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A Note on Surfaces of Revolution Which Have Lightlike Axes of Revolution in Minkowski Space with Density

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Abstract

In this paper, we study surfaces of revolution which have lightlike axes of revolution in Minkowski space with density. The generating curve of these surfaces satisfies a non-linear second order differential equation which describes the prescribed weighted Gaussian curvature. By solving differential equation, we get surfaces of revolution. Also, we draw a graph of the surface of revolution.
Keywords: Minkowski space, manifold with density, weighted curvature, surfaces of revolution

Yoğunluklu Minkowski Uzayında Lightlike Dönme Eksenli Dönel Yüzeyler Üzerine Bir Not

Öz

Bu çalışmada, yoğunluklu Minkowski uzayında lightlike dönme eksenli dönel yüzeyleri çalıştık. Üzerinde çalıştığımız dönel yüzeylerin üreteç eğrisinin, yüzeyin ağırlıklı Gauss eğriliğinden hareketle elde ettiğimiz ikinci dereceden lineer olmayan diferansiyel denklemin bir çözümü olduğunu gördük. Bu diferansiyel denklemi çözerek dönel yüzeyin denklemini elde ettik. Son olarak da elde ettiğimiz dönel yüzeylerin grafiklerini çizdik.
Anahtar Kelimeler: Minkowski uzayı, yoğunluklu manifold, ağırlıklı eğrilik, dönel yüzeyler.

1. Introduction

It is well known that a surface of revolution is a special form of a helicoidal surface. A surface of revolution is a surface generated by rotating a two dimensional curve about an axis. It has been studied in $\mathbb{R}^3$ as well as in the other spaces with special conditions. Firstly, Delaunay has studied a surface of revolution with nonzero constant mean curvature in $\mathbb{R}^3$ (Delaunay, 1841). Kenmotsu has studied surfaces of revolution with prescribed mean curvature (Kenmotsu, 1980). Then Hsiong et. al have generalized theorem of Delaunay (Hsiang and Yu, 1981). Beneki et. al have studied these surfaces in Minkowski 3−space $\mathbb{R}^3_1$ (Beneki et al., 2002). For more details on ruled surfaces and its applications, see (Athoumane, 2004; Chen et al., 2005; Güler, 2007; Güler and Hacısalihoğlu, 2011)

Recently, the studies in Riemannian manifolds with density have arisen. Above problem is extended to manifols with density. It is well known that a manifold with a positive density function e^{φ} used to weight

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the volume and the hypersurface area. For more details on manifolds with density, see (Hıeu and Hoang, 2009; Morgan, 2016; Morgan, 2005; Morgan, 2009; Morgan, 2006; Rayon and Gromov, 2003; Rosales et al., 2008; Yıldız et al., 2018, Yıldız et al., 2018; Yıldız and Akyiğit, 2019).

In Minkowski 3−space with density e^{φ} the weighted mean curvature is given with

$$H_{\varphi} = H - \frac{1}{2}\langle N, \nabla\varphi\rangle$$

where H is the mean curvature of the surface, N is the unit normal vector of the surface and $\nabla\varphi$ is the gradient vector of φ (Rayon and Gromov, 2003). If $H_{\varphi} = 0$ then the surface is called weighted minimal surface. The weighted Gaussian curvature with density e^{φ} is

$$G_{\varphi} = G - \Delta\varphi$$

where G is the Gaussian curvature of the surface and Δ is the Laplacian operator (Corwin et al., 2006).

In this paper, we study surfaces of revolution in the Minkowski 3−space with density e^{φ}, where $\varphi = -y^2 - z^2$. We construct a surface of revolution with prescribed weighted Gaussian curvature. Then we give examples to illustrate our results

2. Preliminaries

The Minkowski 3−space $\mathbb{R}^3_1$ is the real vector space $\mathbb{R}^3$ provided with the standard flat metric given by

$$ds^2 = dx^2 + dy^2 - dz^2$$

where (x, y, z) is a rectangular coordinate system of $\mathbb{R}^3_1$. Let $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3) \in \mathbb{R}_1^3$, then the vector product in $\mathbb{R}^3_1$ is defined by

$$a \wedge b = \begin{vmatrix} e_1 & e_2 & -e_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

In the Minkowski 3−space the axis of revolution is either spacelike, or timelike or lightlike. So there exist four kind of a surface of revolution. In this study, we examine surfaces of revolution with lightlike axis. In addition, the discriminant of the first fundamental form $-(u-g(u))^2(-1+g'(u)^2)$ is negative (Hano and Nomizu, 1984).

