Erzincan Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
2020, 13(ÖZEL SAYI-I),7-17

Erzincan University
Journal of Science and Technology
2020, 13(SPECIAL ISSUE -I),7-17

ISSN: 1307-9085, e-ISSN: 2149-4584

DOI: 10.18185/erzifbed.590950

Araştırma Makalesi

Research Article

Smarandache Curves of Spacelike Salkowski Curve with a Spacelike Principal Normal According to Frenet Frame

Süleyman ŞENYURT[1*], Kemal EREN[2]

[1] Faculty of Arts and Sciences, Department of Mathematics, Ordu University, Ordu, TURKEY

[2] Faculty of Arts and Sciences, Department of Mathematics, Sakarya University, Sakarya, TURKEY

Geliş / Received: 11/07/2019, Kabul / Accepted: 20/01/2020

Abstract

In this study, we define the Smarandache curves depending upon the Salkowski curve with a spacelike principal normal according to Frenet frame. Firstly, the curvature, the torsion and Frenet vectors of the Smarandache curves are calculated. Later, we draw graphic of the obtained Smarandache curves and some related results are given.

Keywords: Minkowski space, spacelike Salkowski curve, spacelike Smarandache curve, Frenet frame.

Frenet Çatısına Göre Spacelike Normalli Spacelike Salkowski Eğrisinden Elde Edilen Smarandache Eğrileri

Öz

Bu çalışmada ilk olarak spacelike normalli spacelike Salkowski eğrisinin Frenet vektörlerinden elde edilen regüler Smarandache eğrileri tanımlandı. Daha sonra her bir Smarandache eğrisinin Frenet vektörleri, eğrilik ve torsiyonu hesaplandı. Son olarak elde edilen eğrilerin Frenet elemanları spacelike Salkowski eğrisinin Frenet elemanları cinsinden yazılarak grafikleri çizildi.

Anahtar Kelimeler: Minkowski uzayı, spacelike Salkowski eğri, spacelike Smarandache eğri, Frenet çatı.

1. Introduction

In the years 1844-1923, Salkowski curves are defined as family of curves with constant curvature and non-constant torsion by E. Salkowski [Salkowski, 1909]. In literature, this curve is known as Salkowski curves. The equation of Salkowski curve is given by J. Monterde and he showed that the principal normal vector of this curve makes a constant angle with a constant direction [Monterde, 2009]. M. Turgut, and S.Yılmaz, described the Smarandache curves in Minkowski space [Turgut and Yılmaz, 2008; Turgut and Yılmaz, 2008]. Later, according to the Darboux frame, Bishop frame and Sabban

*CorrespondingAuthor:senyurtsuleyman52@gmail..com

frame, some features of the Smarandache curves are investigated by [Şenyut and Sivas, 2013; Bektaş and Yüce 2013; Çetin, Tuncer and Karacan, 2014; Taşköprü and Tosun, 2014; Ali, 2010; Çalışkan and Şenyurt, 2015]. The definition of timelike and spacelike Salkowski curve have given by Ali Ahmet T [Ali, 2010; Ali, 2009; Ali, 2011]. Şenyurt S. and Eren K. also studied the Smarandache curves obtained from the Frenet vectors of the timelike Salkowski curve [Şenyurt and Eren, 2019; Şenyurt and Eren, 2019].

In this study, we are define the Smarandache curves γ_{TN}, γ_{TB}, γ_{NB} and γ_{TNB} are drawn by unit vector which is obtained from the linear combination of T, N and B vectors of the spacelike Salkowski curve with a spacelike principal normal. The Frenet apparatus of each curve are calculated and the graph of the Smarandache curves is given.

2. Preliminaries

The Minkowski 3-space R_1^3 has Lorentzian inner product given by

$$\langle , \rangle = -dx_1^2 + dx_2^2 + dx_3^2$$

where, $X = (x_1, x_2, x_3) \in R^3$. Also, the vector product of any vectors $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ in R_1^3 is defined by

$$X \wedge Y = -\begin{vmatrix} -i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

For an arbitrary vector $X \in R_1^3$, if $\langle X, X \rangle > 0$ or $X = 0$ then X is spacelike vector, if $\langle X, X \rangle < 0$, then X is timelike vector, if $\langle X, X \rangle = 0$, $X \neq 0$, then X is lightlike (or null) vector. The norm of an arbitrary vector $X \in R_1^3$ is $\|X\| = \sqrt{|\langle X, X \rangle|}$ [O'Neill, 1983]. The Frenet vectors, the curvatures and the Frenet formula of the spacelike curve $\gamma(t)$ are

$$\begin{aligned} & T(t) = \frac{\gamma'(t)}{\|\gamma'(t)\|}, \; B(t) = \frac{\gamma'(t) \wedge \gamma''(t)}{\|\gamma'(t) \wedge \gamma''(t)\|}, \\ & N(t) = B(t) \wedge T(t), \\ & \kappa(t) = \frac{\|\gamma'(t) \wedge \gamma''(t)\|}{\|\gamma'(t)\|^3}, \\ & \tau(t) = \frac{\langle \gamma'(t) \wedge \gamma''(t), \gamma'''(t) \rangle}{\|\gamma'(t) \wedge \gamma''(t)\|^2}, \end{aligned} \tag{2.1}$$

$$T' = \kappa N, \;\; N' = -\kappa T + \tau B, \;\; B' = \tau N, \tag{2.2}$$

respectively where T and N are spacelike vectors and B is timelike vector [Ali, 2009].

