notation as $\forall x(Cx \rightarrow Mx)$ and reads "for all x, if x is a chimera, x is monster." This statement is vacuously true, on the grounds that there is no chimera, meaning that $[[C]] = \varnothing$. By Subalternation 1, "Every chimera is monster" implies "Some chimera is monster", which is written as $\exists x(Cx \wedge Mx)$ and reads "there is at least one thing which is both chimera and monster". This is false, simply because $[C] = \varnothing$. Thus, the relation of Subalternation yield falsehood from truth. Modern classical logic invalidates the inference $\forall x(Cx \rightarrow Mx) \rightarrow \exists x(Cx \wedge Mx)$, while in traditional logic $CaM \rightarrow CiM$ is regarded as valid.

2) Vacuously true statements might be confusing. Yet, similar inconsistencies can be derived

8 For instance, Copi and Cohen do not include the inference of inversion in their analysis of traditional logic. See: Irving M. Copi, and Carl Cohen., *Introduction to Logic: Study Quide,* (USA: Macmillan, 1994). Mulder renders that "we might just want to avoid considering it [the inference of inversion] as a bona fide part of traditional logic." See: Dwayne Hudson Mulder, "The existential assumptions of traditional logic", *History and Philosophy of Logic* 17(1-2) (1996), 140.

9 The double square-brackets refer, henceforth, to the extension of the term put between them.

from non-vacuously true statements, as well. “No logician has proved Goldbach’s conjecture *(∀x(Lx → ¬Gx))*” is true. By Conversion 1, “No one who has proved Goldbach’s conjecture is logician *(∀x(Gx → ¬Lx*))” must also be true and by Subalternation 2, “Some who has proved Goldbach’s conjecture is not logician” can be deduced: *∃x(Gx ∧ ¬Lx)*, which reads “there is at least one thing (person) who has proved Goldbach’s conjecture and is not logician”, which is obviously false. Thus, traditional logic leads from the true statement that “No logician has proved Goldbach’s conjecture” to the false statement that “Some who has proved Goldbach’s conjecture is not logician”. Note that modern classical logic validates Conversion 1 as used in this proof: *∀x(Lx → ¬Gx) → ∀x(Gx → ¬Lx)*. The problematic inference appears to be, again, the relation of Subalternation. Similar counter-examples against the relation of Subalternation can be found, among many others, for example, in the Kneales[10], Strawson[11], Copi[12] or Morrison[13].

3) Hitherto, in our criticism, the truth value of A and E statements, when [[S]] = Ø, is evaluated according to the modern interpretation. Let us assume, in this instance, that an A statement is false when [[S]] = Ø. “Every chimera is monster” is, then, false. By Contradiction 1, “Some chimera is not monster” must be true. However, it is false because *∃x(Cx ∧ ¬Mx)* implies that [[C]]= Ø.

4) Another aspect of the same problem, which is mostly neglected in such discussions, is that of universal terms, such as “being” or “existent”. The extension of a universal term contains everything in the universe (of discourse). Thus, its complementary term would be empty. Similarly, the complementary term of an empty term is a universal term. For instance, the extensions of “non-chimera” and “non-unicorn” are identical, since they both contain everything in the universe. Let us consider the statement that “Every human is being”. By Contraposition 1, it becomes “Every non- being is non-human”. “Some non-being is non-human” is implied by Subalternation 1, meaning that “there is at least one thing which is non-being and non-human”. The immediate inference of Inversion also might yield problems regarding universal terms. “Every being is existent” implies “Some non-being is non-existent” by Inversion 1.

Therefore, the relations of Subalternation result in inconsistencies when a categorical statement contains empty subject or predicate term. For the modern logician, in traditional logic there is “something wrong”[14]. It contains “contradictions and absurdities”[15] and has been shown to be “confused and inconsistent”[16].

From the beginning of 11th century to the recent times, logicians have tried to save the traditional

10 William Kneale, and Martha Kneale, *The Development of Logic* (London: Oxford University Press, 1962).
11 Peter Frederick Strawson, *Introduction to Logical Theory* (New York: Routledge, 2011).
12 Irving Copi*, Introduction to Logic,* (New York: Macmillian, 1953).
13 John J. Morrison, “The existential import of a proposition in Aristotelian logic,” *Philosophy and Phenomenological Research* 15(3) (1955).
14 Copi et all.., *Introduction to Logic: Study Quide*, 182.
15 Manley Thomson, “On Aristotle’s square of oppositions,” *Philosophical Review* 62 (1953), 264.
16 George Edward Hughes and D. Londey, *The elements of formal logic*. (New York: Harper and Raw, 1965), 331.

square with all its oppositions defined by Aristotle from the logical disaster called inconsistency. One favored and, as far as we know, historically first solution is to distribute existential import among the corners of the square such that it is not possible anymore to derive any inconsistency within the logical system. What is known as O-Corner interpretation of the traditional square of opposition is one where the right side of the square, i.e., the O and E-corners, do not carry existential import, while their affirmative counterparts, i.e., the A and I corners do. The motivation behind this kind of solution is, most probably, that it manages to preserve the extensional bivalent nature of traditional logic, which is also of highest importance for modern classical logic.

Here I will advocate three claims: Firstly, that what I call the Abelardian-Seurenian system is also an O-Corner interpretation and secondly that it is has the same logical power as the main version of O-Corner interpretations originated from Ockham and supported and developed by modern logicians. Thirdly, I will argue that it saves the traditional square of opposition better than the main version of O-corner Interpretation.