Let γ be a C^2−curve on yz−plane of type $\gamma(u) = (0, u, g(u))$ where $u \in I$ for an open interval $I \subset \mathbb{R} - \{0\}$. By applying the rotation with lightlike axis, we can obtain a surface of revolution M as

$$R(u,v) = \begin{bmatrix} 1 & -v & v \\ v & 1-\frac{v^2}{2} & \frac{v^2}{2} \\ v & -\frac{v^2}{2} & 1+\frac{v^2}{2} \end{bmatrix} \begin{bmatrix} 0 \\ u \\ g(u) \end{bmatrix}. \qquad (1)$$

So the parametric equation can be given in the for

$$R(u,v) = \left((g(u)-u)v, (1-\frac{v^2}{2})u + \frac{v^2}{2}g(u), \frac{v^2}{2}u + (1+\frac{v^2}{2})g(u) \right). \qquad (2)$$

We can calculate the mean curvature H, the Gaussian curvature G and the unit normal of surface as

$$H=\frac{(u-g)^2((-1+g')^2(1+g')+(-u+g)g'')}{2((u-g)^2(-1+g'^2))^{3/2}},$$

$$G=-\frac{g''}{(u-g)(-1+g')(1+g')^2}$$

and

$$N=\frac{1}{\sqrt{w}}\Big(v(u-g)(-1+g'),\ \frac{1}{2}(u-g)(-v^2+(-2+v^2)g'),\ \frac{1}{2}(u-g)(-2-v^2+v^2g')\Big)$$

respectively, where $(u-g)^2(-1+g'^2)>0$ and $w=(u-g)^2(-1+g'^2)$. We assume that M is a surface in $\mathbb{R}^3_1$ with density e^{φ} where $\varphi=-y^2-z^2$. By considering density function, we can calculate the weighted mean curvature H_φ and the weighted Gaussian curvature G_φ as

$$H_\varphi=\Big((u-g)^2((-1+g')^2(1+g'))+2(u-g)(-1+g'^2)((1+v^2)g+v^2(u-g)g'-u(v^2+g'))+(-u+g)g''\Big)/2w^{3/2} \tag{3}$$

and

$$G_\varphi=4-\frac{g''}{(u-g)(-1+g')(1+g')^2} \tag{4}$$

respectively.

3. Surface of Revolutıon with Prescribed Weighted Gaussian Curvature

In this section, we construct surface of revolution with prescribed weighted Gaussian curve

Theorem 3.1. Let $\gamma(u)=(0,u,g(u))$ be a profile curve of the surface of revolution give

$$R(u,v)=\left((g(u)-u)v,(1-\frac{v^2}{2})u+\frac{v^2}{2}g(u),\ \frac{v^2}{2}u+(1+\frac{v^2}{2})g(u)\right)$$

in $\mathbb{R}^3_1$ with density $e^{-y^2-z^2}$ and $G_\varphi(u)$ be the weighted Gaussian curvature. Hence, the differential equation of the surfaces of revolutions in $\mathbb{R}^3_1$ with density $e^{-y^2-z^2}$ is (4), the solution of which is given for some particular functional forms of the weighted Gaussian curvature.

Proof. Let's consider the equation (4). If we apply $g(u)=h(u)+u$ into the equation (4), then we get

$$G_\varphi=\frac{4hh'(2+h')^2+h''}{hh'(2+h')^2}. \tag{5}$$

This is second-order nonlinear differential equation. Analytical solution of the equation can't be obtained easily. So, we approach by some special functional forms of the Gaussian curvature.