Definition 2.1. For an arbitrary $m \in R$ and $m > 1$, let us define the space curve

$$\gamma_m(t) = \frac{n}{4m} \begin{pmatrix} 2\cosh(t) - \dfrac{1+n}{1-2n}\cosh\left((1-2n)t\right) \\ -\dfrac{1-n}{1+2n}\cosh\left((1+2n)t\right), \\ 2\cosh(t) - \dfrac{1+n}{1-2n}\sinh\left((1-2n)t\right) \\ -\dfrac{1-n}{1+2n}\sinh\left((1+2n)t\right), \dfrac{1}{m}\cosh\left(2nt\right) \end{pmatrix} \tag{2.3}$$

where $n = \dfrac{m}{\sqrt{m^2 - 1}}$. This curve is called the spacelike Salkowski curve $\gamma_m(t)$. (Figure 1). The curvature and the torsion of the spacelike Salkowski curve is $\kappa(t) = 1$ and $\tau(t) = \tanh(nt)$. In the study [Ali, 2009] the Frenet frame of the spacelike Salkowski curve is given as following

$$T(t)=\begin{pmatrix} n\cosh(t)\sinh(nt)-\sinh(t)\cosh(nt), \\ n\sinh(t)\sinh(nt)-\cosh(t)\cosh(nt), \\ \frac{n}{m}\sinh(nt) \end{pmatrix},$$

$$N(t)=\frac{n}{m}\left(\cosh(t),\sinh(t),m\right), \qquad (2.4)$$

$$B(t)=\begin{pmatrix} \sinh(t)\sinh(nt)-n\cosh(t)\cosh(nt), \\ \cosh(t)\sinh(nt)-n\sinh(t)\cosh(nt), \\ -\frac{n}{m}\cosh(nt) \end{pmatrix}$$

After this definition, the equation (2.2) becomes

$$T'=N,\ \ N'=-T+\tau B,\ \ B'=\tau N \qquad (2.5)$$

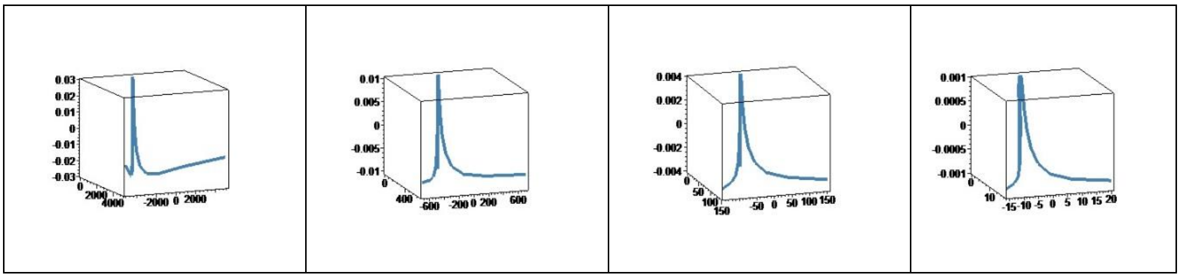

Figure 1. $m=\{3,5,8,16\}$ and $t\in[-5,5]$ for spacelike Salkowski curve

3. Smarandache Curves of Spacelike Salkowski Curve with a Spacelike Principal Normal According to Frenet Frame

In this section, we describe the Smarandache curves of the spacelike Salkowski curve and we calculate Frenet apparatus of the Smarandache curves

Definition 3.1. Let $\gamma_m(t)$ be a spacelike Salkowski curve. The Smarandache curve $\gamma_{TN}(t)$ of the Salkowski curve is defined by frame vectors as follows:

$$\gamma_{TN}(t)=\frac{1}{\sqrt{2}}\left(T(t)+N(t)\right) \qquad (3.1)$$

where, T and N are spacelike vector (Figure 2). Substituting the vectors T and N into the equation (3.1), we get the curve $\gamma_{TN}(t)$ as following:

$$\gamma_{TN}(t)=\frac{1}{\sqrt{2}}\begin{pmatrix} n\cosh(t)\sinh(nt) \\ -\sinh(t)\cosh(nt)+\frac{n}{m}\cosh(t), \\ n\sinh(t)\sinh(nt)- \\ \cosh(t)\cosh(nt)+\frac{n}{m}\sinh(t), \\ \frac{n}{m}\sinh(nt)+n \end{pmatrix} \qquad (3.2)$$