1. The O-Corner Interpretation of Traditional Square of Opposition

To be able to evaluate these attempts, in accordance with Chatti and Schang[17], each corner shall be formalized in modern notation, once with import and once without import, so that these notations could help to evaluate the consistencies of the system. The subscript "$_{imp!}$" shall be understood as that that statement has existential import and the subscript "$_{imp?}$" as that that statement has no explicit existential import. Thus[18];

$A_{imp!}$:	$\exists x(Sx) \wedge \forall x(Sx \rightarrow Px)$	$E_{imp!}$:	$\exists x(Sx) \wedge \forall x(Sx \rightarrow \neg Px)$
$A_{imp?}$:	$\forall x(Sx \rightarrow Px)$	$E_{imp?}$:	$\forall x(Sx \rightarrow \neg Px)$
$I_{imp!}$:	$\exists x(Sx \wedge Px)$	$O_{imp!}$:	$\exists x(Sx \wedge \neg Px)$
$I_{imp?}$:	$\neg\exists x(Sx) \vee \exists x(Sx \wedge Px)$	$O_{imp?}$:	$\neg\exists x(Sx) \vee \exists x(Sx \wedge \neg Px)$

According to the logical analysis Chatti and Schang[19] did with the formulae just provided, only three squares have managed to keep all the oppositions intact without any inconsistency and thus survived the test. Those squares are; 1) A and I carry existential import and E and O do not. 2) Universals are the only ones that carry existential import. 3) While negatives carry existential import, positives do not, which is the exact opposites of the first square. Not surprisingly, the first is the one that is historically proposed and discussed in the literature, which will be the main subject for the present paper. The question of which corners of the

17 Saloua Chatti, and Fabien Schang, "The cube the square and the problem of existential import," *History and Philosophy of Logic* 34(2) (2013), 115.

18 In their articles, some statements are formalized slightly different, however, those that Chatti and Schang provide and the ones provided here are all logically equivalent.

19 Chatti, et all., "The cube the square and the problem of existential import".

traditional square and which sentences[20] carry existential import play a central role in both modern and medieval discussions.

1.1 Abelardian Seurenian System

Although Church declares the 14th-century logician William of Ockham as "the first logician to consider the question of existential import or to propose a tenable theory of it"[21], Seuren maintains that the 11th-century logician Abelard "was [...] probably the first, after Aristotle, to be aware of the problem "[22] and accuses Church of being "keen to erase Abelard's heritage from history". The Kneales seem to agree with Seuren that Abelard is to "have the credits of being the first to worry about the traditional square of opposition", but add that "he did not work out all the consequences of the change he advocated."[23]

Abelard's idea has never been seriously discussed until nine centuries later, Seuren[24] proposed the same particular solution, (as he claims) independently of Abelard. Even then, it is still doubtful to profess that this view got enough attention from modern logicians. One reason may well be, as Parsons contends, that "Abelard's writing was not widely influential." [25] Secondly, in Abelard's works, it seems that the system is not fully developed as the Kneales[26] state. Thirdly, Horn thinks that "Abelard's results [...] were apparently too counter intuitive to be taken seriously."[27]

Let us, now, turn to the details of the system proposed by *Dialectica* of Abelard and Seuren[28]. The first step Abelard took is to differentiate external negation from internal negation. External negation is the one which is put in front of the whole sentence. "Not every S is P" is the externally negated "Every S is P". Internal negation, as can be understood from the text, is the one that negates the copula of the sentence. Thus, "Every S is not P" is the internally negated "Every S is P". External negation shall be symbolized with the usual negation sign "¬" and internal negation

20 Henceforth, I shall use the word `sentence' to refer to a particular structure of surface grammar, while by the words `statement' or `proposition', I shall mean the proposition underlying the sentence. As will be seen later, some different sentences just defined might have the same underlying proposition for some logicians. For the issue at hand, this might even be understood as having the same truth condition: if two sentences p and q have the same underlying proposition, then p and q are true together in a particular state of the world, and are false together in another particular state of the world.

21 Alonzo Church, "The history of the question of existential import of categorical propositions", in *Logic, Methodology, and Philosophy of Science: Proceedings of the 1964 International Congress,* ed. Y. Bar-Hillel (Amsterdam: North-Holland, 1965), 420.

22 Pieter A. Seuren, *The Logic of Language: Language From Within*, volume 2. (New York: Oxford Univerity Press, 2009), 173.

23 Kneale, et all., *The Development of Logic*, 211.

24 Pieter A. Seuren, "The logic of thinking". *Koninklijke Nederlandse Akademie van Wetenschappen, Mededelingen van de Afdelig Letterkunde, Niuwe Reeks*, 65(9) (2002).

25 Terence Parsons, "The traditional square of opposition" in *The Stanford Encyclopedia of Philosophy (Summer 2017 Edition)*, ed. Zalta, E.N., (Metaphysics Research Lab: Stanford University, 2017).

26 Kneale, et all., *The Development of Logic.*

27 Laurence Horn, *A natural history of negation.* (Chicago: University of Chicago Press, 1989), 26.

28 Seuren, "The logic of thinking".

with the tilde sign "∼". For Abelard, these two negations have different logical powers, hence must not be regarded as the same.

> "Aliam itaque vim negatio habet praeposita, aliam interposita"[29]
>
> "The negation therefore has a different power if it is put in front [of the proposition] than if it is put in between. (my translation)"

Thus, Abelard distinguishes not four, but six categorical sentences:

A:	(Every S is P)	*SaP*	**∼*I:***	(Some S is not P)	*∼SiP*
I:	(Some S is P)	*SiP*	**¬A:**	(Not every S is P)	*¬SaP*
∼A:	(Every S is not P)	*∼SaP*	**¬I:**	(Not some S is P)	*¬SiP*

A close scrutiny with respect to these six sentences of Abelard reveals some peculiarities of both modern and traditional logic. In modern logic, $\neg SaP$ and $\sim SiP$ appear to express the same proposition, since $\neg\forall$ is defined as $\exists\neg$ and $\neg\exists$ as $\forall\neg$, which is generally called Law of Quantifier Negation. Thus, modern logic allows the inference from "Not every" to "Some not" (and vice versa) and from "Not some" to "Every not" (and vice versa). Thus we define:

$$\neg SaP \leftrightarrow \sim SiP \qquad \text{(Quan Neg)}$$

$$\neg SiP \leftrightarrow \sim SaP \qquad \textit{(Quan Neg)}$$

The attitude of traditional logic toward Quan Neg is highly complicated. In Apeleius' work (2nd century), one might observe that he merely plays with the idea under the name of 'equipollency'. He uses internal negations, $\sim SaP$ on the E-corner and $\sim SiP$ on the O- corner in the square of opposition and just after the representation of the square, he claims one can get the same propositions if one externally negates their contradictories.

