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# Domination Edge Integrity of Corona Products of $C_n$ with $P_m, C_m, K_{1,m}$

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**Research Article**

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**Abstract**
Vulnerability is the most important concept in analysis of communication networks to disruption. Any network can be modelled by graphs so measures defined on graphs gives an idea in design. Integrity is one of the well-known vulnerability measures interested in remaining structure of a graph after any failure. Domination is also an another popular concept in network design. Nowadays new vulnerability measures take a great role in any failure not only on nodes also on links which have special properties. A new measure domination edge integrity of a connected and undirected graph was defined such as $DI'(G) = min\{\ |S| + m(G - S): S \subseteq E(G)\}$ where $m(G - S)$ is the order of a maximum component of $G - S$ and $S$ is an edge dominating set. In this paper some results concerning this parameter on corona products of graph structures $C_n \odot C_m$, $C_n \odot P_m$, $C_n \odot K_{1,m}$ are presented.

**Keywords:** Corona product, domination, edge domination, edge integrity, edge domination integrity

# $C_n$ ile $P_m, C_m, K_{1,m}$ Graflarının Corona Çarpımlarının Ayrıt Baskın Bütünlüğü

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**Öz**
Zedelenebilirlik, bir iletişim ağının bozulmalara karşı yapılan analizindeki en önemli kavramdır. Herhangi bir ağ graflar ile modellenebilir böylece graflar üzerinde tanımlanan ölçümler tasarımda bir fikir verir. Bütünlük kavramı herhangi bir bozulmadan sonra grafta geriye kalan yapılarla ilgilenen en çok bilinen zedelenebilrlik ölçümlerindendir. Baskınlık da ağ tasarımda yaygın olarak kullanılan önemli bir kavramdır. Günümüzde sadece tepeler üzerinde değil belirli bir özelliğe sahip ayrıtlar üzerinde oluşan hatalarda yeni zedelenebilirlik ölçümleri önemli rol oynamaktadır. Yeni bir ölçüm olan birleştirilmiş yönsüz bir grafın ayrıt baskın bütünlüğü $DI'(G) = min\{\ |S| + m(G - S): S \subseteq E(G)\}$ olarak tanımlanmıştır, burada $m(G - S)$ $G - S$ deki en büyük bileşenin tepe sayısını göstermekte ve $S$ bir ayrıt baskın kümedir. Bu çalışmada bu ölçüm ile ilgili bazı sonuçları $C_n \odot C_m$, $C_n \odot P_m$, $C_n \odot K_{1,m}$ corona çarpımlarının oluşturduğu graf yapılarında gösterilmiştir.

**Anahtar Kelimeler:** Corona çarpımı, baskınlık, ayrıt baskınlık bütünlüğü, ayrıt bütünlük, ayrıt baskınlık



## Introduction

A communication network can be modeled by a graph $G$ where nodes are represented by vertices and links are represented by edges such as $V(G)$, $E(G)$ respectively. Any communication network can be considered to be highly vulnerable to any disruption on its nodes or links. All graphs considered in this paper are connected, undirected and do not contain any loops and multiple edges. First basic vulnerability measures are connectivity or edge connectivity which shows how easily a graph can be broken apart.

Later on, it is observed that these measures are not enough to compare the stability of network structures which have the same order. Most network designers are interested in what happens in the remaining part of the network after failures such as, how many nodes or links are still connected to each other and what is the communication between remaining parts. Integrity and edge integrity concepts are interested in these questions. Both types of integrity were introduced by Barefoot et al. [1] and Goddard and Swart [2] has great contributions for this area. Integrity or edge integrity have been widely studied on specific graph families and relationships with other parameters and bounds were obtained Bagga et al. have presented many results about edge integrity in [3].

The order of a graph $G$ will generally be denoted by $n$. For a real number $x$; $\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$ and $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$.

Domination is another important concept widely studied in graph theory. A subset $S$ of $V$ is called a dominating set of $G$ if every vertex not in $S$ is adjacent to some vertex in $S$. The domination number $\gamma(G)$ (or $\gamma$ for short) of $G$ is the minimum cardinality taken over all dominating sets of $G$ [4].