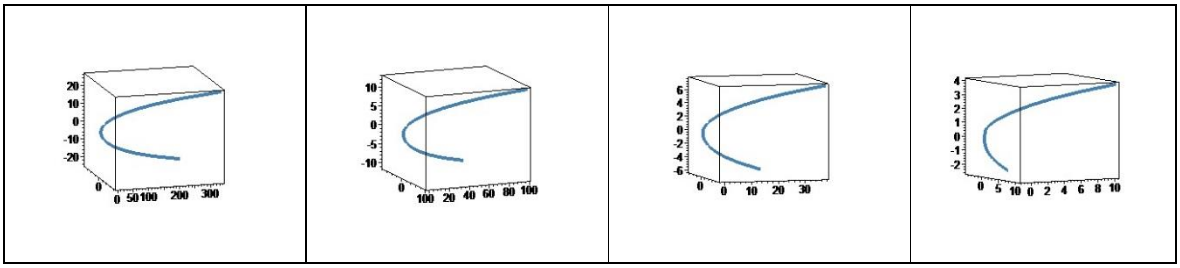

Figure 2. $m=\{3,5,8,16\}$ and $t\in[-5,5]$ for γ_{TN} – Smarandache curve

Theorem 3.1. The Frenet frame $\{T_{TN}, N_{TN}, B_{TN}\}$ of the curve $\gamma_{TN}(t)$ is given

$$T_{TN}=\frac{-T+N+\tau B}{\sqrt{\left|2-\tau^2\right|}},\ \tau\neq\sqrt{2}$$

$$N_{TN}=\frac{\left(\tau^2-\tau\tau'-2\right)T+\left(-\tau^4+3\tau^2+\tau\tau'-2\right)N+\left(-\tau^3+2\tau+2\tau'\right)B}{\sqrt{\left|2-\tau^2\right|\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}},$$

$$\left|2-\tau^2\right|\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|\neq 0$$

$$B_{TN}=\frac{\left(\tau^3-2\tau-\tau'\right)T-\tau'N+\left(2-\tau^2\right)B}{\sqrt{\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}},$$

$$\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\neq 0$$

Proof: Substituting the equation (2.5) into derivative equation of the equation (3.1), we obtain

$$\gamma'_{TN}(t)=\frac{1}{\sqrt{2}}\left(-T+N+\tau B\right) \tag{3.3}$$

If we take the norm in the equation (3.3), then we get

$$\left\|\gamma'_{TN}(t)\right\|=\frac{1}{\sqrt{2}}\sqrt{\left|2-\tau^2\right|}. \tag{3.4}$$

From the equations (3.3) and (3.4), the tangent vector of the curve $\gamma_{TN}(t)$ is found as

$$T_{TN}=\frac{1}{\sqrt{\left|2-\tau^2\right|}}\left(-T+N+\tau B\right),\ \tau\neq\sqrt{2} \tag{3.5}$$

If we take derivative of the equation (3.3), then we get

$$\gamma''_{TN}(t)=\frac{1}{\sqrt{2}}\left(-T+\left(\tau^2-1\right)N+\left(\tau+\tau'\right)B\right) \tag{3.6}$$

From the equations (3.3) and (3.6), we find as

$$\gamma'_{TN}(t)\wedge\gamma''_{TN}(t)=\frac{1}{2}\begin{pmatrix}\left(\tau^3-2\tau-\tau'\right)T\\ -\tau'N+\left(2-\tau^2\right)B\end{pmatrix} \tag{3.7}$$

If we take the norm of the equation (3.7), we get

$$\left\|\gamma'_{TN}(t)\wedge\gamma''_{TN}(t)\right\|=\frac{1}{2}\sqrt{\left|\begin{matrix}\left(\tau^3-2\tau-\tau'\right)^2\\ +\tau'^2-\left(2-\tau^2\right)^4\end{matrix}\right|} \tag{3.8}$$

From the equations (3.7) and (3.8), the binormal vector of the curve $\gamma_{TN}(t)$ is given by

$$B_{TN}=\frac{\left(\tau^3-2\tau-\tau'\right)T-\tau'N+\left(2-\tau^2\right)B}{\sqrt{\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}}, \tag{3.9}$$

$$\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\neq 0.$$

From the equations (3.5) and (3.9), the principal normal vector of the curve $\gamma_{TN}(t)$ is obtained

$$N_{TN}=\frac{\begin{array}{l}\left(\tau^2-\tau\tau'-2\right)T+\left(-\tau^4+3\tau^2+\tau\tau'-2\right)N\\ +\left(-\tau^3+2\tau+2\tau'\right)B\end{array}}{\sqrt{\left|2-\tau^2\right|\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}}, \tag{3.10}$$

$$\left|2-\tau^2\right|\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|\neq 0.$$

Theorem 3.2. The curvature and the torsion of the curve $\gamma_{TN}(t)$ are

$$\kappa_{TN}=\frac{\sqrt{2\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}}{\left(\left|2-\tau^2\right|\right)^{\frac{3}{2}}},$$

$$\tau_{TN}=\frac{\sqrt{2}\left(-\tau^2\tau'+\tau^2\tau''-3\tau\tau'^2-2\tau'-2\tau''\right)}{\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}, \tag{3.11}$$

$$\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|\neq 0,\ \tau\neq\sqrt{2},$$

respectively.

