Teknik Dergi, 2022 12027-12043, Paper 669

Critical Inclination of Failure Surface and Seismic Active Earth Thrust for a Broken Slope Backfill

Umit CALIK[1]

ABSTRACT

This paper presents an analytical approach, based on the limit-equilibrium state, to determine the critical inclination of failure surface and active earth thrust for a cohesive-frictional backfill with a broken slope and a surcharge load at a constant distance from a retaining wall under the seismic condition. A combined analysis using Mononobe-Okabe's and Culmann's methods within a trial and error procedure is performed in this study. The influences of several parameters such as wall height, surcharge magnitude, cohesion, and internal friction angle of the backfill, adhesion between the wall-backfill interface, tension cracks, horizontal and vertical seismic acceleration coefficients have been investigated on the critical inclination of failure surface and the active earth thrust. Additionally, the performance of the proposed approach is explored by geotechnical software (Geo5) and two available methods in the literature. All of the results and the detailed comparisons are given in tabular and graphical forms.

Keywords: Critical inclination of failure surface, Seismic active earth thrust, Cohesive-frictional broken backfill, Adhesion, Tension cracks, Trial and error procedure.

1. INTRODUCTION

In seismic regions, the static and seismic forces act on retaining walls and simultaneously lead to failure. So, they should be taken into account together during the design of retaining structures. Okabe [1] and Mononobe and Matsuo [2] developed a method, the so-called Mononobe-Okabe method, to evaluate lateral earth pressure for cohesionless soils under dynamic conditions. This method was a seismic version of Coulomb's static earth pressure analysis. The earthquake-induced forces on the failure mass are applied by the pseudo-static method using seismic acceleration coefficients in the Mononobe-Okabe method.

The researchers have presented several solutions using the Mononobe-Okabe method, based on the limit-equilibrium of the different failure planes. Düzgün and Bozdağ [3], Ghosh and Sharma [4], and Shao-jun et al. [5] extended the Mononobe-Okabe method to consider the effect of the dynamic behavior of earthquakes with a single critical soil wedge and a planar

Note:

- This paper has been received on February 5, 2020 and accepted for publication by the Editorial Board on January 18, 2021.
- Discussions on this paper will be accepted by September 30, 2022.

- https://doi.org/10.18400/tekderg.684834

1 Dept. of Civil Engineering, Faculty of Technology, Karadeniz Technical University, Trabzon, Turkey - calik@ktu.edu.tr - https://orcid.org/0000-0002-7321-1998

failure surface. In the recent past, the analytical formulations were given in explicit forms to calculate the seismic active earth pressure and the critical angle of the critical failure surface behind retaining walls. Das and Puri [6], Saran and Gupta [7] and Ghosh et al. [8] developed the expressions to cohesive-frictional (c, ϕ) backfills to calculate the static or dynamic active earth pressure. Shukla [9] and Shukla and Bathurst [10] presented the analytical formulations to estimate the dynamic active force and the critical inclination of the failure plane considering the effects of tension cracks, adhesion, and surcharge load. Ghosh and Sengupta [11], Iskander et al. [12], Shukla [13], Lin et al. [14], Zhou et al. [15], Tang and Chen [16], Gupta and Sawant [17], Gupta et al. [18] and Peng and Zhu [19] developed the mathematical expressions taking into account wall height, inclination of the back face of the wall, tension cracks, adhesion, cohesion, internal friction angle of the backfill and surcharge with an infinite sloping surface.

Caltabiano et al. ([20], [21], and [22]) studied the critical angle of failure surface considering the presence of a uniform surcharge at different distances from the retaining wall. Aminpour et al. [23] investigated the effects of intensity, width, height, and location of surcharge related to the edge of the slope as well as the soil properties. Hou and Shu [24] improved the existing trial wedge method to calculate the earth pressure regardless of the irregularity of surcharges at a constant distance from the wall. Arda and Çinicioğlu [25] studied the influences of mean grain size, particle shape, and surcharge on the geometry of the active failure surface.

There is a limited number of studies in the literature regarding the irregular shape of the backfill surface. Yazdani et al. [26] formulated their solution to estimate the seismic active earth pressure for the noncontinuous backfill geometries. Greco [27] derived a formula for the general backfill profile subjected to line and strip surcharges considering soil friction angle, soil unit weight, and friction angle between soil and wall. Afterward, Greco [28] extended his solution to take account of seismic forces using the Mononobe-Okabe method.

Kim et al. [29] solved the problem of determining the critical inclination of the failure plane by considering a trial and error method. However, Greco [30] criticized Kim et al. [29]'s solution for the limitation of the trial and error method. Lu and Yuan [31] constructed a new method using the trial and error method and Culmann's [32] graphical solution with consideration of the inclination of the back face of the wall, the cohesive and adhesive forces in the problem under the static condition.

In the published literature, the common concept for determining critical failure surface and seismic active earth pressure is to accept the soil backfill surface as horizontal or continuously sloping at an angle with the horizontal plane. However, in-situ conditions may generally prevent the construction of backfill like this form. That's why an attempt is made to solve the problem for a retaining wall supporting a broken backfill (in the case of broken terrain). Besides, the chosen specific geometry and the input parameters of the problem in this paper are more realistic and practical to use in the geotechnical analysis of retaining structures. Here, a combined framework which has consisted of Culmann [32]'s graphical solution and Mononobe-Okabe's method is also designed to estimate the critical inclination of failure surface and the active earth thrust by resorting to a trial and error procedure. A script with MATLAB programming language is written to overcome the complex and time-consuming repetitive steps in the calculations. The algorithm of the script is constructed to include all effects of wall height, surcharge magnitude and location, cohesion and internal friction angle of the backfill, horizontal and vertical seismic acceleration coefficients, adhesion between the wall-soil interface, and tension cracks for a cohesive-frictional backfill with the irregular surface profile. Briefly, this study strives to develop an analytical approach, to derive mathematical formulations, and to write its MATLAB code considering all of the mentioned geotechnical methods within the influences of soil, loads, and the surface geometry for the determination of the critical inclination of failure surface and the active earth thrust. Moreover, the proposed approach herein is verified by powerful geotechnical software (Geo5) and two available methods by Shukla [13] and Motta [33]. All of the results and the comparisons are presented in tabular and graphical forms.

2. METHOD OF ANALYSIS

In the development of the analytical approach, the following assumptions have been made:

(a) The backfill is homogeneous, dry, frictional, and cohesive. As known, there is always no access to a cohesionless backfill due to economic aspects. The cohesive-frictional backfills are commonly encountered in the real field applications. The deriving expressions for such a backfill are usable and practicable to estimate the critical inclination of failure surface and the active earth thrust.