> "Every proposition becomes equipollent with its alternate [contradictory opposite], if it takes on a negative particular at the beginning- for example, supposing that it is the universal dedicative: *Every pleasure is good*, if a negation prefixed to it, it will become *Not every pleasure is a good*, which is sound to just the same extent as was its alternate: *Some pleasure it not a good*" [30]

Thus, "Apuleius [...] was well aware of [...] the Laws of Quantifier Negation"[31]. Parsons[32] alleges that Boethius uses both $\sim SiP$ and $\neg SaP$ in his works. If we assume that traditional logic validates Quan Neg, there arises four squares with identical oppositions and entailments as the one in Figure 1 (see Figure 2).

29 Lambertus Marie De Rijk, *Petrus Abaelardus, Dialectica. First Complete Edition of the Parisian Manuscript.* (Assen: Van Gorcum/Hak and Prakke, 1956), 176.

30 David G. Londey and Carmen J. Johanson, *The logic of Apuleius: Including a complete Latin text and English translation of the Peri Hermeneias of Apuleius of Madaura,* (Leiden: Brill Archive,1987), 89.

31 Horn, *A natural history of negation,* 25.

32 Parsons, "The traditional square of opposition".

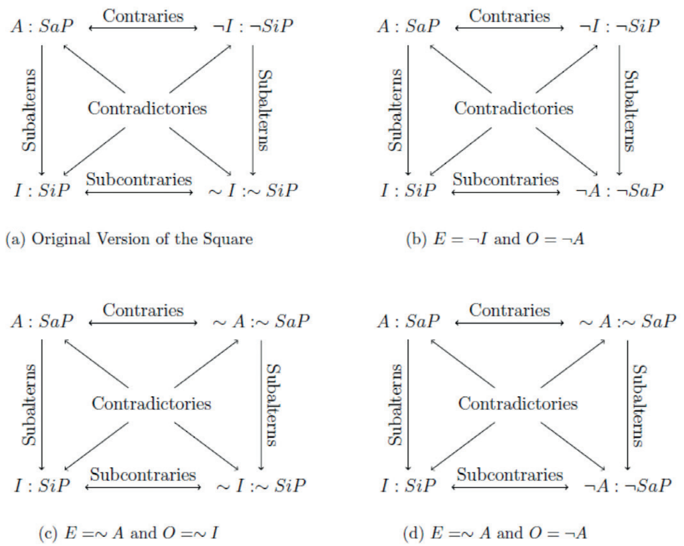

Figure 2. *Variations of the Square with respect to six Abelardian sentences*

As might be seen in Figure 2, with employing Quan Neg, E-corner is divided into ∼*SaP* and ¬*SiP* and O-corner into ¬*SaP* and ∼*SiP*. However, Abelard's attitude toward Quan Neg is different from both modern and traditional logic. Firstly, he rejects the idea that internal negation changes the truth value of a statement:

> "Sic quoque in cathegoricis propositionibus ea tantum propria contradictio ac recte dividens cuilibet affirmationi videtur quae negatione[m] praeposita totam eius sententiam destruit." [33]

> "Thus also with respect to categorical propositions the only right and correctly dividing negation of an arbitrary affirmation seems to be that which destroys its entire meaning by putting the negation sign in front of it. (my translation)"

Thus, with respect to external and internal negations, he adopts the Stoics' understanding that external negation is the only one that inverts the truth value of the sentence in front of which it is put. The internal negation together with changing the quantity of the sentence does not suffice to change truth value. Thus, external negation is the only contradiction inducing negation. His attitude toward negations lets him reject Quan Neg:

33 De Rijk, *Petrus Abaelardus, Dialectica. First Complete Edition of the Parisian Manuscript,* 176

> "Unde et quae dicit: 'omnis homo non est albus' non eadem videtur cum ea: 'non omnis homo est albus' et quae proponit: 'quidam homo non est albus' non eadem es cum ea: 'non quidam homo est albus[...] Re enim hominis prorsus non existente neque ea vera est quae ait: 'omnis homo est homo' nec ea quae proponit: 'quidam homo non est homo'"[34]

> "Therefore the proposition saying "Every man is not white" doesn't seem to be the same as "Not every man is white"; and "Some man is not white" is not the same as "Not: Some man is white". [...] For if the state of affairs is such that men do not exist at all, then neither the proposition 'Every man is a man' is true nor 'Some man is not a man'. (my translation)"

Since from the previous quote we know that *SaP* and ¬*SaP* are contradictories, we understand that *SaP* and ~*SiP* cannot be contradictories. Thus, we can infer easily that ¬*SaP* does not imply ~*SiP* and that he explicitly rejects Quan Neg here and he further provides the relation that *SaP* and ~*SiP* can be both false, meaning that they are contraries. In this system, thus, what carries existence is not the copula but the word "Omni [Every]":

> "Cum auten Quidam homo non est homo semper falsa sit atque Omnis homo est homo homine non existente, patet simul easdem falsas esse: unde nec recte dividentes dici poterunt."[35]

> "But since "Some man is not a man" is always false, if men do not exist, and equally also "Every man is a man", both propositions evidently are false together, so that they cannot be said to be properly dividing. (my translation)"

Various interpretations suggest the same conclusion as well: "We must therefore suppose that in his [Abelard's] view it is the word Omnis [Every], which introduces existential import"[36] Horn agrees that "omnis" involves existence.[37]

34 Ibid, 176.
35 Ibid, 176
36 Kneale, et all., *The Development of Logic*, 211
37 Horn, *A natural history of negation,* 26.