Proof: From the expressions (2.1), the curvature κ_{TN} of the curve $\gamma_{TN}(t)$ can be written

$$\kappa_{TN}=\frac{\sqrt{2\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|}}{\left(\left|2-\tau^2\right|\right)^{\frac{3}{2}}},\ \tau\neq\sqrt{2}\ .$$

If we take the derivative of the equation (3.6), it becomes

$$\gamma'''_{TN}(t)=\frac{1}{\sqrt{2}}\begin{pmatrix}\left(1-\tau^2\right)T\\ +\left(\tau^2+3\tau\tau'-1\right)N\\ +\left(\tau^3-\tau+\tau'+\tau''\right)B\end{pmatrix} \tag{3.12}$$

From the equations (2.1) (3.3), (3.6), (3.8) and (3.12), we get the torsion τ_{TN} of the Smarandahce curve $\gamma_{TN}(t)$ as

$$\tau_{TN}=\frac{\sqrt{2}\left(-\tau^2\tau'+\tau^2\tau''-3\tau\tau'^2-2\tau'-2\tau''\right)}{\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|},$$

$$\left|\left(\tau^3-2\tau-\tau'\right)^2+\tau'^2-\left(2-\tau^2\right)^2\right|\neq 0.$$

Definition 3.2. Let $\gamma_m(t)$ be a spacelike Salkowski curve. The Smarandache curve $\gamma_{TB}(t)$ of the spacelike Salkowski curve is defined by frame vectors as follows:

$$\gamma_{TB}(t)=\frac{1}{\sqrt{2}}\left(T(t)+B(t)\right) \tag{3.13}$$

where, T is spacelike vector B is timelike vector (Figure 3). Substituting the vectors T and B into the equation (2.4), we get the curve $\gamma_{TB}(t)$ as following:

$$\gamma_{TB}(t)=\frac{1}{\sqrt{2}}\begin{pmatrix}n\cosh(t).\sinh(nt)-\sinh(t)\cosh(nt)\\ +\sinh(t)\sinh(nt)-n\cosh(t)\cosh(nt),\\ n\sinh(t)\sinh(nt)-\cosh(t)\cosh(nt)\\ +\cosh(t).\sinh(nt)-n\sinh(t)\cosh(nt),\\ \frac{n}{m}\left(-\cosh(nt)+\sinh(nt)\right)\end{pmatrix} \tag{3.14}$$

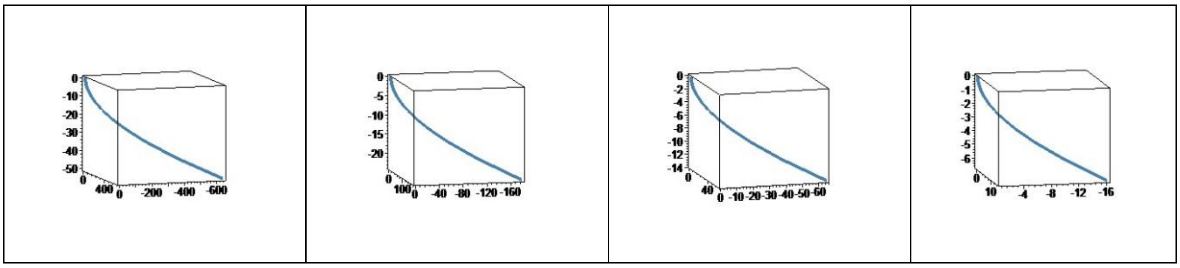

Figure.3. $m=\{3,5,8,16\}$ and $t\in[-5,5]$ for γ_{TB} – Smarandache curve

Theorem 3.3. The Frenet frame $\{T_{TB},N_{TB},B_{TB}\}$ of the curve $\gamma_{TB}(t)$ is given

$$T_{TB}=N\,,\ N_{TB}=\frac{1}{\sqrt{|\tau^2-1|}}(T-\tau B),\ \tau\neq\pm1,$$

$$B_{TB}=\frac{1}{\sqrt{|\tau^2-1|}}(-\tau T+B),\ \tau\neq\pm1.$$

Proof: Substituting the equation (2.5) into derivative equation of the equation (3.13), we find as

$$\gamma'_{TB}(t)=\frac{1}{\sqrt{2}}(1+\tau)N\,. \qquad (3.15)$$

If we take the norm of the equation (3.15), then it becomes

$$\|\gamma'_{TB}(t)\|=\frac{1}{\sqrt{2}}(1+\tau). \qquad (3.16)$$

From the equations (3.15) and (3.16), the tangent vector of curve $\gamma_{TB}(t)$ is found as

$$T_{TN}=N\,. \qquad (3.17)$$

If we take derivate of the equation (3.15), it becomes

$$\gamma''_{TB}(t)=\frac{1}{\sqrt{2}}\left((-1-\tau)T+\tau' N+(\tau+\tau^2)B\right) \ (3.18)$$

From the equations (3.15) and (3.18), we find

$$\gamma'_{TN}(t)\wedge\gamma''_{TN}(t)=\frac{1}{2}\left(-\tau(1+\tau)^2T+(1+\tau)^2B\right).(3.19)$$

If we take the norm of the equation (3.19), this equation becomes

$$\|\gamma'_{TN}(t)\wedge\gamma''_{TN}(t)\|=\frac{1}{2}(1+\tau)^2\sqrt{|\tau^2-1|}\,. \quad (3.20)$$