(b) The failure surface in the backfill is assumed to be planar to simplify the calculations. The failure surface that develops in the backfill is a plane if δ=0. But the failure surface is curved for a rough wall ($\delta \neq 0$). However, the planar failure surface is still used in the practice of Coulomb's theory. It is known to provide acceptable results in the active state (Paik and Salgado [34]).

(c) The retaining wall yields away from the backfill. The displacement of the wall is sufficient enough to cause an active earth pressure condition.

(d) The uniform surcharge load that acts on the backfill at a distance from the wall is long enough to intersect the failure plane.

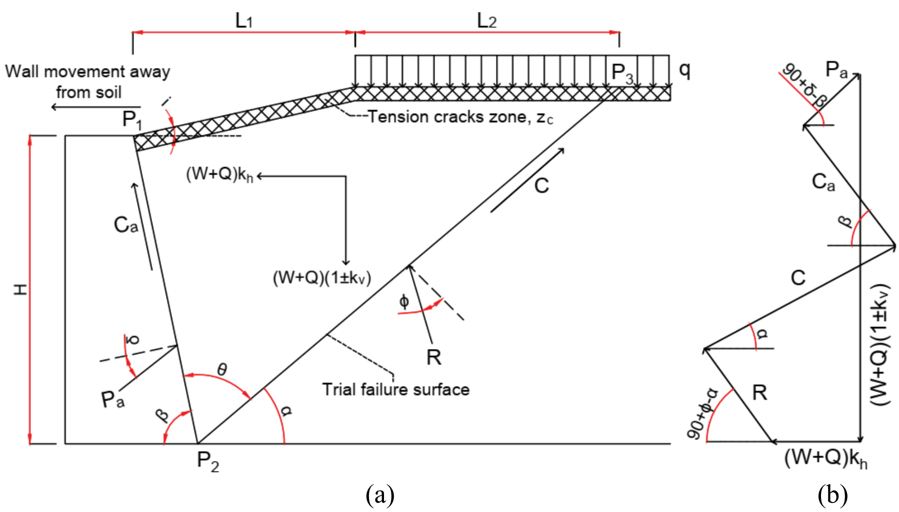

Figure 1 – Geometry of the trial failure wedge and its force polygon

Figure 1(a) shows a general schematic diagram of the cross-section geometry and the forces acting on a trial failure wedge behind the retaining wall for the cohesive-frictional backfill with the broken slope and the uniformly distributed surcharge load at the constant distance in the active state. Also, the force polygon of this trial failure wedge can be seen in Figure 1(b).

Shukla et al. [35] reported that the active earth pressure with tension cracks in the cohesive backfill could increase up to 20%-40% over the case without cracks. Therefore, the effect of tension cracks is taken into consideration. Nian and Han [36] calculated the depth (z_c) of tension cracks zone in seismic condition by using Equation (1). This equation takes the seismic inertial forces and the inclination of backfill into consideration while calculating z_c. Besides, it is assumed that the (unit) cohesion and the (unit) adhesion decrease linearly from their initial values to zero at the surface of the backfill in the tension cracks zone.

$$z_c = \frac{2H\left(\frac{c}{\gamma H}\right)}{\left[(1+\tan\psi\tan i)^2+\frac{4tan^2\psi}{cos^2\varphi}\right](1\mp k_v)cos\varphi}\left[sin\varphi(1+\tan\psi\tan i)+\sqrt{(1+\tan\psi\tan i)^2+4tan^2\psi}\,\right] \quad (1)$$

All of the calculated forces on the trial failure wedge in Figure 1(a) are R, C, C_a, W and Q. Equations (1) and (2) can be written considering the equilibrium of these forces in Figure 1(b) which is drawn in the horizontal [→ + and ← −] and the vertical [↑ + and ↓ -] directions of the force polygon, respectively:

$$P_a\cos(90-\beta+\delta)-C_a\cos\beta+C\cos\alpha-R\cos(90+\phi-\alpha)-(W+Q)k_h = 0 \quad (2)$$

$$P_a\sin(90-\beta+\delta)+C_a\sin\beta+C\sin\alpha+R\sin(90+\phi-\alpha)-(W+Q)(1\pm k_v) = 0 \quad (3)$$

From Equation (2), R is obtained as,

$$R = \frac{P_a\cos(90-\beta+\delta)-C_a\cos\beta+C\cos\alpha-(W+Q)k_h}{\cos(90+\phi-\alpha)} \quad (4)$$

Substituting the R from Equation (4) into Equation (3), the following equation can be derived for the active earth thrust (P_a):

$$P_a = \frac{(W+Q)(1\mp k_v+mk_h)+C_a(m\cos\beta-\sin\beta)-C(m\cos\alpha+\sin\alpha)}{m\cos(90-\beta+\delta)+\sin(90-\beta+\delta)} \quad (5)$$

where $m = \tan(90+\phi-\alpha)$. (6)

The trial failure wedge is taken as an example in Figure 1(a). It is assumed that a downward failure movement of this trial wedge in the backfill occurs along a surface P_2P_3. The failure surface passes through the heel of the wall. It is inclined at an angle (θ) to the back face of the retaining wall.

The magnitude of P_a is calculated from Equations (5) and (6) for the trial wedge. The calculation is repeated many times for the other potential failure surfaces to obtain the maximum magnitude of P_a with resorting to a trial and error procedure. The failure surface that gives the maximum value of P_a is the critical failure plane; the corresponding P_a and θ are the active earth thrust and the critical inclination of the failure surface (θ_{cr}) of the problem. An algorithm in MATLAB programming language is built and a script is coded to estimate P_a and θ_{cr} because of the several input parameters in the problem, the complex and repetitive steps in the calculations. The flowchart of the algorithm in the script is given in Figure 2.

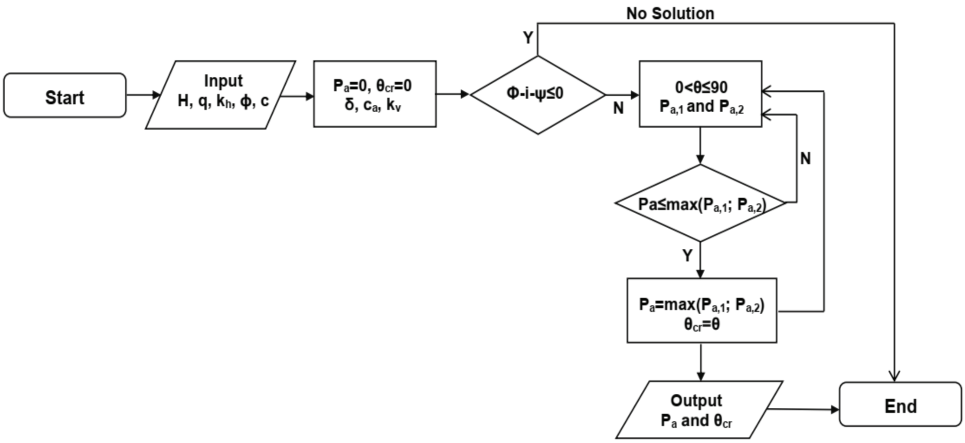

Figure 2 - Flowchart of the algorithm in the script

3. PARAMETRIC STUDY

The parametric study aims to explore the effects of the input parameters on the critical inclination of failure surface with the vertical plane and the active earth thrust in the static and seismic conditions. The details of the input parameters are shown in Table 1.