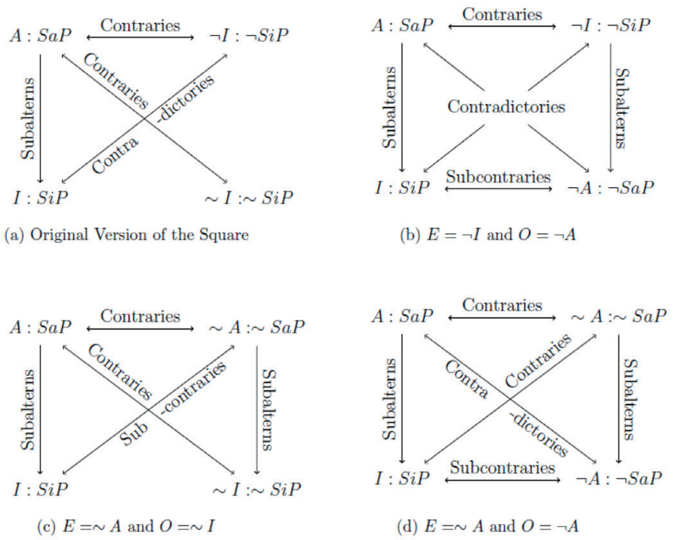

Figure 3. *Abelardian-Seurenian Squares of Opposition*

Given that the word "Omnis" carries existential import unless it is externally negated, contradictory relations between "*SaP* and ¬*SaP*" and "*SiP* and ¬*SiP*", Subalternation relations between "*SaP* and *SiP*", "¬*SaP* and ¬*SiP*" and "∼*SaP* and ∼*SiP*" together with the contrary relations just given in the quotation above reveal the relations in the Figure (3)[38].

Seuren's way to represent those entailments is to use an octagon of opposition. However, its readability is quite low and it is highly demanding to compare his octagon of opposition with the traditional squares in Figure 2. Thus, sacrificing the representations of some of the entailments, four squares corresponding to those in Figure 2 shall serve better for the present purpose (see Figure 3). In Abelardian-Seurenian system Quan Neg is only partly rejected. While the entailments that ∼*SiP* → ¬*SaP* and that ∼*SaP* → ¬*SiP* are preserved, the other entailments that ¬*SaP* → ∼ *SiP* and that ¬*SiP* → ∼ *SaP* are given up.

The system explained here and what Seuren[39] proposes is equivalent to each other. What Seuren[40] contributes to the system is to provide the truth conditions of quantifiers in terms of class inclusion as follows:

"SOME: [[S]] ∩ [[P]] = ∅

38 Seuren in "The logic of thinking", calls this system "Aristotelian-Abelardian" since he thinks that this is Abelard's interpretation of Aristotelian. However, whether either Aristotle or Abelard constructs the system as exactly as explicated here is controversial. Thus both to be on the secure side and to credit Seuren (since he also proposes the same system in "The logic of thinking", independently of Abelard), I shall call it "Abelardian-Seurenian".

39 Seuren, "The logic of thinking".

40 Seuren, *The Logic of Language: Language From Within*, volume 2.

NO [Not some]: [[S]] ∩ [[P]] = Ø

ALL [Every] : [[S]] ⊆ [[P]] and [[S]] = Ø”[41]

Thus, “what Abelard proposed was that, for cases where [[F]] = Ø[42], A- and A* [~A]- type sentences, as well as I- and I* [~I]- sentences, should be considered false, while their negations should be true”[43] since the external negation “¬” is the only truth value inverting negation. These statements must be formalized in modern notation as follows:

A:	$A_{imp!:}$	∃x(Sx) ∧ ∀x(Sx → Px)	¬I:	$E_{imp?:}$	∀x(Sx → ¬Px)
¬A:	$O_{imp?}$	¬∃x(Sx) ∨ ∃x(Sx ∧ ¬ Px)	~I:	$O_{imp!:}$	∃x(Sx ∧ ¬Px)
~A:	$E_{imp!:}$	∃x(Sx) ∧ ∀x(Sx → ¬Px)	I:	$I_{imp!:}$	∃x(Sx ∧ Px)

With these formalization of propositions in modern notation, Figure 3(b) becomes the one in Figure 4, where, as can be clearly seen now, the E- and the O-corner do not imply the existence of their subject terms.

Although Seuren drastically rejects this, some logicians[44] claim that for Abelard, affirmatives carry existential import and negatives do not (this idea will be explored in next section) and wed him to a traditional view of **O**-Corner Interpretation. However, this is now understandable with the help of modern notation and of representing the entailments with four squares, instead of one octagon, because in Figure 3(b) and Figure 4, it can be clearly seen the the **E**- and **O**-corners don’t carry existential import, while **A**- and **I**-corners do and Figure 3(b) is the only square where all the oppositions defined by Aristotle remain intact.

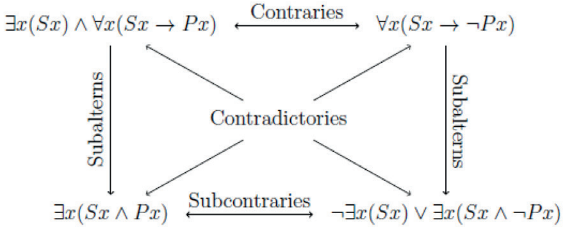

Figure 4. *formalized version of Figure 3(b)*

In rendering Abelard to the view of **O**-corner interpretation, Horn depicts the corresponding square of opposition exactly as the one in Figure 4[45] and in rejecting that Abelard advocates this

41 Ibid., 144-145.
42 “F” is the subject term of the proposition here.
43 Ibid., 174.
44 Allan Bäck, *Aristotle’s theory of predication,* (Leiden: Brill, 2000), 209, and Horn, L. R., “All john’s children are as bald as the king of France: Existential import and the geometry of opposition.” *Chicago Linguistics Society* 33 (1997), 157.
45 Horn, “All john’s children are as bald as the king of France: Existential import and the geometry of opposition”, 157.

view, Seuren claims that for Abelard, **~A** and **~I** do carry existential import[46]. It is evident now, Horn discusses which proposition underlies **O**-corner, not which sentence. Thus, his claim should be understood as that the **O**-corner (of the square that keeps all the oppositions or assuming that Figure 4 is the Abelardian Square) should bear the proposition that $\neg\exists x(Sx) \lor \exists x(Sx \land \neg Px)$. He does not discuss in Horn[47] which sentences should bear this proposition, while Seuren discusses which sentences bear which proposition.

