

Pointwise Bi-Slant Submersions

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Abstract

In the present paper we study pointwise bi-slant submersions from almost Hermitian manifolds onto Riemannian manifolds as a generalization of anti-invariant, semi-invariant, slant, bi-slant, pointwise semi-slant, pointwise slant, pointwise hemi-slant submersions. We mainly focus on pointwise bi-slant submersions from Kaehler manifolds onto Riemannian manifolds. We give some characterizations for such maps. We obtain necessary and sufficient conditions for the totally geodesicness of the distributions D_1 and D_2 mentioned in the definition of the pointwise bi-slant submersions. Also we investigate the geometry of vertical and horizontal distributions. Then we give necessary and sufficient conditions for pointwise bi-slant submersions to be totally geodesic.

Keywords: Almost Hermitian manifold, Pointwise bi-slant submersion.

1. Introduction

Given M and N C^∞ -Riemannian manifold of dimension m and n respectively. A surjective C^∞ -map $\pi: M \rightarrow N$ is a C^∞ -submersion if it has maximal rank at any point of M . According to the conditions on the map π , we have several types the following: Riemannian submersion [9, 13], slant and semi-slant submersions [10, 11, 14, 17], anti-invariant and semi-invariant Riemannian submersions [1, 15, 16], pointwise slant submersions [4, 12], hemi-slant submersions [2, 19], Lagrangian submersions [20], generic submersions [18] etc.

The study of Riemannian submersion between Riemannian manifolds was initiated by O'Neill [13] and Gray [9] in 1960s. Later such submersions were considered by Watson [22] under the name of almost Hermitian submersions between almost Hermitian manifolds. The notions of anti-invariant Riemannian submersions and semi-invariant submersions were introduced by B. Sahin. Also, Sahin investigated slant submersion from almost Hermitian manifold and obtained some characterizations. Pointwise slant submersions which extend slant submersion were studied by J.W. Lee and B. Sahin. They examined the relation between slant submanifolds and pointwise slant submanifolds and found some important results.

On the other hand, as a generalization of CR-submanifolds, slant and semi-slant submanifolds, bi-slant submanifolds in almost contact metric manifolds

defined by Cabrerizo et al. [6]. In Carriazo [7] studied bi-slant immersions in almost Hermitian manifolds. Uddin et al. [21] studied warped product bi-slant immersions in Kaehler manifolds and obtained some

results. Also Alqahtani et al. [3] investigate geometric properties warped product bi-slant submanifolds of cosymplectic manifolds and gave an example for warped product pseudo-slant submanifolds.

In purpose of the present paper is to examine pointwise bi-slant submersions from almost Hermitian manifolds onto Riemannian manifolds. In section 2, we review and collect some basic properties about almost Hermitian manifolds and Riemannian submersions. In section 3 we define pointwise bi-slant submersions and investigate the geometry of leaves of distributions and we a need and sufficient condition to be geodesic of such submersions.

2. Preliminaries

In this section we recall some information about almost Hermitian manifolds and Riemannian submersions. A $2n$ -dimensional smooth manifold M is called an almost Hermitian manifold with a tensor field J of type (1,1) and a Riemannian metric g such that

$$J^2 = -I \quad \text{and} \quad g(X, Y) = g(JX, JY).$$

An almost Hermitian manifold is said to be Kahler manifold if

$$(\nabla_X J)Y = 0, \quad \forall X, Y \in \Gamma(TM)$$

where ∇ is the Levi-Civita connection on M .

$$\nabla_{\xi}U = \mathcal{A}_{\xi}U + \mathcal{V}\nabla_{\xi}U \quad (2.7)$$

Let $\pi: M \rightarrow N$ be a C^{∞} -map between (M, g_M) and (N, g_N) C^{∞} -Riemannian manifolds of dimension m and n respectively. A C^{∞} -submersion $\pi: M \rightarrow N$ is called a Riemannian submersion if at each point p of M , π_{*p} preserves the length of the horizontal vectors [8].