Table 1 - Details of the input parameters

Input parameter	Values
H (m)	6, 8 and 10
q (kN/m^2)	15, 30 and 45
c (kN/m^2)	0, 10 and 20
ϕ (°)	25, 30 and 35
k_h	0, 0.1 and 0.2

The three parameters in the study are correlated with the parameters in Table 1 (e.g., $\delta = (2/3)\phi$, $c_a = (2/3)c$ and $k_v = (2/3)k_h$). The four parameters are kept constant (e.g., $\gamma = 18$ kN/m^3, $i = 15°$, $\beta = 90°$ and $L_1 = 5$ m) during the problem-solving process for simplicity.

3.1. Effects of the Parameters on the Critical Inclination of Failure Surface

The first step of the parametric study is to determine the critical inclinations of the failure surface with the vertical plane. The coded MATLAB script is run through all of the analyses for θ_{cr}. The calculated values of θ_{cr} are shown in Figures 3, 4, and 5. Mononobe-Okabe's method has no result when $\phi - i - \psi \leq 0$. Therefore, the approach in this study does not apply to the problem at $\phi = 25°$ and $k_h = 0.2$ in Figure 5.

In the static condition, the critical inclination decreases with the increase in the values of wall height, cohesion, and internal friction angle. On the other hand, the surcharge load increases the critical inclination. In the seismic condition, the critical inclination has a decreasing tendency with increasing the wall height, the cohesion, and the internal friction angle. The critical inclination shows an increment due to the horizontal seismic acceleration coefficient and the surcharge load. There are the same trends with the parameters, such as the internal friction angle and the seismic acceleration coefficient, in the papers which are studied by Shukla et al. [35], Ghosh and Sengupta [11], and Ghosh and Sharma [37].

The shear strength parameters (c and ϕ) of the backfill soil lead to an increase in the resisting forces along the potential failure surface. Also, increasing the height of the wall geometrically occurs the longer surface P_2P_3 in Figure 1(a) where c and ϕ make the soil-wall system safer against the shear failure. As a result of this, the lower inclinations of failure surface with the vertical plane are more critical. In other words, the wall-soil system is vulnerable to failure at the smaller θ under both static and seismic loads.

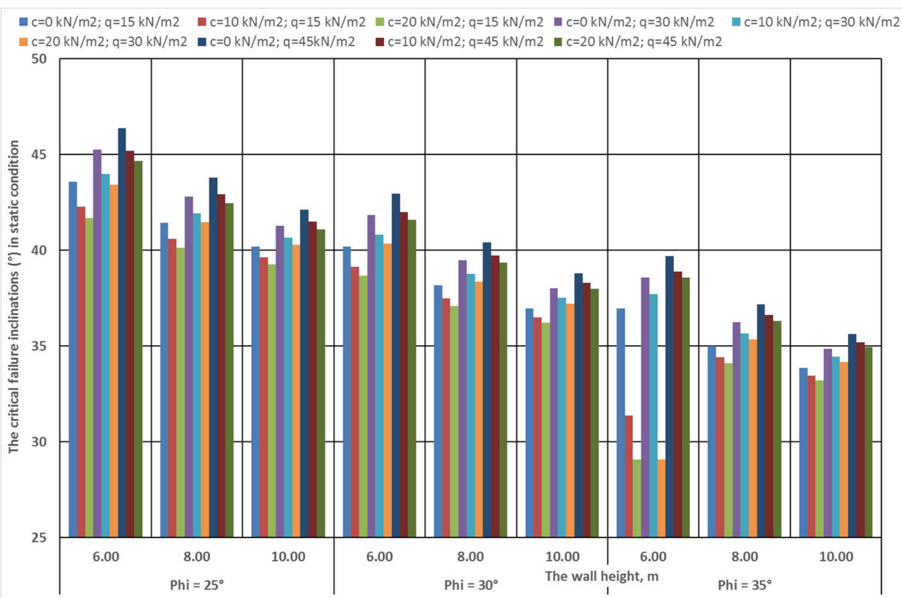

Figure 3 - The critical inclinations in the static condition

The parameters q and k_h contribute P_a to exceed the shear strength of the backfill soil after a certain cross-sectional area of the trial failure wedge. Thus, increasing the critical inclination provides a large enough area to increase the effects of q and k_h on the soil-wall system. As expected, the critical inclinations under the earthquake-induced loads are higher than those under the static loads. According to Figures 3, 4, and 5, the seismic loads increase the critical inclination up to 4-5 degrees each 0.1g increment of k_h.