Thus, it seems that while claiming that affirmatives carry and negatives do not, what these logicians have in their minds is that the Abelardian Square of Opposition is the on Figure 3(b). It wouldn't be misleading to think that Abelard thinks tht the propositions in O- and E-corner do not carry existential import, if the square in Figure 3(b) is considered. Another point to support the idea that for Abelard, only affirmatives have existential import is that , **~A** and **~I** do not have to be taken as negatives, because these statements can easily be obverted into affirmative ones without changing the truth conditions or underlying proposition by the immadiate inference of Observation: "Some human is not white" and "Some human is non-white". All statements except ¬**A** and ¬**I** can be regarded as affirmatives. Thus, if one assumes that Figure 3(b) is *the* Abelardian square of opposition, claiming that for Abelard negatives do no carry existential import while affirmatives do is not to be regarded as misleading.

This particular arrangement of existential import and rejecting *Quan Neg* in favor of one way entailments helps to solve the problem and to save the relation of *Subalternation*. Let us check if any inconsistencies can be derived, depending on the counter-examples given in the previous chapter:

1) In Abelardian-Seurenian system, one cannot derive from the truth of "Every chimera is monster" to the truth of "Some chimera is monster", since "Every chimera is monster" is already false, because SaP has existential import.

2) From the truth of " No logician has proved Goldbach conjecture" to the truth of "There is at least one logician who proves the Goldbach conjecture" cannot be derived. Firstly, "No logician has proved Goldbach conjecture" is true, because $[[L]] \cap [[G]] = \emptyset$. "No one who has proved the Goldbach conjecture is logician" is also true, because $[[L]] \cap [[G]] = [[G]] \cap [[L]] = \emptyset$. However, at this point, the entailment from the truth of "No one who has proved the Goldbach conjecture is logician" to the truth of "There is at least one who has proved the Goldbach conjecture and is logician" is not valid, because the subaltern of the former proposition is " not every one who proved the Golbach conjecture is logician", not "Some who proved the Goldbach conjecture is not logician" and the former is formalized in modern notation as $\neg\exists x(Gx) \lor \exists x(Gx \land \neg Lx)$, from which one cannot derive $\exists(Gx)$.

3) "Every chimera is monster" is false in this system. Its contradictory, however, is not "Some chimera is not monster", but "Not every chimera is monster". Thus we cannot derive the truth

46 Seuren, *The Logic of Language: Language From Within volume 2*, 160.

47 Horn, "All john's children are as bald as the king of France: Existential import and the geometry of opposition."

of the former but that of the latter and the latter has no existential import, to the effect that declaring "Not every chimera is monster" as true does not yield inconsistency.

4) "Every being is existent" is true. By *Contraposition* 1, it becomes "Every non-existent is non-being", which is false. Similarly, "Every being is existent" be- comes "Some non-existent is non-being" by *Inversion* 1, which is also false. Thus, in Abelardian-Seurenian system, *Contraposition* and *Inversion* must be given up for consistency.

Consistency achieved. But at what cost? It has already been shown that contraposition and inversion must be given up to secure the consistency and obversion and conversion still holds. However, contraposition and inversion are not the only ones to be given up, but also some intuitiveness. For example, "Some unicorns are horse" and Some unicorns are not horse" are both false[48]. The other counter-intuitive examples come in when we consider what rejecting *Quan Neg* amounts to in natural languages. While "Not every chimera is monster" is true, "Some chimera is not monster" is false. Moreover, the only state of the world where ¬*SaP* and ~*SiP* have different truth value is when [[S]] = Ø. Under any condition, under any state of the world, ¬*SaP* and ~*SiP* have the same truth value. So is the case for ~ SaP and ¬SiP. Rejecting Quan Neg seems, thus, an ad hoc solution designed for a particular situation, namely [[S]] = Ø. It makes no use for anything other than this particular problem.

Nevertheless, for any logical system purporting to account for logical inferences, being counter-intuitive and having less logical tools must be favored over being incon- sistent. While traditional logic seems inconsistent, Abelardian-Seurenian Logic casts off the inconsistencies resulting from employing empty term into the system at the cost of the immediate inferences of contraposition and inversion and yielding some counter-intuitive results.

1.2 Main Version of the O-Corner Interpretation

Klima claims that "Abelard's distinction did not really catch on, and gave way to stipulation that these two form of negation [external negation and internal negation plus changing the quantity] are equivalent and […] equally canceling its existential import"[49], that is, that logicians after Abelard do not reject *Quan Neg*. They reject the Stoics' and Abelard's understanding of negation that internal negation together with changing the quantity of the sentence does not establish the contradiction. For the proponents of this system, "Not every human is white" and "Some human is not white" are equivalent and both are the contradictories of "Every human is white" as the intuition suggests and the negations in the former two statements cancels the existential import. Thus, the quintessence of the resulting system is that negatives do not carry existential import, while affirmatives do. Its proponents adopt the motto that "existence goes with quality, not quantity", where "quality" refers to the affirmative-negative statements, and "quantity" to the universal-particular statements.

48 Note, however, that these sentences are also both false in the first order predicate logic.

49 Gyula Klima, "Consequence." In *The Cambridge Companion to Medieval Logic*, eds. by Catarina Dutilh Novaes and Stephen Read. (Cambridge: Cambridge University Press, 2016), 329.

Although Horn[50] traces the history of this kind of solution back to Apuleius in the 2nd century, the 14th century logician William of Ockham is generally thought be the first to propose it, though he does not explicitly state the problem. In Summa Logicae II.3, he seems to deal with the problem;

> "it is sufficient for the truth of such a proposition [a particular proposition] that the subject and predicate supposit for some same thing if the proposition is affirmative and a universal sign is not added to the predicate [...]. On the other hand, if such a proposition is negative, then it is required that the subject and predicate not supposit for all the same things. In fact, it is required either that the subject supposit for nothing or that it supposit for something for which the predicate does not supposit. [...] Thus, if there are no men and if there are no animals except for a donkey, then this con- sequence is not valid : 'A man is not a donkey; therefore some animal is not a donkey'"[51]

Similar passages that serve the same purpose can also be found in some other late medieval logicians in 14th century, such as Buridan and Burlay[52] and a group of medieval logicians[53]. According to Ashworth[54], this approach is lost after the 16th century. In the 19th century, Keynes[55] doesn't even mention this particular solution while exploring the possible ways out of this problem. Only at the beginning of the 20th century logicians, for example Caroll[56]and Johnson[57], have begun expressing this idea again, however, it became fashionable only after 1950s with Moody[58] and Thomson[59]. More recently, this idea was advocated by Wedin[60], Klima[61], Parsons [62] and Read[63].