$$\nabla_{\xi}\eta = \mathcal{H}\nabla_{\xi}\eta + \mathcal{A}_{\xi}\eta \quad (2.8)$$

For each $q \in N$, $\pi^{-1}(q)$ is an $(m - n)$ -dimensional submanifold of M called fiber. If a vector field on M is always tangent (or orthogonal) to fibers then it is said to be vertical (or horizontal) [13]. A vector field X on M is called basic if it is horizontal and $\pi_*X_p = X_*\pi(p)$ for $X_* \in \Gamma(TN)$ all $p \in M$. We will show the projection morphisms on the distributions $(\ker\pi_*)$ and $(\ker\pi_*)^{\perp}$ by \mathcal{V} and \mathcal{H} , respectively.

for $\xi, \eta \in \Gamma((\ker\pi_*)^{\perp})$ and $U, V \in \Gamma(\ker\pi_*)$, where $\bar{\nabla}_U V = \mathcal{V}\nabla_U V$. Moreover, if ξ is basic then $\mathcal{H}\nabla_U \xi = \mathcal{A}_{\xi}U$.

Let (M, g_M) and (N, g_N) be Riemannian manifolds and $\psi: M \rightarrow N$ is a smooth mapping between them. The second fundamental form of ψ is given by

$$\nabla\psi_*(X, Y) = \nabla_X^{\psi}\psi_*(Y) - \psi_*(\nabla_X^M Y) \quad (2.9)$$

for $X, Y \in \Gamma(TM)$, where ∇^{ψ} is the pullback connection. The smooth map ψ is said to be harmonic if $\text{trace}\nabla\psi_* = 0$ and ψ is called a totally geodesic map if $(\nabla\psi_*)(X, Y) = 0$ for $X, Y \in \Gamma(TM)$ [5].

A Riemannian submersion $\pi: M \rightarrow N$ determines two (1,2) tensor fields \mathcal{T} and \mathcal{A} on M . These tensor fields are called the fundamental tensor fields or the invariants of π . For $X, Y \in \Gamma(TM)$, these tensor fields can be given by the formulae

$$\mathcal{T}_X Y = \mathcal{H}\nabla_{\mathcal{V}X}\mathcal{V}Y + \mathcal{V}\nabla_{\mathcal{V}X}\mathcal{H}Y \quad (2.1)$$

$$\mathcal{A}_X Y = \mathcal{V}\nabla_{\mathcal{H}X}\mathcal{H}Y + \mathcal{H}\nabla_{\mathcal{H}X}\mathcal{V}Y \quad (2.2)$$

where ∇ is the Levi-Civita connection of (M, g_M) . On the other hand for $U, V \in \Gamma(\ker\pi_*)$ and $\xi, \eta \in \Gamma((\ker\pi_*)^{\perp})$ the tensor fields satisfy the following equations

$$\mathcal{T}_U V = \mathcal{T}_V U \quad (2.3)$$

$$\mathcal{A}_{\xi}\eta = -\mathcal{A}_{\eta}\xi = \frac{1}{2}\mathcal{V}[\xi, \eta]. \quad (2.4)$$

Note that a Riemannian submersion $\pi: M \rightarrow N$ has totally geodesic fibers if and only if \mathcal{T} vanishes identically.

We now remember the following lemma.

Lemma 2.1. ([13]) Let $\pi: M \rightarrow N$ be a Riemannian submersion between Riemannian manifolds and suppose that ξ and η are basic vector fields of M related to ξ_* and η_* on N . Then

- i. $g_M(\xi, \eta) = g_N(\xi_*, \eta_*) \circ \pi$,
 - ii. $[\xi, \eta]^{\mathcal{H}}$ is a basic vector field,
 - iii. $[V, \xi]$ is vertical for any vector field V of $(\ker\pi_*)$,
 - iv. $(\nabla_{\xi}^M \eta)^{\mathcal{H}}$ is the basic vector field i.e.
- $$\pi_* \left((\nabla_{\xi}^M \eta)^{\mathcal{H}} \right) = \nabla_{\xi_*}^N \eta_*$$

where ∇^M and ∇^N are the Levi-Civita connection on M and N , respectively.