Hedetniemi and Mitchell [5] have introduced the concept of edge domination. A subset $X$ of $E$ is called an edge dominating set of $G$ if every edge not in $X$ is adjacent to some edge in $X$. The edge domination number $\gamma'(G)$ (or $\gamma'$ for short) of $G$ is the minimum cardinality taken over all edge dominating sets of $G$.

Domination and integrity were examined together and many new vulnerability measures were defined. Some of them are domination integrity [6], domination edge integrity [7], and total domination integrity [8].

The concept of domination edge integrity of a connected graph as a new vulnerability parameter was defined by Kılıç and Beşirik [7] as follows.

The domination edge integrity of a connected graph $G$ denoted by $DI'(G)$ and defined by

$$DI'(G) = \min\{|S| + m(G - S)\}$$

where $S$ is an edge dominating set and $m(G - S)$ is the order of a maximum component of $G - S$.

A subset $S$ of $E(G)$ is a $DI'$-set if

$$DI'(G) = \{|S| + m(G - S) : S \subseteq \mathrm{E}(G)\}$$

where $S$ is an edge dominating set of $G$.

$DI'$ values of $P_n$, $C_n$, $K_{1,n}$, $K_{m,n}$ were presented and some properties for domination edge integrity value of a connected graph were determined in [7].

## $DI'$ of Corona Products of $C_n$ with some graphs

$DI'$ values of some resulting graphs after corona operation of $C_n$ with $P_m$, $C_m$, $K_{1,m}$ are found as follows.

**Definition** The corona product $G_1 \odot G_2$ is defined as $G$ obtained by taking one copy of $G_1$ of order $p_1$ and $p_1$ copies of $G_2$, and then joining the $i$'th node of $G_1$ to every node in the $i$'th copy of $G_2$ [9].

**Proposition 1 [10]** Let $n$ be an integer,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = n.$

In proof of all theorems, for graph $G$ of order $m$, edge dominating sets $X_1$ and $X_2$ are taken which satisfies, $m(\ C_n \odot G - X_1\ ) = 2(m+1)$ and $m(\ C_n \odot G - X_2\ ) = m+1$ respectively (Figure 1). There is no any other possible selection of edge dominating sets which gives $DI'$ to be minimum. If $X_3$ is taken to be another edge dominating set, cardinality of $X_3$ is greater than both $X_1$ and $X_2$. It is easy to observe from structure of corona product of $C_n$ with any graph $G$. And also $m(C_n \odot G - X_3) > 2(m+1)$ since more edges are added. This selection does not give a minimum result.

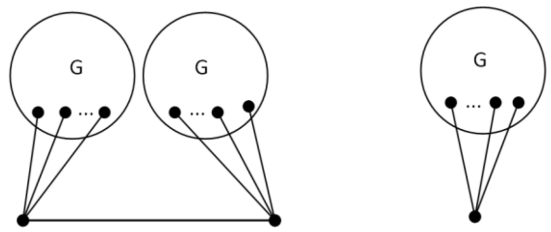


***Figure 1.*** *Maximum components of* $(C_n \odot G) - X_1$ *and* $(C_n \odot G) - X_2$

**Theorem** For $n \geq 3$ and $m \geq 2$, let $n$ to be odd and $A$, $B$ to be as follows,

$A = \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + 2m + 2,$

$B = n + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1.$

Then, $DI'(C_n \odot P_m)$ is obtained as follows,

$$DI'(\ C_n \odot P_m) = \begin{cases} A & ,if\ \ m+1 < \left\lfloor \dfrac{n}{2} \right\rfloor, \\ B & ,if\ \ m+1 > \left\lfloor \dfrac{n}{2} \right\rfloor, \\ A = B & ,if\ \ m+1 = \left\lfloor \dfrac{n}{2} \right\rfloor. \end{cases}$$

**Proof** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $V(P_m) = \{u_1, u_2, \ldots, u_m\}$ vertex sets for cycle graph $C_n$ and path graph $P_m$.