The present system leads us back to the Figure 2, where *Quan Neg* is accepted and all the oppositions remain intact. The only difference is that the **E** and **O**-corners do not carry existential import any more since they are canceled by negation. When [[F]] ≠ Ø, **A** and **I** are false and **E**

50 Horn, "All john's children are as bald as the king of France: Existential import and the geometry of opposition."

51 William of Ockham, W. *Ockham's Theory of Propositions: Part 2 of the Summa Logicae,* Trans. by Alfred J. Freddoso and Henry Schuurman, (Indiana: St.Augustine's Press, 1998), 92.

52 See: Stephan Read, "Aristotle and Lukasiewicz on existential import", *Journal of the American Philosophical Association*, 1(3) (2015).

53 See: E. Jennifer Ashworth., "Existential assumptions in late medieval logic", *American Philosophical Quarterly*, 10(2) (1973) and Ernest A. Moody, *Truth and Consequence in Mediaeval Logic*. (Amsterdam: North-Holland Publishing Company, 1953).

54 Ashworth, "Existential assumptions in late medieval logic".

55 Keynes, *Studies and Exercises in Formal Logic.*

56 Lewis Caroll, *Symbolic Logic, Part I, Elemantary*. (New York: NY Dover Publications, (1958 [1896]).

57 William Ernest Johnson, *Logic, vol. 1.* (Cambridge: Cambridge University Press, 1921).

58 Moody, *Truth and Consequence in Mediaeval Logic.*

59 Thomson, "On Aristotle's square of oppositions".

60 Michael V. Wedin, "Negation and quantification in Aristotle", *History and Philosophy of Logic* 11(2) (1990).

61 Gyula Klima, "Existence and reference in medieval logic". In *New essays in free logic*, eds. Morscher, E. and Hieke, A. (Dortrecht: Springer, 2001) and Gyula Klima., *John Buridan.* (New York: Oxford University Press, 2008)

62 Terence Parsons, "Things that are right with the traditional square of oppositions", *Logica Universalis*, 2(1) (2008) and Terence Parsons, *Articulating medieval logic* (Oxford: Oxford University Press, 2014).

63 Read, "Aristotle and Lukasiewicz on existential import".

and **O** are true. Thus, since *Quan Neg* is accepted in this view, the proposition that Abelard reserved for ¬*SaP* is reserved for both ∼*SiP* and ¬*SaP* and also both ¬*SiP* and ∼*SaP* carries the proposition that Abelardian reserved only for ¬*SiP*. Thus modern formalization of the sentences in this view should be as follows:

A-Corner:	$A_{imp! =}$	$\exists x(Sx) \wedge \forall x(Sx \rightarrow Px)$
E-Corner:	$E_{imp? =}$	$\forall x(Sx \rightarrow \neg Px)$ $(= \sim A = \neg I)$
I-Corner:	$I_{imp! =}$	$\exists x(Sx \wedge Px)$
O-Corner:	$O_{imp? =}$	$\neg\exists x(Sx) \vee \exists x(Sx \wedge \neg Px)$ $(= \sim I = \neg A)$

These propositions result exactly in the square in Figure 4. With these formalizations, let us try to derive some inconsistencies using empty terms within the system.

1) Since "Every chimera is monster" is false, the truth of its subaltern "Some chimera is monster", which implies the existence of at least one chimera, cannot be derived.

2) "No logician has proved the Goldbach conjecture" is true. Its subaltern "Someone logician has not proved the Goldbach conjecture" is also true. The truth of the **O** statement that "Someone who has proved the Goldbach conjecture is not logician" is implied by *Conversion* 2. However, while in modern logic this statement implies the existence of at least one thing that has proved the Goldbach conjecture, in this system, the **O**-corner does not carry existential import and does not imply the existence of its subject term.

3) Since the contradictory of an existential import carrying **A** statement does not imply the existence of its subject term, the inference from the falsity of "Every chimera is monster", the truth of "there is at least one chimera" is not a valid inference.

4) "Every human is being" is true. By *Contraposition* 1, "Every non-being is non-human" must be true, but it is false, for *SaP* has existential import. Thus, in this system while *Contraposition* 1 must be given up to secure consistency, *Contraposition* 2 still holds.

Other than *Contrapositio*n 1, both of *Inversions* and *Obversion* 2 and 4 must also be given up. For example, "Some chimera is not monster" is true, for ∼*SiP* is true when [[C]] = Ø. By Obversion 4 it implies that "Some chimera is non-monster", which is false, because it is an affirmative and affirmatives imply the existence of their subject term. Lastly, consider the true *SaP* statement that "Every being is existent". *Inversion* 1 implies that "Every non-being is non-existent", which is false. Ashworth[64] and Parsons[65] claims that these rules are already explicitly rejected by medieval logicians.

The price to be paid for the consistency is to give up the all the immediate inferences except for *Conversions*, *Contraposition* 2 and *Obversion* 1 and 3. Counter-intuitiveness strikes, however, when one considers what rejecting *Obversion* 2 and 4 amounts to. "Some man is not just" is not

64 E. Jennifer Ashworth, *Language and Logic in the Post-medieval Period*, (Dortrecht: Reidel Publishing, 1974), 199.
65 Parsons, "The traditional square of opposition".

logically equivalent to "Some man is unjust". Seuren's[66] example reveals the oddity more clearly: while "Some mermaids are not married" is true, "Some mermaids are unmarried" is false. Moreover, intuition suggests that "Some chimera is not chimera" must be false since it is self-contradictory. However, in **O**-Corner Interpretation, it is true.