Furthermore from (2.1) and (2.2) we have

$$\nabla_U V = \mathcal{T}_U V + \bar{\nabla}_U V \quad (2.5)$$

$$\nabla_U \xi = \mathcal{H}\nabla_U \xi + \mathcal{T}_U \xi \quad (2.6)$$

3. Pointwise Bi-slant submersions from Almost Hermitian Manifolds

Definition 3.1. Let (M, g_M, J) be an almost Hermitian manifold and (N, g_N) a Riemannian manifold. A Riemannian submersion $\pi: M \rightarrow N$ is called a pointwise bi-slant submersion if for $i = 1, 2$ the angles θ_i between JU_i and a pair of orthogonal distributions D_i , respectively, are independent of the choice of the nonzero vector $U_i \in \Gamma(\ker\pi_*)$ such that $\ker\pi_* = D_1 \oplus D_2$ and $J D_i \perp D_j$ for $j = 1, 2$. Then the angle θ_i is called the slant function of the pointwise slant submersion.

Let $\pi: (M, g_M, J) \rightarrow (N, g_N)$ be a pointwise bi-slant submersion. For $U \in \Gamma(\ker\pi_*)$, we can write

$$U = PU + QU \quad (3.1)$$

where $PU \in \Gamma(D_1)$ and $QU \in \Gamma(D_2)$.

Also, for $U \in \Gamma(\ker\pi_*)$, we obtain

$$JU = \phi U + \omega U \quad (3.2)$$

where $\phi U \in \Gamma(\ker\pi_*)$ and $\omega U \in \Gamma(\ker\pi_*)^{\perp}$.

For $\xi \in \Gamma(\ker\pi_*)^{\perp}$, we have

$$J\xi = B\xi + C\xi \quad (3.3)$$

where $B\xi \in \Gamma(\ker\pi_*)$ and $C\xi \in \Gamma(\ker\pi_*)^{\perp}$.

The horizontal distribution $(\ker\pi_*)^{\perp}$ is decomposed as

$$(\ker\pi_*)^{\perp} = \omega D_1 \oplus \omega D_2 \oplus \mu$$

where μ is the complementary distribution to $\omega D_1 \oplus \omega D_2$ in $(\ker\pi_*)^{\perp}$. Using (3.2) and (3.3) we arrive

$$\phi D_1 = D_1, \phi D_2 = D_2, B\omega D_1 = D_1, B\omega D_2 = D_2$$

Thus we can give the following result.

Theorem 3.1. Let π be a Riemannian submersion from an almost Hermitian manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) . Then π is a pointwise bi-

slant submersion if and only if there exist bi-slant function θ_i defined on D_i such that

$$\phi^2 = -(\cos^2\theta_i)I, \quad i = 1, 2.$$

Proof. The proof of this theorem is the same as slant submersions [17].

Theorem 3.2. Let π be a pointwise bi-slant submersion from Kaehlerian manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) with bi-slant functions θ_1, θ_2 . Then the distribution D_1 defines a totally geodesic foliation if and only if

$$\sin^2\theta_1 g_M([U, \xi], V) = \sin 2\theta_1 \xi [\theta_1] g_M(U, V) + g_M(\mathcal{A}_\xi \omega \phi U, V) - g_M(\mathcal{A}_\xi \omega U, \phi V) - g_M(\mathcal{H}\nabla_\xi \omega U, \omega V)$$

$$\text{and } g_M(\mathcal{T}_U \omega V, \phi W) + g_M(\mathcal{H}\nabla_U \omega V, \omega W) = 0$$

where $U, V \in D_1, W \in D_2$ and $\xi \in \Gamma((\ker\pi_*)^\perp)$.