Let $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1\}$ and $E(P_m) = \{u_1u_2, \ldots, u_{m-1}u_m\}$ be edge sets respectively.

For $n \geq 3$, we have 2 cases as follows.

**Case 1** Let $X_1$ be an edge dominating set of $C_n \odot P_m$. Then the size of maximum component will be as

$m(C_n \odot P_m - X_1) = 2(m+1)$. $X_1$ is obtained as follows.

Let $S_1 \subset E(C_n)$ be an edge dominating set of $C_n$ and $|S_1| = \left\lceil \frac{n}{2} \right\rceil$.

Let $S_{2_i}$ be a minimum edge dominating set of $i$th copy of $P_m$ and $|S_{2_i}| = \left\lceil \frac{m-1}{3} \right\rceil$ and

$S_2 = S_{2_1} \cup S_{2_2} \cup \ldots \cup S_{2_n}$.

$X_1 = S_1 \cup S_2$ is an edge dominating set of $C_n \odot P_m$. Therefore, $|X_1| = \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil$ and $m(C_n \odot P_m - X_1) = 2(m+1)$. Thus,

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$$DI'(C_n \odot P_m) \leq |X_1| + m(C_n \odot P_m - X_1)$$

$$= \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + 2m + 2$$

$$= DI'(C_n \odot P_m)_{X_1}.$$

**Case 2** Let $X_2$ be an edge dominating set of $C_n \odot P_m$ and then size of maximum component will be $m(C_n \odot P_m - X_2) = m + 1$. $X_2$ is obtained as follows.

Let $S_1' = E(C_n)$. $S_1'$ be an edge dominating set of $C_n$ and $|S_1'| = n$.

Let $S_{2_i}$ be a minimum edge dominating set of $i$th copy of $P_m$ and $|S_{2_i}| = \left\lceil \frac{m-1}{3} \right\rceil$ and

$S_2 = S_{2_1} \cup S_{2_2} \cup \ldots \cup S_{2_n}$.

$X_2 = S_1' \cup S_2$ is an edge dominating set of $C_n \odot P_m$. Therefore, $|X_2| = n + n\left\lceil \frac{m-1}{3} \right\rceil$ and $m(C_n \odot P_m - X_2) = m + 1$. Thus,

$$DI'(C_n \odot P_m) \leq |X_2| + m(C_n \odot P_m - X_2)$$

$$= n + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1$$

$$= DI'(C_n \odot P_m)_{X_2}.$$

Because of definition of $DI'$, the values and similarities under corona operation $DI'(C_n \odot P_m)_{X_1}$ and $DI'(C_n \odot P_m)_{X_2}$ must be examined as follows.

**i.** If $m + 1 < \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$DI'(C_n \odot P_m)_{X_1}$ to be

$$= \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + 2m + 2$$

$$= \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1 + m + 1$$

$$< \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + \left\lfloor \frac{n}{2} \right\rfloor + m + 1,$$

by Proposition 1

$$= n + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1$$

$$= n + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1$$

$$= DI'(C_n \odot P_m)_{X_2}.$$

Since $DI'(C_n \odot P_m)_{X_1} < DI'(C_n \odot P_m)_{X_2}$, then

$$DI'(C_n \odot P_m) = DI'(C_n \odot P_m)_{X_1}$$

$$= \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m-1}{3} \right\rceil + 2m + 2.$$

**ii.** If $m + 1 > \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$DI'(C_n \odot P_m)_{X_2} < DI'(C_n \odot P_m)_{X_1}$. It can be proved in a similar way as above. Thus, $DI'(C_n \odot P_m) = DI'(C_n \odot P_m)_{X_2}$

$$= n + n\left\lceil \frac{m-1}{3} \right\rceil + m + 1.$$

**iii.** If $m + 1 = \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$DI'(C_n \odot P_m)_{X_1} = DI'(C_n \odot P_m)_{X_2}$. Hence, $DI'(P_n \odot P_m) = DI'(C_n \odot P_m)_{X_1}$