From the equations (3.19) and (3.20), the binormal vector of curve $\gamma_{TB}(t)$ is found as

$$B_{TB}=\frac{1}{\sqrt{|\tau^2-1|}}(-\tau T+B),\ \tau\neq\pm1. \quad (3.21)$$

Also, from the equations (3.17) and (3.21), the principal normal vector of curve $\gamma_{TB}(t)$ is found as

$$N_{TB}=\frac{1}{\sqrt{|\tau^2-1|}}(T-\tau B),\ \tau\neq\pm1 \qquad (3.22)$$

Theorem 3.4. The curvature and the torsion of the curve $\gamma_{TB}(t)$ are

$$\kappa_{TB}=\frac{\sqrt{2}\sqrt{|\tau^2-1|}}{|1+\tau|}$$
$$\text{and } \tau_{TB}=-\frac{\sqrt{2}\tau'}{|1+\tau|(1+\tau^2)},\ \tau\neq-1, \qquad (3.23)$$

respectively.

Proof: From the expressions (2.1), the curvature κ_{TB} of the curve $\gamma_{TB}(t)$ can be written

$$\kappa_{TB} = \frac{\sqrt{2}\sqrt{\left|\tau^2 - 1\right|}}{\left|1+\tau\right|},\ \tau \neq -1.$$

If we take derivative of the equation (3.18), it becomes

$$\gamma'''_{TB}(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} -2\tau' T \\ +\left(\tau^3 + \tau^2 - \tau - 1 + \tau''\right) N \\ +\left(\tau' + 3\tau\tau'\right) B \end{pmatrix} \quad (3.24)$$

From the equations (3.15), (3.18), (3.20) and (3.24), the torsion of the curve $\gamma_{TB}(t)$ is got

$$\tau_{TB} = -\frac{\sqrt{2}\tau'}{\left|1+\tau\right|\left(1+\tau^2\right)},\ \tau \neq -1.$$

Definition 3.3. Let $\gamma_m(t)$ be a spacelike Salkowski curve. The Smarandache curve $\gamma_{NB}(t)$ of the spacelike Salkowski curve is defined by frame vectors as follows:

$$\gamma_{NB}(t) = \frac{1}{\sqrt{2}}\left(N(t) + B(t)\right), \quad (3.25)$$

where, N is spacelike vector and B is timelike vector (Figure 4). Substituting the vectors N and B into the equation (2.4), we get the curve $\gamma_{NB}(t)$ as following:

$$\gamma_{NB}(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} \sinh(t)\sinh(nt) \\ -n\cosh(t)\cosh(nt) + \dfrac{n}{m}\cosh(t), \\ \cosh(t)\sinh(nt) \\ -n\sinh(t)\cosh(nt) + \dfrac{n}{m}\sinh(t), \\ n - \dfrac{n}{m}\cosh(nt) \end{pmatrix} \quad (3.26)$$

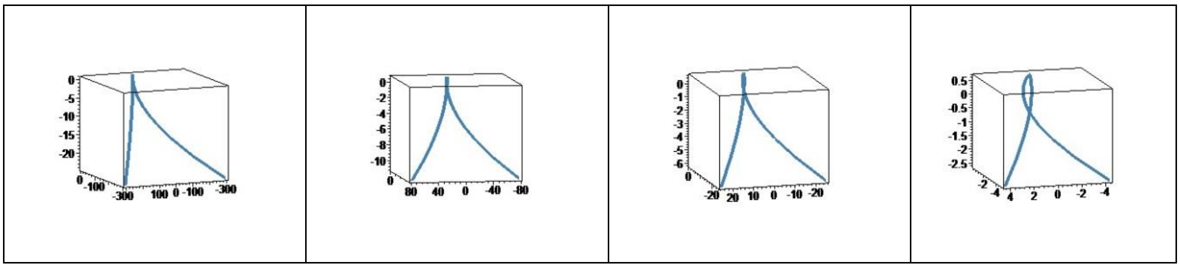

Figure 4. $m = \{3, 5, 8, 16\}$ and $t \in [-5, 5]$ for γ_{NB} – Smarandache curve

Theorem 3.5. The Frenet frame $\{T_{NB}, N_{NB}, B_{NB}\}$ of the curve $\gamma_{NB}(t)$ is

$$T_{NB} = -T + \tau N + \tau B,$$

$$N_{NB} = \frac{1}{\sqrt{\left|\tau^2 + 2\tau' - 1\right|}}\begin{pmatrix} \tau T - \left(\tau^2 + \tau' - 1\right) N \\ -\left(\tau^2 + \tau'\right) B \end{pmatrix},$$

$$B_{NB} = \frac{-\tau T - \tau' N + \left(1 - \tau'\right) B}{\sqrt{\left|\tau^2 + 2\tau' - 1\right|}},$$

$\tau^2 + 2\tau' - 1 \neq 0.$

Proof: if we take derivative of the equation (3.25), we get

$$\gamma'_{NB}(t) = \frac{1}{\sqrt{2}}\left(-T + \tau N + \tau B\right). \quad (3.27)$$

