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A UTILITARIAN RESOLUTION OF THE PARETIAN LIBERAL PARADOX

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ABSTRACT: ***Bu makale, kişi hak ve özgürlükleri ile Pareto etkinlik arasında belirli şartlarda bir çelişki öngören "Paretian Liberal Paradoksu"nun, ilgili hak ve özgürlüklerin pozitif net toplam (toplumsal) fayda üretmeleri koşulu altında ortadan kalkacağını kanıtlamaktadır.***

I-INTRODUCTION

In the history of social choice paradigm, several theorems played a key role in setting the tone and the direction of research on social choice problems. Amartya Sen's theorem in his paper on the "Impossibility of a Paretian-liberal" (Sen, 1970) is unarguably one of them. It is quite fascinating to see how a seemingly ordinary theorem could initiate a line of inquiry that is so rich and productive that it still inspires many researchers in the field. Apparently, A. Sen has touched upon something quite profound in our intellectual psyche so that after a quarter of a century the controversy surrounding his paper still remains at the center of social choice analysis. Among the works that explore the controversy in question are Aldrich [2], Breyer [3], Gaertner, Pattanaik and Suzumara [4], Gibbard [5], Kelsey [6], Pressler [7] and Seidl [8]. A part of the reason for this ongoing interest in Sen's theorem lies in the importance we attach to the issues and concepts the theorem deals with, namely, on the one hand individual rights and freedoms which are at the very foundations of the vision of society that underlies the liberal democratic discourse, and on the other hand the Paretian principle which is still a core concept in the conventional economic analysis.

The importance individuals attach to certain rights has a lot to do with with the intensity with which they prefer right-chosen alternatives to others. Preference intensities for such alternatives are, therefore, likely to serve as a discursive key to the understanding and resolution of the conflicts over those rights. Paradoxically, however, the theoretical treatment and the proposed resolutions on the subject, with a few exceptions such as that of Ng [9] and Mueller [10], placed little or no emphasis on the role preference intensities could play in explaining and resolving the conflicts in question. This paper takes a step towards filing this gap in the literature by exploring a preference intensity-based, utilitarian resolution of the Paretian liberal paradox.

II-THE GENERAL FRAMEWORK

Let E be the set of a finite number of individuals forming a society, and let Z be the set of mutually exclusive social alternatives. Assume that the cardinalities of E and Z, denoted by, respectively, $|E|$ and $|Z|$, are finite, and $|E|>1$, $|Z|>2$. Each individual i in the society has a preference ordering R^i, which is a binary relation on Z such that $R^i \subseteq \{(x,y)$: *x, y are in Z}, and* $i=1,\ldots,n$. For any x, y in Z, $(x,y) \in R^i$ means the same thing as xR^iy which will be interpreted as "x is preferred to y" by individual i. Define strict preference (P^i) and indifference (I^i) relations on $\{x,y\}$ as follows: xP^iy *if and only if* xR^iy *and not* yR^ix. xI^iy *if and only if* xR^iy *and* yR^ix.

A preference relation R^i *on Z is said to be* <u>complete</u> *if and only if* xP^iy *or* yP^ix *or* xI^iy *for all x, y in Z such that* $x \neq y$. R^i *on Z is* <u>incomplete</u> *if it is not complete.* R^i *on Z is* <u>transitive</u> *if and only if for all x, y, z in Z,* (xP^iy *and* yP^iz *implies* xP^iz), *and* (xI^iy *and* yI^iz *implies* xI^iz). R^i *on Z is* <u>intransitive</u> *if it is not transitive.* R^i *on Z is* <u>acyclical</u> *over an m-set* $\{x_1,\ldots,x_m\}$ *in Z if and only if the following condition holds: For all* $x_1,\ldots,x_m$ *in Z, if* [$x_1P^ix_2$, *and* $x_2P^ix_3$, *and...and* $x_{m-1}P^ix_m$], *then* $x_1R^ix_m$. R^i *is* <u>cyclical</u> *over an m-set, if and only if for all* $x1,\ldots,xm$ *in Z,* $x_1P^ix_2$, *and* $x_2P^ix_3$, *and...and* $x_{m-1}P^ix_m$ *and* $x_mP^ix_1$. *Clearly, if* R^i *is not cyclical over any m-set in Z, none of its subsets is.*

The utilitarian resolution of the Paretian liberal paradox presented in this paper requires an explicit account of individuals' preference intensities which we will introduce as follows: Each individual k in E is endowed with a total of M (M>0) points which she allocates among m alternatives in Z in proportion to her utility index over those m alternatives. Let a^k_j be the number of points individual k assigns to alterntive x_j in Z. Thus, $\Sigma^m_{j=1} a^k_j = M$, for each individual in E. The mumber of points an individual assigns to an alternative reflects her intensity of preference for that alternative. Thus, the more intense is her preference for an alternative, the greater is the number of points she assigns to that alternative. For instance, if she prefers $x_i \in Z$ to $x_j \in Z$, then $a^k_i > a^k_j$.