1.3 Problem of Providing a Unified Semantics:

At this point, a brief detour must be taken to illustrate the challenge proponents of this view face when aiming to provide a uniform semantics for its quantifiers whose meanings apparently depends on their relations to negation in a sentence. In this system, obversion must be given up for while "Some chimera is not monster" is true, "Some chimera is non-monster" must be false because it is affirmative and has existential import. This creates a problem for a logical system. The quantifiers "Every" and "Some" cannot easily have a unified semantics because the meaning of a sentence depend on how the quantifiers and "Not" are related. Seuren rightly appreciates if "one takes [*SaP*] to be true in case $[[F]] \neq \emptyset$ and $[[F]] \cap [[G]] = \emptyset$, then [~*SaP*] must be taken to be true just in case $[[F]] \neq \emptyset$ and $[[F]] \cap [[G]] = \emptyset$, which gives both [*SaP*] and [~*SaP*] existential import"[67] Parsons seems to agree that "it is apparent that the particular quantifier **'Some'** and the negation sign **'not'** now have independent meanings, and that the truth conditions for particular negative propositions are determined by how these meanings interact with one another"[68] The first attempt to give a uniform and systematic account is provided by Klima[69] and, later on, in Klima[70].

Aside from the technical and formal details, the core of the idea is to use the devices called 'restricted variables' and 'zero-entity'. Restricted variables are "variables, which take their values not from the whole universe of discourse, but from the extension of an open sentence"[71]. For instance, the variable '*x.Hx*' takes its values from the extension of the open sentence '*Hx*'. Let '*H*' stand for "human", and '*W*' stand for "white", then *W(x.Hx)* ranges not over the whole universe, but over the extension of "human". If the extension of "human" is empty, then it takes, as its value, the zero-entity, which is not in the universe of discourse, to the effect that it makes the sentence false. Additionally, if '*H*' stands for "human", then [~*H*] stands for "non-human", meaning that [~*H*] stands for anything else in the universe except for humans.

Let us, now, check if the difference between the scopes of negations in the sentences "Some chimera is not monster" and "Some chimera is non-monster" is really appreciated in this semantics or not. The former should be formalized as $(\exists x.Cx) \sim (M(x.Cx)$, while the latter as $(\exists x.C x)([\sim M](x.C x))$. Since in $(\exists x.C x) \sim (M (x.C x)$, Cx is empty, $(M(x.Cx))$ takes zero-entity as its value

66 Pieter A. Seuren, "Does a leaking o-corner save the square?" in *Around and beyond the square of opposition*, eds. Béziau, J.Y., and Jacquette, D., (Basel: Springer,2012), 132.

67 Seuren *The Logic of Language: Language From Within*, 169.

68 Parsons, "Things that are right with the traditional square of oppositions", 6.

69 Gyula Klima, *Ars artium: essays in philosophical semantics, mediaeval and modern*, (Budepest: Instute of Philosopher, Hungarian Academy of Sciences, 1988)

70 Klima,, *John Buridan*

71 Klima, *Ars artium: essays in philosophical semantics, mediaeval and modern,* 12.

and it is false. If *(M(x.Cx))* is false, then ~ *(M(x.Cx))* is true. *∃x.Cx* requires that *~(M(x.Cx))* must be true at least for one substitution, which makes the whole statement *(∃x.Cx) ~ (M(x. Cx))* true. In the latter sentence, *(∃x.Cx)([~ M](x.Cx)),* since *Cx* is empty, *([~M](x.Cx))* takes the value of zero-entity and is false. Thus, *(∃x.Cx)([~M](x.Cx))* must be false because existential quantifier requires that there must be at least one substitution that makes *([~M](x.Cx))* true. Similar symbolization and proofs can also be given for the universals. Thus, Klima concludes that "it provides us with a uniform, systematic account of relative scope relations of negation an all sorts of determiners in categorical propositions."[72] Klima also believes that this semantics gives a proper account of why "Some chimera is not chimera" is true, while it seems self-contradictory. The essence is that "Some chimera is non-chimera" is the self- contradictory one, not "Some chimera is not chimera". While the former is symbolized as *(∃x.Cx)([~C](x.Cx)),* the latter *(∃x. Cx)~(C(x.Cx)).* However, although in the formalism he provides, the difference of scopes of negations is promising and suggestive, whether any formalism is able to correct the natural intuition still remains an open question awaiting to be answered.

In similar vain, but in much more complicated formalism Parsons[73] provides another semantics, which also boils down to one with restricted variables given by Klima, if his strained formalism is modified. His semantics is given in terms of truth values according to the assignments to variable. A sentence is σ-true, when the assignment σ assigns things to the variables. In Parsons, every categorical statement has the form of "x=y". Both variables 'x' and 'y' must be bound by either (Some) or (Every) or (No). For each quantifier, he provides two cases where the sentence is true; one when its subject terms is empty and one when its subject term is not empty. The crucial difference is that while in Klima's semantic, when the subject term is empty, zero-entity is assigned to the variable, in Parsons if the subject term is empty, the assignment σ assigns nothing at all to the variable and he claims "if σ assings nothing to 'x' or to 'y', 'x=y' is not σ –true."[74] However, it seems that the contrast between "nothing at all" and "zero-entity" does not make any difference for Parsons, since he claims that "this method of handling common [general] terms so as to get the right truth conditions with respect to existential import is equivalent to a method first proposed (so far as I know) in Klima (1988)"[75]

Seuren[76] raises critiques for both semantics. The focus of the problem is 'zero- entity'. He poses a paradox similar to Russell's paradox of naive set theory. He defines a predicate "is zero-entity". Then, "∅ is zero-entity" must be false since when ∅ is assigned to variable, it delivers falsity. Moreover, he asks whether ∅ belongs to the extention of the predicate "is zero-entity". "If it does, why it produces falsity? If it does not, why is it an admissible substitution within the range of ["is zero-entity"]? "[77]. For Parsons' semantics, the situation is as complicated as his formalism, since he does not employ 'zero-entity' explicitly. If his "assigning nothing at all to a variable" is equal

72 Klima, *John Buridan,* 151.
73 Parsons, "Things that are right with the traditional square of oppositions" and Parsons, *Articulating medieval logic.*
74 Parsons, "Things that are right with the traditional square of oppositions", 7.
75 Parsons, *Articulating medieval logic*, 105.
76 Seuren, "*Does a leaking o-corner save the square?"*
77 Ibid, 135.

to "assigning zero-entity", then he must face the same criticism. However, if we take "assigning nothing" literally, then, consider the case where in 'x=y', an entity A is assigned to 'x' and nothing is assigned to 'y'. Then what results from this assignment is 'A=y'. This must be false according to Parsons. However, 'A=y' is not a proposition, nor even a sentence. It is just a sentential function sentential functions do not have a truth value.