First case, we assume that $G_\varphi(u)=0$, then equation (5) takes the form

$$4hh'(2+h')^2+h''=0. \tag{6}$$

Solution of (6) is

$$\frac{h}{2}+\frac{c_1}{8}\ln\left|\frac{1+2c_1h}{1-2c_1h}\right|+u+c_2=0,\quad |2c_1h|<0$$ and $c_1,c_2\in\mathbb{R}$, i.e ,

$$\frac{g+u}{2}+\frac{c_1}{8}\ln\left|\frac{1+2c_1(g-u)}{1-2c_1(g-u)}\right|+c_2=0$$

If $c_1 = 0$ the $g(u) = 2c_2 - u$. So we obtain the parametrization of the surface as follow

$$R(u,v) = \left((2c_2 - u - u)v, (1 - \frac{v^2}{2})u + \frac{v^2}{2}(2c_2 - u), \right.$$
$$\left. \frac{v^2}{2}u + (1 + \frac{v^2}{2})(2c_2 - u) \right)$$

and the figure of the domain

$$\begin{cases} 0 < u < 20 \\ -5 < v < 5 \end{cases}$$

is given in figure 1.

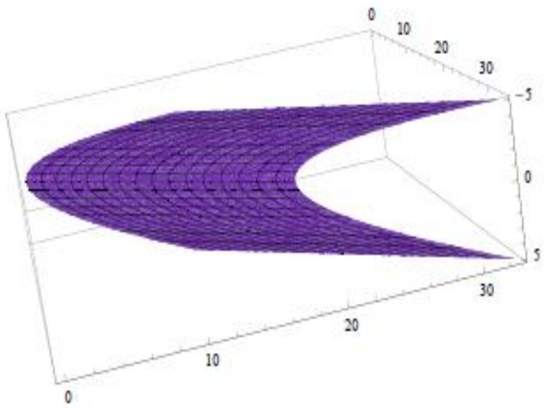

Figure 1

Second case, we assume that $G_\varphi(u) = 4$ which is satisfied by the function $g(u) = (c_1 + 1)u + c_2$, then we obtain the parametrization of the surface as follow

$$R(u,v) = \left((c_1 u + c_2 - u)v, (1 - \frac{v^2}{2})u + \frac{v^2}{2}(c_1 u + c_2), \right.$$
$$\left. \frac{v^2}{2}u + (1 + \frac{v^2}{2})(c_1 u + c_2) \right).$$

We assume that

$$G_\varphi = 4 - \frac{g''}{(u - c_1 u^2 - c_2 u - c_3)(-1 + 2c_1 u + c_2)(1 + 2c_1 u + c_2)^2}$$

which is satisfied by the function $g(u) = c_1 u^2 + (c_2 + 1)u + c_3; c_1, c_2, c_3 \in \mathbb{R}$ then we obtain the parametrization of the surface as follow

$$R(u,v) = \left((c_1 u^2 + (c_2 - 1)u + c_3 - u)v, (1 - \frac{v^2}{2})u + \frac{v^2}{2}g(u), \right.$$
$$\left. \frac{v^2}{2}u + (1 + \frac{v^2}{2})g(u) \right).$$

The figure of the domain

$$\begin{cases} -3 < u < 3 \\ -2 < v < 2 \end{cases}$$

and for $c_1 = c_2 = c_3 = 1$ is given in figure 2

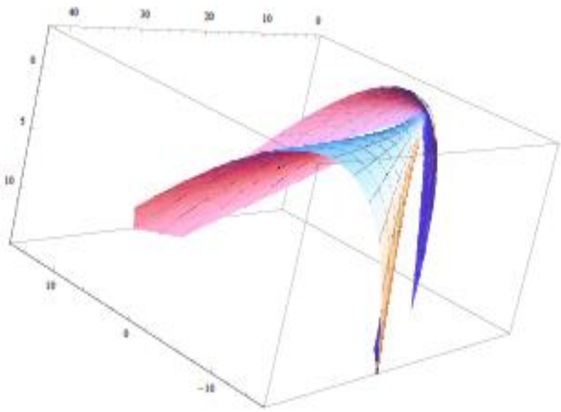

Figure 2

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