$$= DI'(C_n \odot P_m)_{X_2}.$$

**Theorem** For $n \geq 3$ and $m \geq 3$, let $n$ to be odd and $A$, $B$ to be as follows,

$$A = \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m}{3} \right\rceil + 2m + 2,$$

$$B = n + n\left\lceil \frac{m}{3} \right\rceil + m + 1.$$

Then, $DI'(C_n \odot C_m)$ is obtained as follows,

$$DI'(C_n \odot C_m) = \begin{cases} A \quad , if \ \ m+1 < \left\lfloor \frac{n}{2} \right\rfloor, \\ B \quad , if \ \ m+1 > \left\lfloor \frac{n}{2} \right\rfloor, \\ A = B, if \ \ m+1 = \left\lfloor \frac{n}{2} \right\rfloor. \end{cases}$$

**Proof** Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $V(C_m) = \{u_1, u_2, \dots, u_m\}$ be vertex sets for for cycle graphs $C_n$ and $C_m$. And let

$E(C_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$ and $E(C_m) = \{u_1u_2, \dots, u_{m-1}u_m, u_mu_1\}$ be edges sets of cycle graphs $C_n$ and $C_m$.

For $n \geq 3$, we have 2 cases as follows.

**Case 1** Let $X_1$ be an edge dominating set of $C_n \odot C_m$ and $m(C_n \odot C_m - X_1) = 2(m+1)$. $X_1$ is obtained as follows.

Let $S_1 \subset E(C_n)$ be an edge dominating set of $C_n$ and $|S_1| = \left\lceil \frac{n}{2} \right\rceil$.

Let $S_{2_i}$ be a minimum edge dominating set of $i$th copy of $C_m$ and $|S_{2_i}| = \left\lceil \frac{m}{3} \right\rceil$ and

$S_2 = S_{2_1} \cup S_{2_2} \cup \dots \cup S_{2_n}$.

$X_1 = S_1 \cup S_2$ is an edge dominating set of $C_n \odot C_m$. Therefore, $|X_1| = \left\lceil \frac{n}{2} \right\rceil + n \left\lceil \frac{m}{3} \right\rceil$ and $m(C_n \odot C_m - X_1) = 2(m+1)$. Thus,

$$DI'(C_n \odot C_m) \leq |X_1| + m(C_n \odot C_m - X_1)$$
$$= \left\lceil \frac{n}{2} \right\rceil + n \left\lceil \frac{m}{3} \right\rceil + 2m + 2$$
$$= DI'(C_n \odot C_m)_{X_1}.$$

**Case 2** Let $X_2$ be an edge dominating set of $C_n \odot C_m$ and $m(C_n \odot C_m - X_2) = m+1$. $X_2$ is obtained as follows.

Let $S_1' = E(C_n)$. $S_1'$ is an edge dominating set of $C_n$ and $|S_1'| = n$.

Let $S_{2_i}$ be a minimum edge dominating set of $i$th copy of $C_m$ and $|S_{2_i}| = \left\lceil \frac{m}{3} \right\rceil$ and

$S_2 = S_{2_1} \cup S_{2_2} \cup \dots \cup S_{2_n}$.

$X_2 = S_1' \cup S_2$ is an edge dominating set of $C_n \odot C_m$.Therefore, $|X_2| = n + n \left\lceil \frac{m}{3} \right\rceil$ and

$m(C_n \odot C_m - X_2) = m+1$. Thus,

$$DI'(C_n \odot C_m) \leq |X_2| + m(C_n \odot C_m - X_2)$$
$$= n + n \left\lceil \frac{m}{3} \right\rceil + m + 1$$
$$= DI'(C_n \odot C_m)_{X_2}.$$

Because of definition of $DI'$, the relationship between $DI'(C_n \odot C_m)_{X_1}$ and $DI'(C_n \odot C_m)_{X_2}$ must be examined as follows.