The ontological nature of 'zero-entity' is not clear in Klima[78]. The only clarifications he provides about it are that "the only requirement concerning [∅] is not an element of the universe of discourse of that model"[79] and that "a term has the zero-entity as its value means no more nor less, than that the term refers to nothing."[80] However, Seuren seems to be right in that 'A=y' is a sentential function and cannot have a truth value and thus, he concludes that only "with the introduction of the ontologically vicious element ∅, uniform definitions become possible. **LOCA [O**-Corner Interpretation] is, therefore, spoiled by its ontology."[81]

Thus, while one must accept the ontological oddity of zero-entity, it seems to an impartial reader that both semantics save the traditional square of opposition with all its oppositions. However, Seuren renders it "logically and ontologically vicious." [82]

2.Abelardian-Seurenian Logic vs O-Corner Interpretation

Historical adequacy and roots of both systems have already been lengthily discussed in the literature. Yet, the analysis of their logical merits somehow remained perfunctory. Let us now try to compare the systems.

The fact that *Obversion* 2 and 4 do not hold in **O**-Corner Interpretation gives rise to two more propositions; $Sa\overline{P}$, $Si\overline{P}$. Note that since Aberlardian-Seurenian system validates *Obversions,* in that system $\sim SaP$ and $Sa\overline{P}$ are equal, just as $\sim SiP$ and $Si\overline{P}$. According to the analysis in Read (2015), $Sa\overline{P}$ and $Si\overline{P}$ are affirmative propositions and thus carry existential import. These propositions can be formalized in our notation as follows:

$$Sa\overline{P}: \exists x(Sx) \land \forall x(Sx \to Px)$$

$$Si\overline{P}: \exists x(Sx \land \neg Px)$$

Table 1.
Comparison of the sentences and propositions in both systems.

	Abelardian-Seurenian	O-Corner Interpretation

78 Klima. *John Buridan.* and Klima, *Ars artium: essays in philosophical semantics, mediaeval and modern*
79 Klima, *John Buridan,* 149.
80 Klima, *Ars artium: essays in philosophical semantics, mediaeval and modern,* 28.
81 Seuren,, "Does a leaking o-corner save the square?", 136.
82 Pieter A. Seuren, *From Whorf to Montague: Explorations in the theory of language*. (Oxford: Oxford University Press, 2013), 277.

SaP	$\exists x(Sx) \land \forall x(Sx \rightarrow Px)$	$\boldsymbol{A}_{imp!}$	$\exists x(Sx) \land \forall x(Sx \rightarrow Px)$	$\boldsymbol{A}_{imp!}$
SiP	$\exists x(Sx \land Px)$	$\boldsymbol{I}_{imp!}$	$\exists x(Sx \land Px)$	$\boldsymbol{I}_{imp!}$
$\neg SaP$	$\neg\exists x(Sx) \lor \exists x(Sx \land \neg Px)$	$\boldsymbol{O}_{imp?}$	$\neg\exists x(Sx) \lor \exists x(Sx \land \neg Px)$	$\boldsymbol{O}_{imp?}$
$\neg SiP$	$\forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{E}_{imp?}$	$\forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{E}_{imp?}$
$\sim SaP$	$\exists x(Sx) \land \forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{E}_{imp!}$	$\forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{E}_{imp?}$
$\sim SiP$	$\exists x(Sx \land \neg Px)$	$\boldsymbol{O}_{imp!}$	$\neg\exists x(Sx) \lor \exists x(Sx \land \neg Px)$	$\boldsymbol{O}_{imp?}$
SaP	$\exists x(Sx) \land \forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{E}_{imp!}$	$\exists x(Sx) \land \forall x(Sx \rightarrow \neg Px)$	$\boldsymbol{A}_{imp!}$
SiP	$\exists x(Sx \land \neg Px)$	$\boldsymbol{O}_{imp!}$	$\exists x(Sx \land \neg Px)$	$\boldsymbol{I}_{imp!}$

These are, unsurprisingly, the propositions that are given to the sentence $\sim SaP$ and $\sim SiP$ respectively, in Aberlardian-Seurenian system. As yet, it should be clear that Abelardian-Seurenian system and **O**-Corner Interpretation have exactly the same set of propositions and these propositions have exactly the same relations to each other. This means that both systems have the same logical power. Table 1 offers a summary of propositions and sentences in both systems, which enables us to check all of them at a glance.

Thus, the antagonism between both systems does not depend on the question which corner carries which propositions (or which meaning), but on the question which sentence should bear which propositions. Since this is the case, we do not have to discuss the entailments among the propositions, in order to be able to favor one system over the other and the only point to discuss is how these sentences should be formalized. This provides us with the luxury of comparing these logical systems with modern classical logic and of appealing to the intuition in natural languages, which would be meaningless, if their logical power were unequal. However, neither natural languages nor classical predicate logic can help us in favoring one system over the other because what both suggest is different than the ones suggested by these systems.

As the saying goes "for all they that take the sword shall perish with the sword". Ockham's razor shall be employed against his own theory: thus, considering the ontological oddity of zero-entity and the unnecessary complicatedness of semantics for the main version of O-corner interpretation, Aberlardian-Seurenian system shall be favored in saving the traditional square of opposition.

Conclusion

In the first section, the traditional square of opposition has been described in detail and its weakness are pointed out. In section 2.1, the Abelardian- Seurenian system is explicated and it has been shown that it is actually an O-corner interpretation since the square which manages to keep all the oppositions intact does not endow its negative statement with existential import. Later, the system originated from Ockham and generally regarded as the O-corner interpretations is analyzed and the difficulty of providing a unified semantics for the system is shown. Lastly in the last section, it is shown that both systems have the same logical power and Abelardian-Seurenian system shall be favored because of the problems in O-corner interpretation.

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