Define a collective choice rule, which we will call <u>point voting rule</u>, that takes into account the individuals' preference intensities over m alternatives in the following manner:

x_iPx_j iff $\Sigma^n_{k=1} a^k_i > \Sigma^n_{k=1} a^k_j$

x_iIx_j iff $\Sigma^n_{k=1} a^k_i = \Sigma^n_{k=1} a^k_j$,

where P and I indicate, respectively social strict preference and social indifference. Alternatively,

x_iRx_j iff $\Sigma^n_{k=1} a^k_i \geq \Sigma^n_{k=1} a^k_j$,

where R indicates social preference.

II-THE PARETIAN LIBERAL PARADOX: A RESOLUTION

Sen's theorem establishing a conflict between the Pareto principle and Liberalism involves two central concepts, which we will define as follows:

Definition: *An individual i is decisive for an ordered pair (x,y) if xP^iy implies xPy. The individual i is said to have a right over {x,y} if she is decisive for (x,y) and (y,x).*

The formal conditions of Sen's theorem are:

Condition U (Unrestricted Domain): *Every logically possible combination of individual orderings is included in the domain of the collective choice rule.*

Condition P (Pareto Efficiency): *Let {x,y} be any pair contained in Z. If for every i in E xP^iy, then xPy.*

Condition L (Liberalism): *There are at least two individuals in E, each of whom has a right over at least one pair of alternatives*

Sen proves that given Condition U, Condition P and Condition L are incompatible, i.e., they together imply the possibility of cyclical social preferences.

To set the stage for a utilitarian resolution of the paradox, we will present two concepts, one of which defines right-induced gains and losses, and the other formalizes the idea of a conflict between Pareto efficiency and individual rights (liberalism).

Definition: *A right of an individual, say g, over $\{x_i,x_j\}$ is said to create, for individual g, a gain of $|a^g_i - a^g_j|$. The exercise of the right over $\{x_i,x_j\}$ by individual g is said to create, for individual k, k≠g, k=1,...,n, a gain of $|a^k_i-a^k_j|$ if $a^k_i-a^k_j$ is positive and a loss of $|a^k_i-a^k_j|$ if $a^k_i-a^k_j$ is negative. $|\Sigma^n_{k=1}(a^k_i-a^k_j)|$ represents the aggregate gain (or loss) the right over $\{x_i,x_j\}$ creates for individuals in E.*

Definition: *For a given configuration of individual preferences, a conflict is said to exist between Condition P and Condition L with respect to an m-set, m>2, in Z if the simultaneous (joint) application of both conditions results in a social preference relation R that is cyclical over that m-set while the individual application of each condition in the absence of the other does not.*

Given an unrestricted domain of individual preferences, Sen's theorem implies the existence of at least one m-set in Z with respect to which a conflict exists between Condition P and Condition L. However, there are conditions under which such conflicts cease to exist. The following theorem presents a utilitarian condition that eliminates all such conflicts in the entire domain of alternatives.

Theorem 3.1: If the net aggregate gains any right creates are greater zero, there is no conflict between individual rights and Pareto efficiency with respect to any m-set in Z.

Proof: Let R^P be the set of ordered pairs over which the social preference is determined by Condition P, and R^l be the set of ordered pairs over which the social preference is determined by (Condition L) that satisfy the condition stated in the theorem. Let R^* be the the social preference relation induced by the point voting rule. We will first show that $R^P \subset R^*$ and $R^l \subset R^*$.

Let $(x_i,x_j) \in R^P$. Then, $a^k_i > a^k_j$ for every k in E. Thus,

$\Sigma^n_{k=1} a^k_i > \Sigma^n_{k=1} a^k_j$, which implies that $(x_i,x_j) \in R^*$. Hence, $(x_i,x_j) \in R^P$implies $(x_i,x_j) \in R^*$, i.e., $R^l \subset R^*$.

Let $(x_s,x_t) \in R^l$. Then, there is an individual in E, say 1, who has a right over $\{x_s,x_t\}$ and who strictly prefers x_s to x_t. Such a right creates a gain of x_s-x_t for individual 1. Without the loss of generality, assume that the right over (x_s,x_t) also creates gains for individuals 2 to h, losses for individuals h+1 to l, and neither gains nor losses for individuals l+1 to n. By assumption, the net aggregate gains of any right are greater than zero, implying that the net aggregate gains the right over $\{x_s,x_t\}$ creates are greater than the aggregate losses it induces. Thus,

$$|\Sigma^h_{k=1} (a^k_s-a^k_t)| > |\Sigma^l_{k=h+1} (a^k_s-a^k_t)|$$

$$\Rightarrow \Sigma^h_{k=1} (a^k_s-a^k_s) > - \Sigma^l_{k=h+1} (a^k_s-a^k_t)$$

$$\Rightarrow \Sigma^h_{k=1} (a^k_s-a^k_t) + \Sigma^l_{k=h+1} (a^k_s-a^k_t) > 0.$$