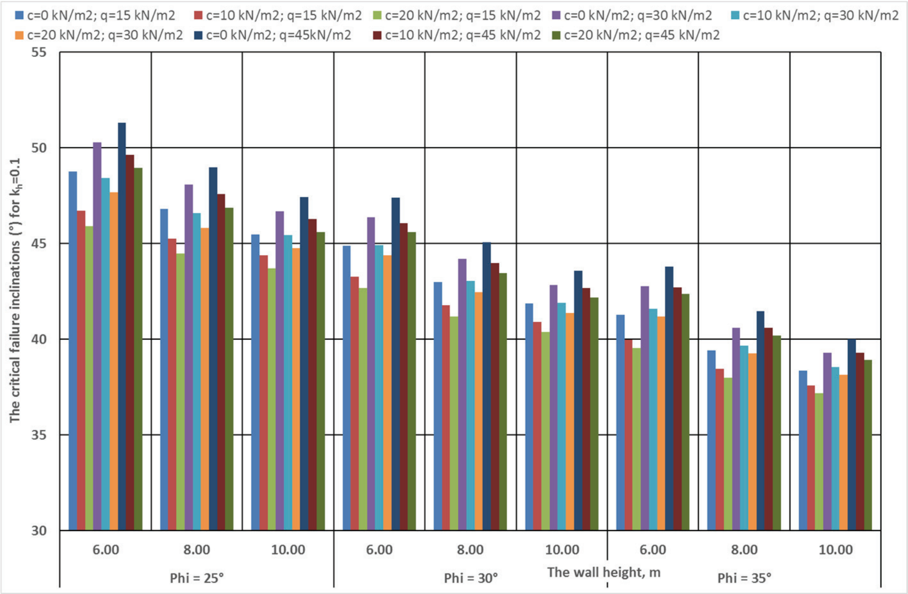

Figure 4 - The critical inclinations in the seismic condition, for k_h = 0.1

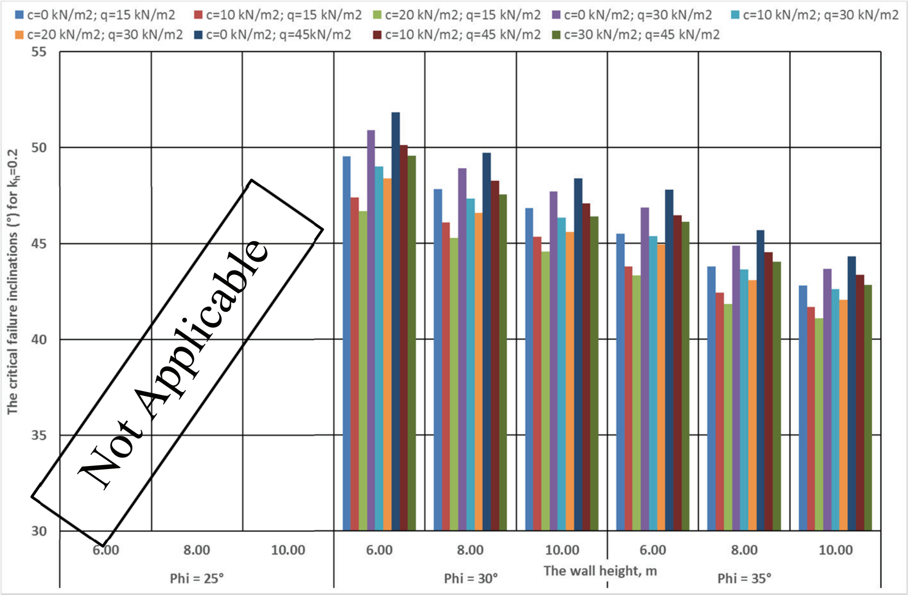

Figure 5 - The critical inclinations in the seismic condition, for k_h = 0.2

3.2. Effects of the Parameters on the Active Earth Thrusts

The second step of the parametric study is to estimate the active earth thrusts obtained from Equations (5) and (6) in the state of failure. The computed results of active earth thrust are listed in Figures 6, 7, and 8. At $\phi = 25°$ and $k_h = 0.2$, the proposed approach has still no solution in Figure 8 since $\phi - i - \psi \leq 0$.