Proof. For any $U, V \in D_1$ and $\xi \in D_2$ we can write

$$\begin{aligned} g_M(\nabla_U V, \xi) &= -g_M([U, \xi], V) - g_M(\nabla_\xi U, V) \\ &= -g_M([U, \xi], V) - g_M(\nabla_\xi \phi U, JV) \\ &\quad - g_M(\nabla_\xi \omega U, JV) \\ &= -g_M([U, \xi], V) + g_M(\nabla_\xi \phi^2 U, V) \\ &\quad + g_M(\nabla_\xi \omega \phi U, V) - g_M(\nabla_\xi \omega U, JV). \end{aligned}$$

From Theorem 3.1, the above equation takes the form

$$\begin{aligned} \sin^2\theta_1 g_M(\nabla_U V, \xi) &= -\sin^2\theta_1 g_M([U, \xi], V) \\ &\quad + g_M(\nabla_\xi \omega \phi U, V) - g_M(\nabla_\xi \omega U, JV) \\ &\quad + \sin^2\theta_1 X[\theta_1] g_M(U, V). \end{aligned}$$

Thus using (2.8) we have

$$\begin{aligned} \sin^2\theta_1 g_M(\nabla_U V, \xi) &= -\sin^2\theta_1 g_M([U, \xi], V) \\ &\quad + g_M(\mathcal{A}_\xi \omega \phi U, V) - g_M(\mathcal{A}_\xi \omega U, \phi V) \\ &\quad + \sin^2\theta_1 X[\theta_1] g_M(U, V) \\ &\quad - g_M(\mathcal{H}\nabla_\xi \omega U, \omega V). \end{aligned}$$

Furthermore, for $W \in \Gamma(D_2)$

$$\begin{aligned} g_M(\nabla_U V, W) &= g_M(\nabla_U \phi V, JW) + g_M(\nabla_U \omega V, JW) \\ &= -g_M(\nabla_U \phi^2 V, W) + g_M(\nabla_U \omega V, JW) \end{aligned}$$

From the equation (2.6) and Theorem 3.1 we arrive at

$$\begin{aligned} \sin^2\theta_1 g_M(\nabla_U V, W) &= g_M(\mathcal{T}_U \omega V, \phi W) \\ &\quad + g_M(\mathcal{H}\nabla_U \omega V, \omega W). \end{aligned}$$

Theorem 3.3. Let π be a pointwise bi-slant submersion from Kaehlerian manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) with bi-slant functions θ_1, θ_2 . Then the distribution D_2 defines a totally geodesic foliation if and only if

$$\begin{aligned} \sin^2\theta_2 g_M([W, \xi], Z) &= \sin 2\theta_2 \xi [\theta_2] g_M(W, Z) + \\ &\quad g_M(\mathcal{A}_\xi \omega \phi W, Z) - g_M(\mathcal{A}_\xi \omega W, \phi Z) - \\ &\quad g_M(\mathcal{H}\nabla_\xi \omega W, \omega Z) \end{aligned}$$

and

$$g_M(\mathcal{T}_W \omega Z, \phi U) + g_M(\mathcal{H}\nabla_W \omega Z, \omega U) = 0$$

where $U \in D_1, W, Z \in D_2$ and $\xi \in \Gamma((\ker\pi_*)^\perp)$.

Proof. By using similar method in Theorem 3.2 the proof of this theorem can be easily made.

Theorem 3.4. Let π be a pointwise bi-slant submersion from Kaehlerian manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) with bi-slant functions θ_1, θ_2 . Then the distribution $(\ker\pi_*)^\perp$ defines a totally geodesic foliation on M if and only if

$$\begin{aligned} g_M(\mathcal{H}\nabla_\xi \eta, \omega \phi U) &= (\cos^2\theta_1 - \cos^2\theta_2) g_M(\mathcal{A}_\xi \eta, QU) \\ &\quad + g_M(\omega \mathcal{A}_\xi \eta, \omega U) + g_M(C\mathcal{H}\nabla_\xi \eta, \omega U). \end{aligned}$$

where $\xi, \eta \in \Gamma(\ker\pi_*)^\perp$ and $U \in (\ker\pi_*)$.