The norm of this equation is found as

$$\|\gamma'_{NB}(t)\| = \frac{1}{\sqrt{2}} \tag{3.28}$$

From the equations (3.27) and (3.28), the tangent vector of the curve $\gamma_{NB}(t)$ is

$$T_{NB} = -T + \tau N + \tau B \tag{3.29}$$

If we take derivative of the equation (3.27), it becomes

$$\gamma''_{NB}(t) = \frac{1}{\sqrt{2}}\left(-\tau T + \left(\tau^2 + \tau' - 1\right)N + \left(\tau' + \tau^2\right)B\right) \tag{3.30}$$

From the equations (3.27) and (3.30), we found

$$\gamma'_{NB}(t) \wedge \gamma''_{NB}(t) = \frac{1}{2}\left(-\tau T - \tau' N + \left(1 - \tau'\right)B\right) \tag{3.31}$$

The norm of the equation (3.31) is

$$\|\gamma'_{NB}(t) \wedge \gamma''_{NB}(t)\| = \frac{1}{2}\sqrt{\left|\tau^2 + \tau'^2 - \left(1 - \tau'\right)^2\right|}\,. \tag{3.32}$$

From the equations (3.31) and (3.32), the binormal vector of the curve $\gamma_{NB}(t)$ is found as

$$B_{NB} = \frac{-\tau T - \tau' N + \left(1 - \tau'\right)B}{\sqrt{\left|\tau^2 + 2\tau' - 1\right|}}, \tag{3.33}$$

$\tau^2 + 2\tau' - 1 \neq 0.$

From the equations (3.29) and (3.33), the principal normal vector of the curve $\gamma_{NB}(t)$ is

$$N_{NB} = \frac{\tau T - \left(\tau^2 + \tau' - 1\right)N - \left(\tau^2 + \tau'\right)B}{\sqrt{\left|\tau^2 + 2\tau' - 1\right|}}, \tag{3.34}$$

$\tau^2 + 2\tau' - 1 \neq 0.$

Theorem 3.6. The curvature and the torsion of the curves $\gamma_{NB}(t)$ are

$$\kappa_{NB} = \sqrt{2}\sqrt{\left|\tau^2 + \tau'^2 - \left(1 - \tau'\right)^2\right|}$$

and ,(3.35)

$$\tau_{NB} = \frac{\sqrt{2}\left(\tau - \tau'\tau - \tau''\right)}{\left|\tau^2 + \tau'^2 - \left(1 - \tau'\right)^2\right|},\ \tau^2 + 2\tau' - 1 \neq 0,$$

respectively.

Proof: From the equations (2.1), (3.28) and (3.32) the curvature of the curve $\gamma_{NB}(t)$ is found as

$$\kappa_{NB} = \sqrt{2}\sqrt{\left|\tau^2 + \tau'^2 - \left(1 - \tau'\right)^2\right|}$$

If we take derivative of the equation (3.30), this equation is found as

$$\gamma'''_{TN}(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} \left(-\tau^2 - 2\tau'\right)T \\ +\left(\tau^3 - \tau + 3\tau\tau' + \tau''\right)N \\ +\left(\tau^3 - \tau + 3\tau\tau' + \tau''\right)B \end{pmatrix}. \tag{3.36}$$

From the equations (3.27), (3.30), (3.32) and (3.36), the torsion of the curve $\gamma_{NB}(t)$ is got

$$\tau_{NB} = \frac{\sqrt{2}\left(\tau - \tau'\tau - \tau''\right)}{\left|\tau^2 + \tau'^2 - \left(1 - \tau'\right)^2\right|},\ \tau^2 + 2\tau' - 1 \neq 0\,.$$

Definition 3.4. Let $\gamma_m(t)$ be a spacelike Salkowski curve. The Smarandache curve γ_{TNB} of the Salkowski curve is defined by frame vectors as follows:

$$\gamma_{TNB}(t) = \frac{1}{\sqrt{3}}\left(T(t) + N(t) + B(t)\right) \tag{3.37}$$

where T is spacelike vector, N is spacelike vector and B is timelike vector (Figure 5). Substituting the vectors T, N and B into the equation (2.4), we get the curve $\gamma_{TNB}(t)$ as following:

$$\gamma_{TNB}(t)=\frac{1}{\sqrt{2}}\begin{pmatrix} n\cosh(t)\sinh(nt)-\sinh(t)\cosh(nt) \\ +\sinh(t)\sinh(nt)-n\cosh(t)\cosh(nt) \\ +\frac{n}{m}\cosh(t), \\ n\sinh(t)\sinh(nt)-\cosh(t)\cosh(nt) \\ +\cosh(t)\sinh(nt)-n\sinh(t)\cosh(nt) \\ +\frac{n}{m}\sinh(t), \\ \frac{n}{m}\sinh(nt)-\frac{n}{m}\cosh(nt)+n \end{pmatrix} \quad (3.38)$$