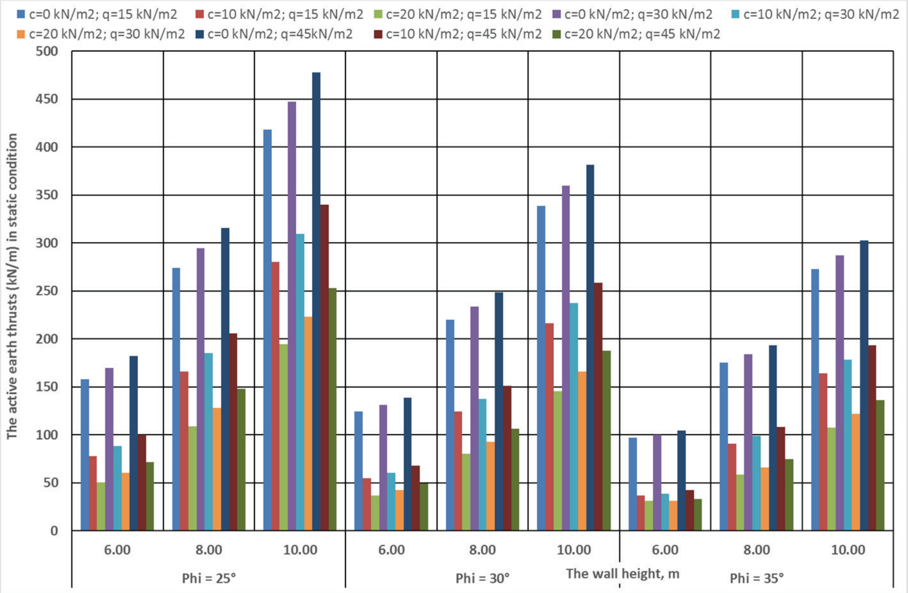

Figure 6 - The active earth thrusts in the static condition

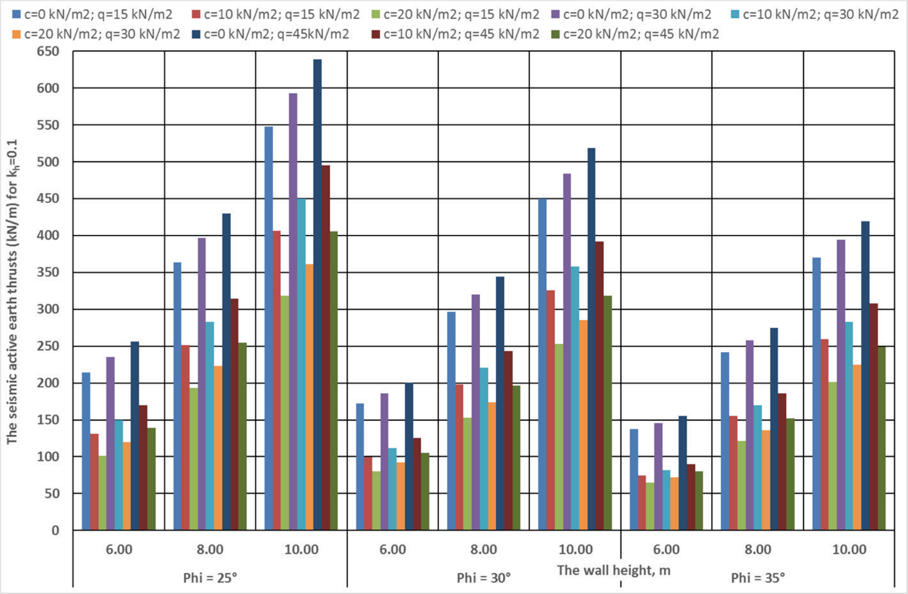

Figure 7 - The active earth thrusts in the seismic condition, for $k_h = 0.1$

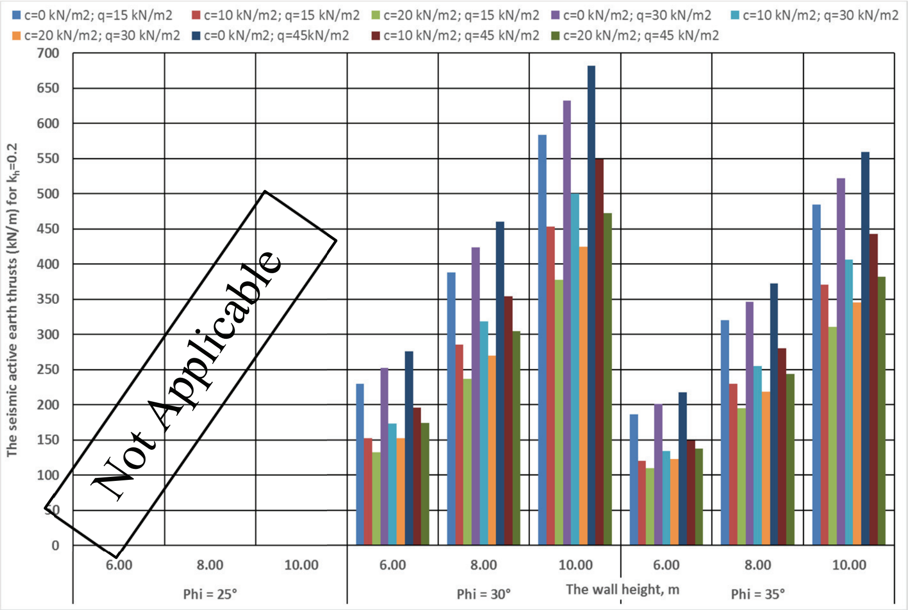

Figure 8 - The seismic active earth thrusts in the seismic condition, for k_h = 0.2

In the static condition, the active earth thrust shows a notable decreasing behavior when the cohesion and the internal friction angle of the backfill increase. On the contrary, the wall height and the surcharge load increase the active earth thrust. Herein, the wall height is more dominant to get a higher active earth thrust. In the seismic condition, the active earth thrust indicates a decreasing trend with the cohesion and the internal friction angle of the backfill, and an increasing trend with the height of the wall, the horizontal seismic acceleration coefficient, and the surcharge load. The similar tendency of the seismic active earth thrusts is reported by Shukla et al. [35], Ghosh and Sengupta [11], and Ghosh and Sharma [37].

As shear strength parameters (c and ϕ) increase (i.e. the backfill soil becomes stronger), Coulomb's active pressure coefficient decreases. Therefore, these two parameters reduce the magnitude of the active earth thrust.

An increase in the height of the wall geometrically induces the higher values of the horizontal surcharge load distance (L_2) and the weight of the failure wedge (W). These new geometrical cases maximize the influences of static and seismic loads (via q and k_h) on the soil-wall system. Hence, the higher loads apply the additional lateral forces along the back of the retaining wall. In other words, the retaining wall is subjected to higher values of the active earth thrust. As seen in Figures 6, 7, and 8, the seismic forces considerably increase P_a up to 300% according to the values under the static condition. It is worthy to note here that H shows a more significant effect on P_a in comparison to the effects of q and k_h.

3.3. Comparison of the Results of Active Earth Thrusts

The performance of the proposed approach is deeply investigated by using geotechnical software (Geo5) and the two previously published studies for the cohesive-frictional backfills.

Firstly, the analysis in this study is verified by the geotechnical software Geo5. Geo5 does not explicitly indicate the critical inclination of the failure surface. That's why the study only compares its computed values of P_a to the results of Geo5. Figure 9 shows this comparison attempt for the case $k_h = 0.2$, $k_v = (2/3)k_h$, $\delta = (2/3)\phi$, $c_a = (2/3)c$, $\gamma = 18$ kN/m^3, $i = 15°$, $\beta = 90°$ and $L_1 = 5$ m.

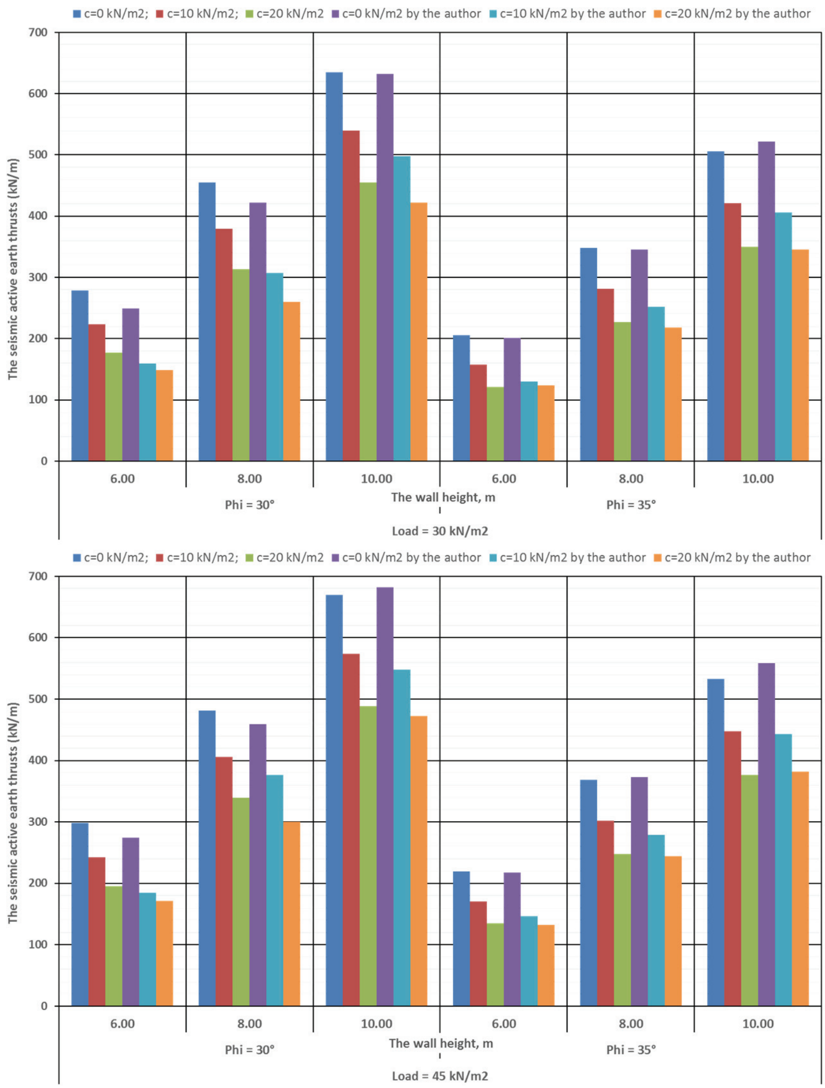

Figure 9 - The seismic active earth thrusts calculated by Geo5 and the study

As seen in Figure 9, the differences in P_a range from 1.1% to 28.6%. The results of the study mostly diverge from the values given by Geo5 at H = 6 m and c = 10 kN/m^2 for q = 30 kN/m^2 and 45 kN/m^2. Furthermore, the comparison of the values of P_a with Geo5 reveals that this study underestimates P_a. At this point, it can be stated the most important reason for the discrepancy in the values that the different computation approaches and equations are used by the study and Geo5 to consider the effects of the soil mass, tension cracks zone, and (static and seismic) loads. On the other hand, the results in Figure 9 depict a convergence with the increase in the values of input parameters.

