

## One Transmitter - Two Receiver Moving Target SAR Imaging

### Bir Gönderici ve Ýki alýcýdan Oluþan Hareketli Hedef SAR Görüntülemesi

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#### Abstract

Synthetic Aperture Radar (SAR) imaging algorithm for the moving targets with some motion irregularities are investigated. For accurate SAR imaging of the moving targets, position and the velocity of the object according to the radar platform must be known accurately. To calculate the instantaneous range and velocity of the object, local radar platform with one transmitter-two receiver (OT-TR) is considered. After the range-bin alignment of the returned signal in each receiver, target's angular position and velocity can be determined by using the phase difference of the receivers. Motion effect free SAR images of moving targets can only be achieved by doing the phase compensation which uses the target's location, direction and speed along the aperture. In this paper, the formulation for rejection of motion irregularities in SAR images is presented and some computer simulations are given.

#### 1. Introduction

Synthetic aperture radar and inverse synthetic aperture radar are well-known techniques for the reconstruction of high resolution images by coherent data processing which is obtained from many different perspective views of an object region, [1]-[5]. The reconstruction of accurate SAR images depends to the correct phase compensation of the measured signals. This compensation can be implemented to the returned signal by using the motion information of the object with respect to the radar. To produce a moving targets' image, a basic SAR system should be modified.

In section 2.1, the basic synthetic aperture system to be modeled is the side looking configuration. Radar transmits a linear FM chirp pulse. Under the physical optics approximation, the return signal can be

expressed as an integral form by means of the location, speed, reflectivity and shape of the target. A set of complex samples of each pulse return are recorded for a range domain processing. For preprocessing, range bins are first aligned according to the correlation feature of the echo envelopes in different pulse returns. The reference envelope is obtained by averaging the exponentially weighted previous envelopes for the tracing of the target. The most strong scatterer is selected as a dominant scatterer. This scatterer is used for phase compensation and angular trajectory computation. In section 2.2, a formulation is given to obtain the moving target's angular position and velocity along the aperture base on the phase difference of the OT-TR system. In the trajectory computation, the dominant point position in range, combining with the phase information gives an accurate measure of position and velocity. In order to eliminate motion-induced phase errors, the phase

information of the dominant point is used for a phase compensation. The coordinate system of the moving target is determined by means of the trajectory computations along the synthetic aperture.

## 2. SAR Imaging System

### 2.1 Basic SAR

Let we consider a two-dimensional SAR imaging system as shown in Fig.1. A radar platform moves along the  $x$ -axis direction with velocity  $\mathbf{v}_p$ . Radar illuminates the target area with spherical radiation the pattern which depends on the radar antenna diameter, frequency and the range of the target area, and corresponding reception at discrete locations for  $x_p \in [-L, L]$ . Radar transmits a linear FM chirp pulse as

$$g(t) = \begin{cases} \exp[j(\mathbf{w} + \mathbf{a}t^2)] & t < |T/2| \\ 0 & t > |T/2| \end{cases} \quad (1)$$

where  $\mathbf{w}$  is the RF carrier frequency and  $2\mathbf{a}$  is the FM rate of the reference waveform  $g(t)$ . For a given radar position  $x_p$ , the roundtrip phase delay of the echoed signal from a point scatterer at  $(x, y)$  is  $2R/c$  (where  $c$  is the speed of the electromagnetic wave and  $R$  is the range of the target with respect to the radar,  $(x - x_p)^2 + y^2)^{1/2}$ . The total received echo signal can be expressed by using the target reflectivity function  $b(x, y, t)$  within the radar investigation area  $\mathbf{A}$  as follows:

$$S(x_p, t) = \iint_{\mathbf{A}} b(x, y, t) g\left(t - \frac{2R}{c}\right) dx dy \quad (2)$$

For stationary targets  $\partial b(x, y, t)/\partial t = 0$ , and one can obtain the unknown reflectivity function by using the Fourier transform relation [6]

$$b(x, y) = F_2^{-1}\{C_q(t)\} \quad (3)$$

where is two-dimensional inverse Fourier

transform and  $C_q(t)$  is the post-processed received signal. Detailed evaluation of the inversion algorithm is described in the following sections.