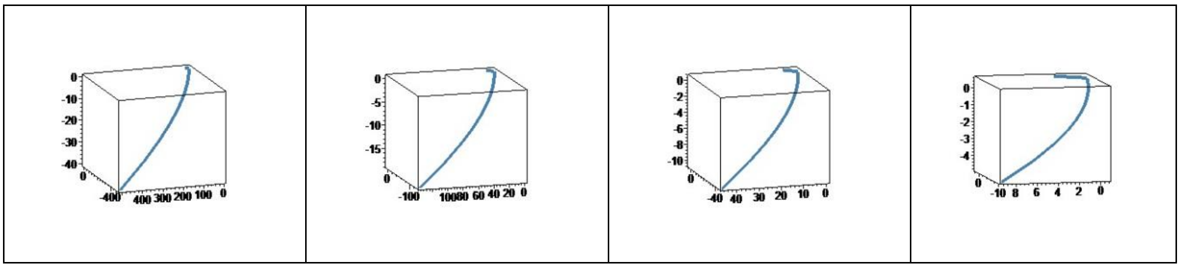

Figure 5. $m=\{3,5,8,16\}$ and $t\in[-5,5]$ for γ_{TNB} – Smarandache curve

Theorem 3.7. The Frenet frame $\{T_{TNB},N_{TNB},B_{TNB}\}$ of the Smarandache curve $\gamma_{TNB}(t)$ is

$$T_{TNB}=\frac{-T+(1+\tau)N+\tau B}{\sqrt{|2+2\tau|}},\ \tau\neq -1,$$

$$N_{TNB}=\frac{\begin{array}{c}\left(2\tau^2+4\tau+2-\tau'\right)T+\left(-3\tau^3-2\tau^2+2\tau+2-\tau\tau'-\tau'\right)N \\ -\left(2\tau^3+4\tau^2+2\tau+\tau\tau'+2\tau'\right)B\end{array}}{\sqrt{|2+2\tau|\left|\left(-2\tau^2-2\tau-\tau'\right)^2+\tau'^2-\left(2\tau+2-\tau'\right)^2\right|}},$$

$$|2+2\tau|\left|\left(-2\tau^2-2\tau-\tau'\right)^2+\tau'^2-\left(2\tau+2-\tau'\right)^2\right|\neq 0,$$

$$B_{TNB}=\frac{-\left(2\tau^2+2\tau+\tau'\right)T-\tau'N+\left(2\tau+2-\tau'\right)B}{\sqrt{\left|\left(-2\tau^2-2\tau-\tau'\right)^2+\tau'^2-\left(2\tau+2-\tau'\right)^2\right|}},$$

$$\left|\left(-2\tau^2-2\tau-\tau'\right)^2+\tau'^2-\left(2\tau+2-\tau'\right)^2\right|\neq 0.$$

Proof: If we take derivative of the equation (3.37), then we get

$$\gamma'_{TNB}(t)=\frac{1}{\sqrt{3}}\left(-T+(1+\tau)N+\tau B\right). \quad (3.39)$$

If we take the norm of this equation, we find

$$\|\gamma'_{TNB}(t)\|=\frac{1}{\sqrt{3}}\sqrt{|2+2\tau|}. \quad (3.40)$$

From the equations (3.39) and (3.40), the tangent vector of the Smarandache curve $\gamma_{TNB}(t)$ is

$$T_{TNB}=\frac{1}{\sqrt{|2+2\tau|}}\left(-T+(1+\tau)N+\tau B\right),\ \tau\neq -1. \quad (3.41)$$

If we take derivative of the equation (3.39), we get

$$\gamma''_{TNB}(t)=\frac{1}{\sqrt{3}}\begin{pmatrix}(-1-\tau)T\\ +(\tau^2-1+\tau')N\\ +(\tau^2+\tau+\tau')B\end{pmatrix} \qquad (3.42)$$

From the equations (3.39) and (3.42), we have

$$\gamma'_{TNB}(t)\wedge\gamma''_{TNB}(t)=\frac{1}{3}\begin{pmatrix}(-2\tau^2-2\tau-\tau')T\\ -\tau'N+(2\tau+2-\tau')B\end{pmatrix} \qquad (3.43)$$

The norm of the equation (3.43) is

$$\|\gamma'_{TNB}(t)\wedge\gamma''_{TNB}(t)\|=\frac{1}{3}\sqrt{\left|\begin{matrix}(-2\tau^2-2\tau-\tau')^2\\ +\tau'^2-(2\tau+2-\tau')^2\end{matrix}\right|} \qquad (3.44)$$

From the equations (3.43) and (3.44), the binormal vector of the curve $\gamma_{TNB}(t)$ is

$$B_{TNB}=\frac{-(2\tau^2+2\tau+\tau')T-\tau'N+(2\tau+2-\tau')B}{\sqrt{\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|}}, \qquad (3.45)$$

$$\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|\neq 0.$$

From the equations (3.41) and (3.45), we obtain the principal normal vector of the curve $\gamma_{TNB}(t)$ as following:

$$N_{TNB}=\frac{(2\tau^2+4\tau+2-\tau')T+\begin{pmatrix}-3\tau^3-2\tau^2+2\tau\\ +2-\tau\tau'-\tau'\end{pmatrix}N-\begin{pmatrix}2\tau^3+4\tau^2+2\tau\\ +\tau\tau'+2\tau'\end{pmatrix}B}{\sqrt{|2+2\tau|\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|}},$$