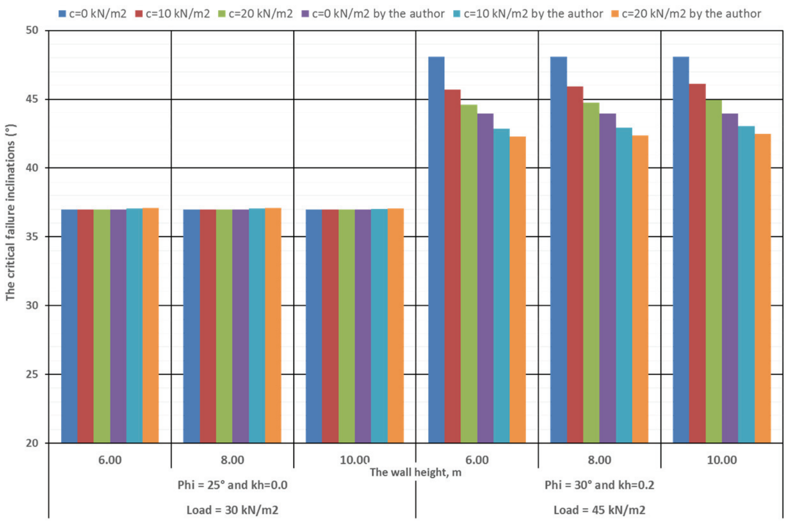

Figure 10 - The critical failure inclinations calculated by Shukla [13] and the study

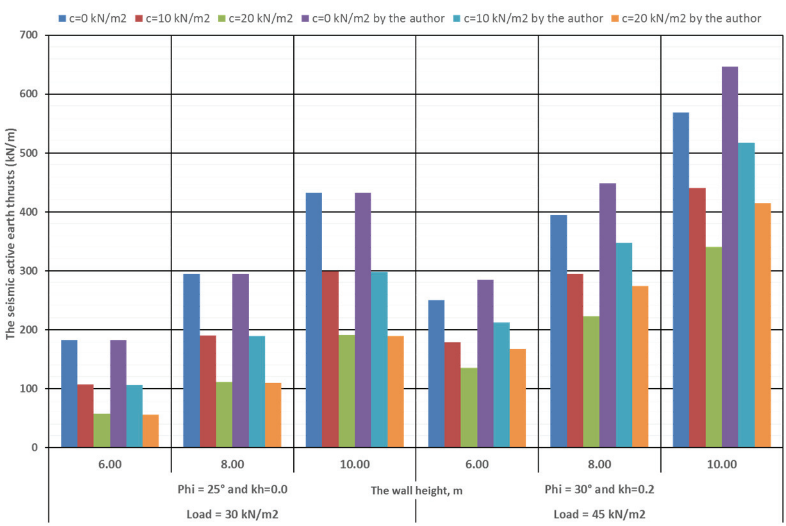

Figure 11 - The seismic active earth thrusts calculated by Shukla [13] and the study

Secondly, the study also performs another comparison of the seismic active earth thrusts and the critical failure inclinations with the results calculated by Shukla [13] for the case $k_v = (2/3)k_h$, $\delta = (2/3)\phi$, $c_a = (2/3)c$, $\gamma = 18$ kN/m^3, $\beta = 90°$. L_1 and i are equal to 0 to ignore the geometry differences between the two studies. The bar graphs of θ_{cr} and P_a are shown in Figures 10 and 11, respectively.

The two studies approximately give the same results of θ_{cr} and P_a for cohesionless and cohesive backfill soils in the static condition. The established minor difference between the values of θ_{cr} and P_a barely increases up to 1.8% with increasing cohesion. Hence, it could be concluded that θ_{cr} and P_a are calculated in quite a proximity in these two studies.

The differences in θ_{cr} and P_a given by these two studies are noticed in all cases of the soil-wall system subjected to the earthquake loads in addition to the static surcharge load. Under the seismic condition, the proposed approach in this study slightly underestimates the angles of θ_{cr} and significantly overestimates the values of P_a in comparison to Shukla [13].

The computed values of θ_{cr} by the two studies just show a few degrees of variation (max 4 degrees) in the case of cohesionless soils. However, the increasing cohesion decreases the difference in the values of θ_{cr} to 1 degree. Therefore, Shukla [13]'s study confirms the present θ_{cr} findings of this study.

A similar convergence does not exist when the values of P_a, calculated by this study, are compared with the results given by Shukla [13]. The proposed approach estimates higher values of P_a (from 13.7% to 23.7%) with increasing cohesion. Herein, the problem is solved by Geo5 once again to investigate the accuracy of the two studies at $k_h = 0.2$, $q = 45$ kN/m^2, $\phi = 30°$, and $c = 20$ kN/m^2 for H = 6 m, 8 m, and 10 m. The results of P_a calculated by Geo5, this study, and Shukla [13] are summarized in Table 2.

Table 2 - P_a (kN/m) calculated by Geo5, this study, and Shukla [13]

	The software Geo5	This study	Shukla [13]
H = 6 m	195.732	177.407	135.410
Normalized P_a by this study	**1.10**	**1.00**	**0.76**
H = 8 m	319.483	289.085	223.000
Normalized P_a by this study	**1.11**	**1.00**	**0.77**
H = 10 m	488.218	435.871	340.755
Normalized P_a by this study	**1.12**	**1.00**	**0.78**

In light of the results of P_a in Table 2, this study gives 23% more values than Shukla [13]'s results but 11% fewer values than the results of Geo5. The proposed approach in this study shows a better convergence to the values of the substantial commercial software Geo5 which is currently available to use for geotechnical engineers. It can be said from this viewpoint that this study is appropriate to estimate the values of P_a in geotechnics.

The two approaches, presented by this paper and Shukla [13], have not considered the total adhesive force between the wall and the backfill, the total cohesive force along the failure plane, the depth of tension crack zone, and the loads (especially seismic forces) in a similar way throughout the process of developing their expressions and the calculations. Moreover, Shukla [13] presented his generalized expression in terms of dynamic active earth pressure coefficients without a trial and error procedure. Therefore, more or less variation exists between the given values by the two studies.