### 2.2 Moving Target Imaging

Instantaneous knowledge of the target position and motion is generally unknown but is required for SAR imaging. The main problems in moving target SAR imaging are coherent processing for the target motion compensation, the computation of target angular trajectory to determine the angular positions of the target with respect to the radar and to provide tracing information. It is well known that using a single-antenna radar, it is difficult to determine target angular trajectory due to coarse angle resolution. On the other hand, it can be determined by using the subaperture [5] or multi-receiver system. One Transmitter-Two Receiver (OT-TR) angular trajectory and position determining local SAR system is shown in Fig.2.

The primary antenna transmits reference signals, then primary and secondary antennas, which are separated as  $d$ , receive the echoes from the targets. For the tracing of a specific target, range-bin alignment implemented in each receiver by using the iterative envelop correlation method for each pulse return. After the target tracing, dominant scattering center can be selected as a strong scatterer within the range window. This target center is used for coherent demodulation and position determination.

The phase difference of the primary and secondary receivers may be expressed in terms of ranges and wave-length by ignoring small modulating terms as

$$\Delta f(t) = \frac{2p}{I} [R_p(t) - R_s(t)] \quad (4)$$

From Fig.2, the range difference  $[R_p(t)-R_s(t)]$  is formulated via primary range  $R_p(t)$ , distance of the two receivers  $d$  and target position  $\hat{e}_p(t)$  as follows,

$$\begin{aligned} & [R_p(t) - R_s(t)] = \\ & \left[ R_p(t) - \sqrt{R_p^2(t) + d^2 + 2dR_p(t) \cos \mathbf{q}(t)} \right] \end{aligned} \quad (5)$$

By substituting eqn(5) into eqn(4), a relation between the measured phase difference  $\Delta\phi(t)$  and angular position  $\hat{e}(t)$  is obtained. This relation is called the angular trajectory equation. Initial angular position and angular velocity may be extracted from this relation by using the optimization procedures. The range difference between the primary and secondary antennas takes a value between the  $-d < R_p - R_s < d$ . This range difference corresponds to phase difference  $-2\delta d/c < \Delta\phi < 2\delta d/c$  and angular position  $0 < \hat{e} < \delta$ . On the other hand, the wrapped measured phase difference takes place many angular positions on a semi-circle with constant range. Forward and backward arrangements of the wrapped phase difference locations over the pulse returns enable the angular position of the target. After the initial position of the target is determined, target angular velocity can be calculated by using the spectral-domain Fourier analysis or the least-square method.

### 2.3 Imaging Algorithm

Received echo signal from the targets, which contains number of  $N$  distributed points, is the form of

$$S_q(t) = \sum_{n=1}^N b_n g\left(t - \frac{2R_n}{c}\right) \quad (6)$$

where  $b_n$  represent the intensities of the point targets and  $R_n$  is the range of the points with respect to the radar.  $R_n$  can be expressed as

$$R_n = \left| \vec{r}_p + \vec{r}_n \right| \quad (7)$$

where  $\vec{r}_p$  is the location vector of the target coordinate system  $(x,y)$  in the radar coordinate system  $(x,y)$ , and  $\vec{r}_n$  is the location vector of the point scatterer in the coordinate system  $(x,y)$ . We can say that all the point scatterers lie in a circular area which radius is  $A$ , and  $|\vec{r}_n| < A$ . In the radar applications, distance of the target to the radar is much greater than the object size  $A$ , i.e.  $R_p \gg A$ . Taking into account this assumption, the point scatterer range  $R_n$  can be approximately expressed as

$$R_n \cong R_p + \vec{r}_n \cdot \frac{\vec{r}_p}{R_p} \quad (8)$$

where  $\vec{r}_p / R_p$  is the unit vector in the direction of  $\theta$  as shown in Fig.2 and  $\vec{r}_n \cdot \vec{r}_p / R_p$  is the projection of the point scatterer along the  $\theta$  direction. The returned echo signal can be written as the line integral of the projection  $P_\theta(u)$  as

$$S_q(t) = \int_{-A}^A P_q(u) g\left(t - \frac{2(R+u)}{c}\right) du \quad (9)$$

After the range-bin alignment is implemented, and trajectory and position are calculated, the received signal is coherently demodulated by rising the shifted reference waveform  $g^*(t - 2R_p/c)$ . Then it is applied to the low pass filter and the output signal will be

$$C_q(t) = S_q(t) \times g^*(t - 2R_p/c) \quad (10)$$

or equivalently,

$$C_q(t) = \int_{-A}^A P_q(u) e^{-j(2\mathbf{w}+4\mathbf{a}t)u/c} du \quad (11)$$