Proof. Suppose that $\xi, \eta \in \Gamma(\ker\pi_*)^\perp$ and $U \in (\ker\pi_*)$. So we can write

$$\begin{aligned} g_M(\nabla_\xi \eta, U) &= -g_M(\nabla_\xi \eta, \phi^2 PU) + g_M(J\nabla_\xi \eta, \omega U) \\ &\quad - g_M(\nabla_\xi \eta, \omega \phi U) - g_M(\nabla_\xi \eta, \phi^2 QU). \end{aligned}$$

From Theorem 3.1 we obtain

$$\begin{aligned} g_M(\nabla_\xi \eta, U) &= \cos^2\theta_1 g_M(\nabla_\xi \eta, PU) - g_M(\nabla_\xi \eta, \omega \phi U) \\ &\quad + g_M(J\nabla_\xi \eta, \omega U) + \cos^2\theta_2 g_M(\nabla_\xi \eta, QU). \end{aligned}$$

Using the equations (2.8) and $PU = U - QU$ we have

$$\begin{aligned} \sin^2\theta_1 g_M(\nabla_\xi \eta, U) &= -g_M(\mathcal{H}\nabla_\xi \eta, \omega \phi U) + \\ &\quad g_M(\omega \mathcal{A}_\xi \eta, \omega U) + g_M(C\mathcal{H}\nabla_\xi \eta, \omega U) + (\cos^2\theta_2 - \\ &\quad \cos^2\theta_1) g_M(\mathcal{A}_\xi \eta, QU). \end{aligned}$$

Considering above equation we have the desired equation.

Theorem 3.5. Let π be a pointwise bi-slant submersion from Kaehler manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) with bi-slant functions θ_1, θ_2 . Then the distribution $(\ker\pi_*)$ defines a totally geodesic foliation on M if and only if

$$\begin{aligned} g_M(\mathcal{A}_\xi \omega U, \phi V) &= -\sin^2\theta_1 g_M([U, \xi], V) + \\ &\quad \sin 2\theta_1 \xi [\theta_1] g_M(PU, PV) - g_M(\mathcal{H}\nabla_\xi \omega U, \omega V) + \\ &\quad \sin 2\theta_2 \xi [\theta_2] g_M(QU, QV) - g_M(\mathcal{A}\nabla_\xi \omega \phi U, V) \\ &\quad + (\cos^2\theta_1 - \cos^2\theta_2) g_M(\mathcal{V}\nabla_\xi U, V) \end{aligned}$$

where $\xi \in \Gamma(\ker\pi_*)^\perp$ and $U, V \in (\ker\pi_*)$.

Proof. Given $\xi \in \Gamma(\ker\pi_*)^\perp$ and $U, V \in (\ker\pi_*)$. Then we derive

$$\begin{aligned} g_M(\nabla_U V, \xi) &= -g_M([U, \xi], V) - g_M(J\nabla_\xi U, JV) \\ &= -g_M([U, \xi], V) - g_M(\nabla_\xi \phi^2 PU, V) \\ &\quad - g_M(\nabla_\xi \phi^2 QU, V) + g_M(\nabla_\xi \omega \phi U, V) \\ &\quad - g_M(\nabla_\xi \omega U, JV). \end{aligned}$$

By using Theorem 3.1 we have

$$g_M(\nabla_U V, \xi) = \sin 2\theta_1 \xi [\theta_1] g_M(PU, PV)$$

$$\begin{aligned}
 & -g_M([U, \xi], V) - \cos^2 \theta_2 g_M(\nabla_U QV, \xi) \\
 & + g_M(\nabla_\xi \omega \phi U, V) - \cos^2 \theta_1 g_M(\nabla_\xi PV, \xi) \\
 & + \sin 2\theta_2 \xi[\theta_2] g_M(QU, QV) \\
 & - g_M(\nabla_\xi \omega U, JV)
 \end{aligned}$$

Making use of equations (2,7), (2.8) and $PU = U - QU$ we arrive

$$\begin{aligned}
 \sin^2 \theta_1 g_M(\nabla_U V, \xi) &= (\cos^2 \theta_1 - \cos^2 \theta_2) g(\mathcal{V} \nabla_\xi QU, V) \\
 & - \sin^2 \theta_1 g_M([U, \xi], V) - g_M(\mathcal{A}_\xi \omega U, \phi V) \\
 & + g_M(\mathcal{A}_\xi \omega \phi U, V) - g_M(\mathcal{H} \nabla_\xi \omega U, \omega V) \\
 & + (\sin 2\theta_1) \xi[\theta_1] g_M(PU, PV) \\
 & + (\sin 2\theta_2) \xi[\theta_2] g_M(QU, QV)
 \end{aligned}$$

Using above equation we obtain the desired equation.