Thirdly, the examples, presented by Motta [33] to determine the values of θ_{cr} and P_a, are calculated again for a cohesionless backfill with a uniform surcharge load at a constant distance from the wall under the static and seismic conditions. The results are shown in Table 3 for H = 4 m, γ = 20 kN/m^3, c = 0 kN/m^2 and β = 90°.

Table 3 - P_a and θ_{cr} calculated by Motta [32] and this study, respectively

q	40 kN/m^2	20 kN/m^2
ϕ	30°	35°
δ	0°	17.5°
i	0°	15°
L_1	1 m	2 m
	P_a (kN/m) for k_h = 0 and k_v = 0	P_a (kN/m) for k_h = 0.1 and k_v = 0
c = 0 kN/m^2	85.83 and 85.83	79.36 and 70.46
Δ, %	**0.0**	**-12.6**
	θ_{cr} (°) for k_h = 0 and k_v = 0	θ_{cr} (°)for k_h = 0.1 and k_v = 0
c = 0 kN/m^2	34.85 and 34.85	43.31 and 39.69
Δ, %	**0.0**	**-9.1**

The values of θ_{cr} and P_a computed by Motta [33] and this study are the same in the first example for the case of static analysis. About 10% variation is observed in the second example for the case of dynamic analysis. Motta [33] considered that the backfill was continuously sloping at an angle (i) with the horizontal plane. However, this study solves the second example for the case of the broken slope of the backfill. The difference between the slope geometries in the studies causes the different results of P_a and θ_{cr}.

4. CONCLUSIONS

This paper firstly presents an analytical approach, based on Culmann's and Mononobe-Okabe's methods within a trial and error procedure, to calculate the critical inclination of failure surface and the active earth thrust for a cohesive-frictional backfill with a broken slope and a surcharge load at a constant distance under seismic conditions. Then, a parametric attempt, using its derived equations and coded MATLAB script, is made to explore the effects of the major field parameters in the design of a retaining structure, such as wall height, surcharge magnitude, cohesion, and internal friction angle of the backfill, horizontal and vertical seismic acceleration coefficients, adhesion between wall-soil interface and tension cracks. Finally, the results of the proposed approach are verified by the comparison with Geo5 and previously published two studies.

By the parametric attempt in this study, it is found that the critical inclination of the failure surface increases with an increase in the surcharge load and the horizontal seismic acceleration coefficient, while it decreases with the wall height, the cohesion, and the internal friction angle. The critical inclination of the failure surface is highly sensitive to the horizontal seismic acceleration coefficient according to the effects of the other parameters.

The present work shows that the active earth thrust increases with increasing the wall height, the surcharge magnitude, and the horizontal seismic acceleration coefficient. Herein, it is worth noting that wall height has a greater influence on the active earth thrust. The cohesion and the internal friction angle of the backfill soil decrease the values of active earth thrust.

Under the static condition, the results of the critical inclination and the active earth thrust given by this work are the same for the cohesionless backfill when compared to the values of Shukla [13]'s and Motta [32]'s studies. However, the study shows significant differences in the results of the active earth thrust with increasing the cohesion and the horizontal seismic acceleration coefficient. It overestimates the seismic active earth thrust compared to Shukla [13]'s study but underestimates compared to Motta [32]'s study and Geo5. Besides, the study still results in fairly close to the critical inclinations of the failure surface estimated by Shukla [13] and Motta [32] under the seismic condition.

The approach presented by this paper about deriving the equations and the algorithm of the coded MATLAB script herein can be employed by engineers in any geotechnical problems associated with a critical inclination of failure surface and an active earth thrust for a cohesive-frictional backfill with a broken slope and a surcharge load at a constant distance under seismic conditions.

Symbols

c : the (unit) cohesion of the backfill

c_a : the (unit) adhesion between the retaining wall and the backfill

C : the total cohesion along the failure plane

C_a : the total adhesion along the back face of the wall

H : the height of the retaining wall

i : the inclination of the backfill surface

k_h : the horizontal seismic acceleration coefficient

k_v : the vertical seismic acceleration coefficient

m : a dimensionless factor in Equation (5)

L_1 : the horizontal distance from the wall to the surcharge load

L_2 : the horizontal surcharge load distance on the failure wedge

P_a : the active earth thrust

$P_{a,1}$	: the calculated active earth thrusts from Equation (5) considering positive (downward) vertical seismic acceleration coefficient
$P_{a,2}$	: the calculated active earth thrusts from Equation (6), considering negative (upward) vertical seismic acceleration coefficient
q	: the magnitude of the surcharge load
Q	: the resultant force of the surcharge load on the trial failure wedge (= qL_2)
R	: the soil reaction on the failure plane
W	: the weight of the trial failure wedge (= $\gamma Area_{wedge}$)
z_c	: the depth of tension cracks zone
β	: the obliquity of the back face of the retaining wall
θ	: the angle between the back face of the retaining wall and the failure surface
θ_{cr}	: the critical inclination of failure surface
Δ	: $(P_{a,\text{ by the study}} - P_{a,\text{ by Motta}}) / P_{a,\text{ by the study}}$; $(\theta_{cr,\text{ by the study}} - \theta_{cr,\text{ by Motta}}) / \theta_{cr,\text{ by the study}}$
α	: the inclination of the failure plane with the horizontal
δ	: the friction angle between the retaining wall and the backfill
γ	: the unit weight of the backfill
ϕ	: the internal friction angle of the backfill
ψ	: the seismic inertia angle which is defined as $\arctan(k_h/(1\pm k_v))$

References

[1] Okabe, S., General theory of earth pressure and laboratory testings on seismic stability of retaining walls. Journal of the Japanese Society of Civil Engineers, 12, 1, 123-134, 1926.

[2] Mononobe, N., Matsuo, H., On the determination of earth pressure during earthquakes. Proceedings of the World Engineering Congress Vol 9, Tokyo-Japan, 177-185, 1929.

[3] Düzgün, M., Bozdağ, Ö., İstinat duvarlarına etkiyen sismik zemin basıncının dağılımı için geliştirilen yeni bir yöntem. Teknik Dergi, 14, 66, 2819-2834, 2003.

[4] Ghosh, S., Sharma, R.P., Pseudo-dynamic response of non-vertical retaining wall supporting c-ϕ soil backfill. Geotechnical and Geological Engineering, 28, 5, 633-641, 2010.