**i.** If $m+1 < \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$$DI'(C_n \odot C_m)_{X_1}$$
$$= \left\lceil \frac{n}{2} \right\rceil + n \left\lceil \frac{m}{3} \right\rceil + 2m + 2$$
$$= \left\lceil \frac{n}{2} \right\rceil + n \left\lceil \frac{m}{3} \right\rceil + m + 1 + m + 1$$
$$< \left\lceil \frac{n}{2} \right\rceil + n \left\lceil \frac{m}{3} \right\rceil + \left\lfloor \frac{n}{2} \right\rfloor + m + 1$$

by Proposition 1

$$= n + n \left\lceil \frac{m}{3} \right\rceil + m + 1$$
$$= n + n \left\lceil \frac{m}{3} \right\rceil + m + 1 = DI'(C_n \odot C_m)_{X_2}.$$

Since $DI'(C_n \odot C_m)_{X_1} < DI'(C_n \odot C_m)_{X_2}$, then

$$DI'(C_n \odot C_m) = DI'(C_n \odot C_m)_{X_1}$$

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$$= \left\lceil \frac{n}{2} \right\rceil + n\left\lceil \frac{m}{3} \right\rceil + 2m + 2.$$

**ii.** If $m+1 > \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$DI'(C_n \odot C_m)_{X_2} < DI'(C_n \odot C_m)_{X_1}$.

It can be proved in a similar way as above. Therefore,

$$DI'(C_n \odot C_m) = DI'(C_n \odot C_m)_{X_2}$$
$$= n + n\left\lceil \frac{m}{3} \right\rceil + m + 1.$$

**iii.** If $m+1 = \left\lfloor \frac{n}{2} \right\rfloor$, then we have

$DI'(C_n \odot C_m)_{X_1} = DI'(C_n \odot C_m)_{X_2}$.

Hence,

$$DI'(C_n \odot C_m) = DI'(C_n \odot C_m)_{X_1}$$
$$= DI'(C_n \odot C_m)_{X_2}.$$

**Theorem** For $n \geq 3$, let $A = \left\lceil \frac{n}{2} \right\rceil + n + 2m + 4$ and $B = 2n + m + 2$. Then,

$DI'\big(C_n \odot K_{1,m}\big)$ is obtained as follows,

$$DI'\big(C_n \odot K_{1,m}\big) = \begin{cases} A & ,if\ \ m+2 < \left\lfloor \frac{n}{2} \right\rfloor, \\ B & ,if\ \ m+2 > \left\lfloor \frac{n}{2} \right\rfloor, \\ A = B, if & m+2 = \left\lfloor \frac{n}{2} \right\rfloor. \end{cases}$$

**Proof** Let $V(C_n) = \{\, v_1,\ v_2,\ \ldots\ ,\ v_n\}$ for cycle graph $C_n$ and for each $i$th copy of $K_{1,m}$ with the vertex set

$V_i(K_{1,m}) = \{u_{1_i}, u_{2_i}, \ldots, u_{m_i}, u_{m+1_i}\}$. ($u_{1_i}$ is central vertex of $i$th copy of $K_{1,m}$) and edge sets be $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1\}$, $E_i(K_{1,m}) = \{u_{1_i}u_{2_i}, \ldots, u_{1_i}u_{m_i}, u_{1_i}u_{m+1_i}\}$.

For $n \geq 3$ we have 2 cases as follows.

**Case 1** Let $X_1$ be an edge dominating set of $C_n \odot K_{1,m}$ and $m(C_n \odot K_{1,m} - X_1) = 2(m+2)$. $X_1$ is obtained as follows.

Let $S_1 \subset E(C_n)$ be $S_1$ an edge dominating set of $C_n$ and $|S_1| = \left\lceil \frac{n}{2} \right\rceil$.

$S_2 = \{v_1u_{1_1}, v_2u_{1_2}, \ldots, v_nu_{1_n}\}$is a minimum edge dominating set of $C_n \odot K_{1,m} - E(C_n)$ and $|S_2| = n$.