Since the right over (x_s,x_t) creates neither gain nor loss for individuals l+1 to n, $\Sigma^n_{k=l+1} (a^k_s-a^k_t) = 0$. Thus, adding this zero-sum to the left-hand side of the inequality does not affect the inequality, i.e.,

$$\Sigma^h_{k=1} (a^k_s-a^k_t) + \Sigma^l_{k=h+1} (a^k_s-a^k_t) + \Sigma^n_{k=l+1} (a^k_s-a^k_t) > 0$$

$$\Rightarrow \Sigma^n_{k=1} (a^k_s-a^k_t) > 0$$

$$\Rightarrow \Sigma^n_{k=1} a^k_s > \Sigma^n_{k=1} a^k_t,$$

which implies that $(x_s,x_t) \in R^*$. Thus, $(x_s,x_t) \in R^l$ implies $(x_s,x_t) \in R^*$, i.e., $R^l \subset R^*$. Since $R^p \subset R^*$ and $R^l \subset R^*$, $(R^p \cup R^l) \subset R^*$.

It is straightforward to show that R* is transitive over every triple in Z: Take an arbitrary triple $\{x_1,x_2,x_3\}$ in Z and let $(x_1,x_2) \in R^*$ and $(x_2,x_3) \in R^*$. Then, $\Sigma^n_{k=1} a^k_1 \geq \Sigma^n_{k=1} a^k_2$ and $\Sigma^n_{k=1} a^k_2 \geq \Sigma^n_{k=1} a^k_3$. Thus, $\Sigma^n_{k=1} a^k_1 \geq \Sigma^n_{k=1} a^k_3$, which implies that $(x_1,x_3) \in R^*$. Therefore R^* is transitive over $\{x_1,x_2,x_3\}$. This property of R^* holds for every triple in Z. Since R^* is transitive over every triple in Z, it is acylical over every m-set in Z. Thus, none of the subsets of R^* is cyclical over any m-set in Z. Since $R^p \cup R^l$ is a subset of R^*, it is not cylical over any m-set in Z, which implies that individual rights and Pareto efficiency do not generate a cyclical social preference over any m-set in Z, ruling out any conflict betweeen them over such m-sets in Z.

Q.e.d.

IV-CONCLUDING REMARKS

The result contained in the theorem above is significant for two reasons. First, it indicates an alternative, utilitarian way out of the Paretian liberal dilemmas, i.e., the Paretian liberal conflicts would not arise if rights were assigned in such a way that the net aggregate gains they create are greater than zero. Second, it provides insights into the nature of rights-induced externalities that give rise to the paradox. With a minor reformulation of the theorem, it is possible, for instance, to show how predominantly negative externality inducing rights could render social choice contexts susceptible to the conflicts in question and how the condition stated in the theorem could resolve such conflicts. A detailed examination of the relation between rights, Pareto efficiency and externalities, which is beyond the scope of this paper, is a potentially fruitful exercise for future research.

REFERENCES

[1] Sen, A. 1970. *Collective Choice and* Social Welfare. San Francisco: Holden-Day.

[2] Aldrich, J. 1977. "The Dilemma of a Paretian Liberal: Some Consequences of Sen's Theorem." *Public Choice* 30: 1-22.

[3] Breyer, F. 1977. "The liberal Paradox, Decisiveness over Issues and Domain Restrictions." *Z. Nationalokonomie* 37: 45-60.

[4] Gaertner, W., P. K. Pattanaik and K. Suzumura. 1992. AIndividual Rights Revisited.@ Economica 59(234): 161-77.

[5] Gibbard, A. 1974. "A Pareto Consistent Libertarian Claim." *Journal of Economic Theory* 7: 388-410.

[6] Kelsey, D. 1988. "What is Responsible for the Paretian Epidemic." *Social Choice and Welfare* 5:303-306.

[7] Pressler, J. 1987. "Rights and Social Choice: Is There a Paretian Libertarian Paradox." *Economics and Philosophy* 3:1-22.

[8] Seidl, C. 1990. AOn the Impossibility of a Generalization of the Libertarian Resolution of the Liberal Paradox.@ Journal-of-Economics (Zeitschrift- fur-Nationalokonomie) 51(1): 71-88.

[9] Mueller D.C. 1996. "Constitutional and Liberal Rights." *Analyse and Kritik* 18: 96-117.

[10] Ng, Y.K. 1971. "The Possibility of a Paretian Liberal: Impossibility Theorems and Cardinal Utility." *Journal of Political Economy* 79: 1397-1402.

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