$$|2+2\tau|\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|\neq 0.$$

$$(3.46)$$

Theorem 3.8. The curvature and the torsion of the curve $\gamma_{TNB}(t)$ are

$$\kappa_{TNB}=\frac{\sqrt{3}\sqrt{\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|}}{(|2+2\tau|)^{\frac{3}{2}}},$$

$$\tau_{TNB}=\frac{\sqrt{3}(-\tau^4-2\tau^2\tau'-4\tau\tau'+3\tau'^2-\tau'-2\tau''-2\tau\tau'')}{\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|}, \qquad (3.47)$$

$$\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|\neq 0,\ \tau\neq -1,$$

respectively.

Proof: From the equations (2.1), (3.40) and (3.44) we get the curvature of the Smarandache curve $\gamma_{TNB}(t)$ as

$$\kappa_{TNB}=\frac{\sqrt{3}\sqrt{\left|(-2\tau^2-2\tau-\tau')^2+\tau'^2-(2\tau+2-\tau')^2\right|}}{(|2+2\tau|)^{\frac{3}{2}}},\ \tau\neq -1.$$

The derivate of the equation (3.42) is

$$\gamma'''_{TN}(t)=\frac{1}{\sqrt{3}}\begin{pmatrix}(1-\tau^2-2\tau')T\\ +(\tau^3+\tau^2-\tau-1+3\tau\tau'+\tau'')N\\ +(\tau^3-\tau+3\tau\tau'+\tau'+\tau'')B\end{pmatrix} \qquad (3.48)$$

From the equations (3.39), (3.42), (3.44) and (3.48), we obtain the torsion of the curve $\gamma_{TNB}(t)$.

Corollary 3.1. The Smarandache curves of spacelike Salkowski curve are spacelike curves with a timelike principal normal.

Corollary 3.2. $\gamma_{TB}-$Smarandache curve is evolute of spacelike Salkowski curve.

Proof: From the equations (2.4) and (3.17), we get $\langle T, T_{TB} \rangle = \langle T, N \rangle = 0$. In that case, we call that γ_{TB} $-$Smarandache curve is evolute of spacelike Salkowski curve.

4. References

Ali, A.T. (2010) Special Smarandache Curves in the Euclidian Space, *International Journal of Mathematical Combinatorics,* 2,30-36.

Ali, A.T. (2010). Timelike Salkowski and anti-Salkowski curves in Minkowski 3- space. *J. Adv. Res. Dyn. Cont. Syst*., 2, 17–26.

Ali, A.T. (2009). Spacelike Salkowski and anti-Salkowski curves with spacelike principal normal in Minkowski 3-space. *Int. J. Open Problems Comp. Math.* 2 451–460.

Ali, A.T. (2011). Spacelike Salkowski and anti-Salkowski curves with timelike principal normal in Minkowski 3-space. *Mathematica Aeterna,* Vol.1, No.04, 201-210.

Bektaş, Ö. and Yüce, S. (2013) Special Smarandache Curves According to Darboux Frame in Euclidean 3-Space, *Romanian Journal of Mathematics and Computer sciencel*, 3(1), 48-59.

Çalışkan, A. and Şenyurt, S. (2015). Smarandache Curves in Terms of Sabban Frame of Spherical Indicatrix Curves, *Gen. Math. Notes,* 31(2),1-15.

Çetin, M. Tuncer, Y. and Karacan, M.K. (2014) Smarandache Curves According to Bishop Frame in Euclidean 3-Space, *Gen. Math. Notes*, 20, 50-66.

Monterde, J. (2009). Salkowski curves revisited: A family of curves with constant curvature and non-constant torsion, *Computer Aided Geometric Design,* 26(3), 271- 278.

O'Neill, B. (1983). Semi-Riemannian Differential Geometry, *Academic Press,* USA.

Salkowski, E. (1909). Zur Transformation von Raumkurven, *Math. Ann*., 66, 517-557.

Şenyurt, S. and Sivas, S. (2013). An Application of Smarandache Curve, *University of Ordu Journal of Science and Technology*, 3(1), 46-60.

Şenyurt, S. and Eren, K. (2019). Smarandache curves of timelike anti-Salkowski curve according to Frenet frame, *Blacksea 1. International Multidisciplinary Scientific Works Congress*, 667-679.

Şenyurt, S. and Eren, K. (2019). Smarandache curves of timelike Salkowski curve according to Frenet frame, *Blacksea 1. International Multidisciplinary Scientific Works Congress,* 680-692.

Taşköprü, K. and Tosun, M. (2014) Smarandache Curves on S^2 , *Boletim da Sociedade Paranaense de Matematica 3 Srie*.,32(1), 51-59.

Turgut, M. and Yılmaz, S. (2008). Smarandache Curves in Minkowski Spacetime, *International J.Math. Combin*., 3, 51-55.

Turgut, M. and Yılmaz, S. (2008) On the Differential Geometry of the curves in Minkowski spacetime I, *Int. J. Contemp. Math. Sci*. 3(27), 1343-1349.