$S_1 \cup S_2$ is an edge dominating set of $C_n \odot K_{1,m}$. Therefore, $|X_1| = \left\lceil \frac{n}{2} \right\rceil + n$ and

$m(C_n \odot P_m - X_1) = 2(m+2)$. Thus,

$DI'\big(C_n \odot K_{1,m}\big) \leq |X_1| + m\big(C_n \odot K_{1,m} - X_1\big) = \left\lceil \frac{n}{2} \right\rceil + n + 2m + 4 = DI'(C_n \odot P_m)_{X_1}$.

**Case 2** Let $X_2$ be an edge dominating set of $C_n \odot K_{1,m}$ and $m(C_n \odot K_{1,m} - X_2) = m+2$. $X_2$ is obtained as follows.

Let $S_1' = E(C_n)$. $S_1'$ is an edge dominating set of $C_n$ and $|S_1'| = n$.

$S_2 = \{v_1u_{1_1}, v_2u_{1_2}, \ldots, v_nu_{1_n}\}$is a minimum edge dominating set of $C_n \odot K_{1,m} - E(C_n)$ and $|S_2| = n$.

$X_2 = S_1' \cup S_2$ is an edge dominating set of $C_n \odot K_{1,m}$. Therefore, $|X_2| = n + n = 2n$ and $m\big(C_n \odot K_{1,m} - X_2\big) = m+2$. Thus, $DI'\big(C_n \odot K_{1,m}\big) \leq |X_2| + m\big(C_n \odot K_{1,m} - X_2\big) = 2n + m + 2 = DI'\big(C_n \odot K_{1,m}\big)_{X_2}$.

Because of definition of $DI'$, the relationship between $DI'\big(C_n \odot K_{1,m}\big)_{X_1}$ and $DI'\big(C_n \odot K_{1,m}\big)_{X_2}$ must be examined as follows.

**i.** If $m+2<\left\lfloor \frac{n}{2} \right\rfloor$, then we have

$$DI'\left(C_n \odot K_{1,m}\right)_{X_1} = \left\lceil \frac{n}{2} \right\rceil + n + 2m + 4$$

$$= \left\lceil \frac{n}{2} \right\rceil + n + m + 2 + m + 2$$

$$< \left\lceil \frac{n}{2} \right\rceil + n + \left\lfloor \frac{n}{2} \right\rfloor + m + 2,$$

by Proposition 1,

$$= n + n + m + 2 = 2n + m + 2 = DI'\left(C_n \odot K_{1,m}\right)_{X_2}.$$

Since,

$DI'\left(C_n \odot K_{1,m}\right)_{X_1} < DI'\left(C_n \odot K_{1,m}\right)_{X_2}$, then

$$DI'\left(C_n \odot K_{1,m}\right) = DI'\left(C_n \odot K_{1,m}\right)_{X_1}$$

$$= \left\lceil \frac{n}{2} \right\rceil + n + 2m + 4.$$

**ii.** If $m+2>\left\lfloor \frac{n}{2} \right\rfloor$, then we have $DI'\left(C_n \odot K_{1,m}\right)_{X_2} < DI'\left(C_n \odot K_{1,m}\right)_{X_1}$. It can be proved in a similar way as above. Therefore,

$$DI'\left(C_n \odot K_{1,m}\right) = DI'\left(C_n \odot K_{1,m}\right)_{X_2}$$

$$= 2n + m + 2.$$

**iii.** If $m+2=\left\lfloor \frac{n}{2} \right\rfloor$, then we have

$$DI'\left(C_n \odot K_{1,m}\right)_{X_1} = DI'\left(C_n \odot K_{1,m}\right)_{X_2}.$$

Hence,

$$DI'\left(C_n \odot K_{1,m}\right) = DI'\left(C_n \odot K_{1,m}\right)_{X_1}$$

$$= DI'\left(C_n \odot K_{1,m}\right)_{X_2}.$$

## Conclusion

Edge domination and integrity are important measures for network designers. Domination edge integrity [7, 11] is a new measure which combines these two concepts. In this paper, domination edge integrity of some graphs under corona operation is examined such as $C_n \odot P_m$, $C_n \odot C_m$, $C_n \odot K_{1,n}$ and some results are obtained.

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