[5] Shao-jun, M., Kui-hua, W., Wen-bing, W., Pseudo-dynamic active earth pressure behind retaining wall for cohesive soil backfill. Journal of Central South University of Technology, 19, 3298-3304, 2012.

[6] Das, B.M., Puri, V.K., Static and dynamic active earth pressure. Geotechnical and Geological Engineering, 14, 4, 353-366, 1996.

[7] Saran, S., Gupta, R.P., Seismic earth pressures behind walls. Indian Geotechnical Journal, 33, 3, 195-213, 2003.

[8] Ghosh, S., Dey, G.N., Datta, B., Pseudo-static analysis of rigid retaining wall for dynamic active earth pressure. 12th International Conference of International Association for Computer Methods and Advances in Geomechanics (IACMAG) 1-6 October, Goa-India, 4122-4131, 2008.

[9] Shukla, S.K., Dynamic active thrust from c-ϕ soil backfills. Dynamic Earthquake Engineering, 31, 3, 526-529, 2011.

[10] Shukla, S.K., Bathurst, R. J., An analytical expression for the dynamic active thrust from c-ϕ soil backfill on retaining walls with wall friction and adhesion. Geomechanics and Engineering, 4, 3, 209-218, 2012.

[11] Ghosh, S., Sengupta, S., Extension of Mononobe-Okabe theory to evaluate seismic active earth pressure supporting c-ϕ soil backfill, Electronic Journal of Geotechnical Engineering, 17(D), 495-504, 2012.

[12] Iskander, M., Chen, Z., Omidvar, M., Guzman, I., Rankine pseudo-static earth pressure for c-ϕ soils. Mechanics Research Communications, 51, 51-55, 2013.

[13] Shukla, S.K., Generalized analytical expression for dynamic active thrust from c-ϕ soil backfills. International Journal of Geotechnical Engineering, 9, 4, 416-421, 2015.

[14] Lin, Y., Leng, W., Yang, G., Zhao, L., Li, L., Yang, J., Seismic active earth pressure of cohesive-frictional soil on retaining wall based on a slice analysis method. Soil Dynamics and Earthquake Engineering, 70, 133-143, 2015.

[15] Zhou, Y., Chen, F., Wang, X., Seismic active earth pressure for inclined rigid retaining walls considering rotation of the principal stresses with pseudo-dynamic method. International Journal of Geomechanics, 18, 7: 04018083, 1-9, 2018.

[16] Tang, Y., Chen, J., A computational method of active earth pressure from finite soil body. Mathematical Problems in Engineering, Vol. 2018: 9892376, 1-7, 2018.

[17] Gupta, A., Sawant, V.A., Effect of soil amplification on seismic earth pressure using pseudo-dynamic approach. International Journal of Geotechnical Engineering, 4, 2, 1-12, 2018.

[18] Gupta, A., Yadav, V., Sawant, V.A., Agarwal R., Development of design charts considering the effect of backfill inclination and wall inclination on the seismic active pressure for c-ϕ soil. International Journal of Mathematical, Engineering and Management Sciences, 4, 2, 409-419, 2019.

[19] Peng, J., Zhu, Y., Derivation of Shukla's generalized expression for dynamic active thrust by inclined slice element method. Soil Mechanics and Foundation Engineering, 56, 2, 77-81, 2019.

[20] Caltabiano, S., Cascone, E., Maugeri, M., Seismic stability of retaining walls with surcharge. Soil Dynamics and Earthquake Engineering, 20, 469-476, 2000.

[21] Caltabiano, S., Cascone, E., Maugeri, M, A procedure for seismic design of retaining walls, seismic prevention of damage: A case study in a mediterranean city. WIT Trans. State Art Sci. Eng., 8, 263-277, 2005.

[22] Caltabiano, S., Cascone, E., Maugeri, M., Static and seismic limit equilibrium analysis of sliding retaining walls under different surcharge conditions. Soil Dynamics and Earthquake Engineering, 37, 38-55, 2012.

[23] Aminpour, M.M., Maleki, M., Ghanbari, A., Investigation of the effect of surcharge on behavior of soil slopes, Geomechanics and Engineering, 13, 4, 653-669, 2017.

[24] Hou, G., Shu, S., Trial wedge approach to determine lateral earth pressures. International Journal of Geomechanics, 19, 1: 06018035, 1-16, 2019.

[25] Arda, Ç., Çinicioğlu, Ö., Kohezyonsuz zeminlerin tane dağılım ve şekil özelliklerinin aktif göçme yüzeyi geometrisine etkileri. Teknik Dergi, 30, 5, 9399-9420, 2019.

[26] Yazdani, M., Azad A., Farshi, A.H., Talatahari, S., Extended 'Mononobe-Okabe' method for seismic design of retaining walls. Journal of Applied Mathematics, Vol. 2013, 1-10, 2013.

[27] Greco, V.R., An algorithm for the evaluation of active thrust for backfill with irregular profile. Geotechnique Letters, 5, 2, 1-6, 2015a.

[28] Greco, V.R., Seismic active thrust due to backfill of general topographic profile. Soil Dynamics and Earthquake Engineering, 79, 66-70, 2015b.

[29] Kim, W.C., Park, D., Kim, B. Development of a generalized formula for dynamic active earth thrust. Geotechnique, 60, 9, 721-727, 2010.

[30] Greco, V.R., Discussion of 'Development of a generalized formula for dynamic active earth thrust by Kim, W.C., Park, D. and Kim, B.'. Geotechnique, 62, 4, 365-366, 2012.

[31] Lu, H., Yuan, B., Calculation of passive earth pressure of cohesive soil based on Culmann's method. Water Science and Engineering, 4, 1, 101-109, 2011.

[32] Culmann, K., Die graphische statik, Meyer&Zeller, Zurih, 1866.

[33] Motta, E., Generalized coulomb active-earth pressure for distanced surcharge. Journal of Geotechnical Engineering, 120, 6, 1072-1079, 1994.

[34] Paik, K.H., Salgado, R., Estimation of active earth pressure against rigid retaining walls considering arching effects. Geotechnique, 53, 7, 643-653, 2003.

[35] Shukla, S.K., Gupta, S.K., Sivakugan, N., Active earth pressure on retaining wall for c-ϕ soil backfill under seismic loading condition. Journal of Geotechnical and Geoenvironmental Engineering, 135, 5, 690-696, 2009.

[36] Nian, T., Han, J., Analytical solution for Rankine's seismic active earth pressure in c-ϕ soil with infinite slope. Journal of Geotechnical and Geoenvironmental Engineering, 1399, 1611-1616, 2013.

[37] Ghosh, S., Sharma, R.P., Seismic active earth pressure on the back of battered retaining wall supporting inclined backfill. International Journal of Geomechanics, 12, 1, 54-63, 2012.