Theorem 3.6. Let π be a pointwise bi-slant submersion from Kaehler manifold (M, g_M, J) onto a Riemannian manifold (N, g_N) with bi-slant functions θ_1, θ_2 . Then π is totally geodesic if and only if

$$\cos^2 \theta_1 \mathcal{T}_U PV + \cos^2 \theta_2 \mathcal{T}_U QV = C\mathcal{H} \nabla_U \omega V + \omega \mathcal{T}_U \omega V$$

and

$$\cos^2 \theta_1 \mathcal{A}_\xi PU + \cos^2 \theta_2 \mathcal{A}_\xi QU = -C\mathcal{H} \nabla_\xi \omega V - \omega \mathcal{A}_\xi \omega V$$

where $\xi \in \Gamma(\ker \pi_*)^\perp$ and $U, V \in (\ker \pi_*)$.

Proof. Firstly since π is a Riemannian submersion for $\xi, \eta \in \Gamma(\ker \pi_*)^\perp$ we have

$$(\nabla \pi_*)(\xi, \eta) = 0.$$

Therefore for $\xi, \eta \in \Gamma(\ker \pi_*)^\perp$ and $U, V \in (\ker \pi_*)$ it is enough to show that $(\nabla \pi_*)(U, V) = 0$ and $(\nabla \pi_*)(\xi, U) = 0$. So we can write

$$(\nabla \pi_*)(U, V) = -\pi_*(\nabla_U V).$$

Then using the equation (2.6) and (2.7), we obtain

$$\begin{aligned}
 (\nabla \pi_*)(U, V) &= -\pi_*(\nabla_U V) = -\pi_*(J \nabla_U \phi V + J \nabla_U \omega V) \\
 &= -\pi_*(\nabla_U \phi^2 PV + \nabla_U \phi^2 QV + \\
 & \quad J \nabla_U \omega V).
 \end{aligned}$$

From Theorem 3.1 we find

$$\begin{aligned}
 (\nabla \pi_*)(U, V) &= -\pi_*(-\cos^2 \theta_1 \nabla_U PV - \cos^2 \theta_2 \nabla_U QV \\
 & \quad + J \nabla_U \omega V) \\
 &= \pi_*(\cos^2 \theta_1 (\mathcal{T}_U PV + \bar{\nabla}_U PV) + \\
 & \quad \cos^2 \theta_2 (\mathcal{T}_U QV + \bar{\nabla}_U QV) - J \mathcal{T}_U \omega V - \\
 & \quad J \mathcal{H} \nabla_U \omega V).
 \end{aligned}$$

Therefore we arrive at the first equation of Theorem 3.6.

On the other hand, we have

$$(\nabla \pi_*)(\xi, U) = -\pi_*(\nabla_\xi U).$$

Then using the equation (2.7) and (2.8), we arrive

$$\begin{aligned}
 (\nabla \pi_*)(\xi, U) &= \pi_*(\cos^2 \theta_1 (\mathcal{A}_\xi PU + \mathcal{V} \nabla_\xi PU) \\
 & \quad + \cos^2 \theta_2 (\mathcal{A}_\xi QU + \mathcal{V} \nabla_\xi QU) - J \mathcal{A}_\xi \omega U \\
 & \quad - J \mathcal{H} \nabla_\xi \omega U).
 \end{aligned}$$

This concludes the proof.

Author's Contributions

S. Aykurt Sepet: Drafted and wrote the manuscript, performed the experiment and result analysis.

Ethics

There are no ethical issues after the publication of this manuscript.

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