By writing  $(2\mathbf{w}+4\mathbf{a}t)/c = W_u$ , eqn(11) can be given as

$$C_q(t) = \int_{-A}^A P_q(u) e^{-jW_u u} du \quad (12)$$

The Fourier transform of the projection  $P_\theta(t)$  is defined as

$$\overline{P}_q(W_u) = \int_{-A}^A P_q(u) e^{-jW_u u} du \quad (13)$$

By substituting equation(13) into equation(12), the relation between the Fourier transform of the projection of the reconstructed object and the demodulated received echo signal is found as

$$\overline{P}_q(W_u) = C_q(t) \quad (14)$$

According to the Fourier projection-slice theorem [7], unknown object reflectivity density function  $b(x, y)$  can be reconstructed by using the two dimensional inverse Fourier transform of the demodulated echo signal  $C_\theta(t)$ . However, received echo signal is only available for

$$-\frac{T}{2} + \frac{2(R_p - A)}{c} \leq t \leq \frac{T}{2} + \frac{2(R_p + A)}{c} \quad (15)$$

So, two dimensional Fourier data of the reconstructed object is obtained for the different angles,  $\theta$ , which are the radar viewing angles of the target along the synthetic aperture. The inner and outer radii,  $W_1$  and  $W_2$  are proportional to the radar center frequency, FM modulation rate and pulse duration. By substituting lower and upper limits of the time interval in eqn(15) into the  $W_u = (2\mathbf{w}+4\mathbf{a}t)/c$ , (11) one can obtain the minimum  $W_1$  and maximum  $W_2$  Fourier domain radii as

$$\begin{aligned} W_1 &= \left( \frac{2\mathbf{w}}{c} - \frac{2\mathbf{a}T}{c} + \frac{8\mathbf{a}(R-A)}{c^2} \right) \\ W_2 &= \left( \frac{2\mathbf{w}}{c} + \frac{2\mathbf{a}T}{c} + \frac{8\mathbf{a}(R+A)}{c^2} \right) \end{aligned} \quad (16)$$

The aperture length, sample-spacing, operation frequency, pulse duration and the frequency modulation rate of FM chirp-pulse are effective on the resolution.

### 3. Numerical examples

In this section, we give results obtained from a computer simulation using the imaging algorithm explained above. In this computer simulation, we choose a target moving along the line given by the equations,

$$\begin{aligned} \overline{x}(t) &= x(0) + \vec{v} \cdot \vec{e}_x \cdot t \cdot \sin(0.08)t - 130 \sin 4t \\ \overline{y}(t) &= y(0) + \vec{v} \cdot \vec{e}_y \cdot t - 170 \cos 9t \end{aligned} \quad (17)$$

and the location of the target coordinate system is shown in Fig.3. The target contains 9 point scatterers as given in Fig.4. Numerical simulation parameters of the target and SAR system are given in Table.1.

Table 1. Numerical Simulation Parameters

T a r g e t		9 point scatterers
	Velocity	$v = -150 e_x + 200 e_y$
	$x(t=0\text{sec})$	8660m
	$y(t=0\text{sec})$	5000m
R a d a r		OT-TR system
	Frequency	$F=3.0\text{GHz}$
	Modulation	$\alpha=500\text{MHz/s}$
	Direction	+x-axis
	OT-TR distance	$D=5\text{m}$
	Aperture	$2L=10\text{km}$
	$X_p(t=0\text{sec})$	-5000m
	$Y_p(t=0\text{sec})$	5000m

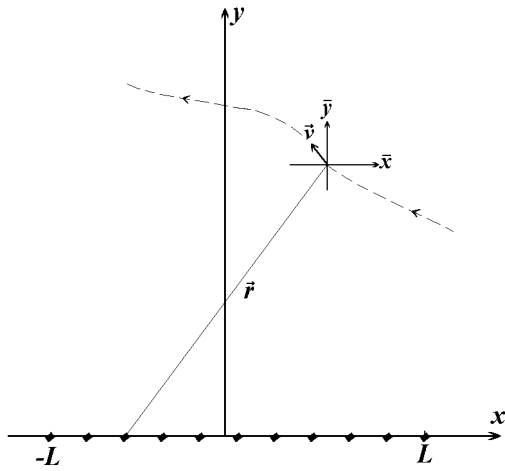
The location of the frequency space data, which is coherently processed, along the aperture is given in Fig.5. Two dimensional inverse Fourier transform of the data given in Fig.5, forms the image of the target, as shown in Fig.6.

#### 4. Conclusion

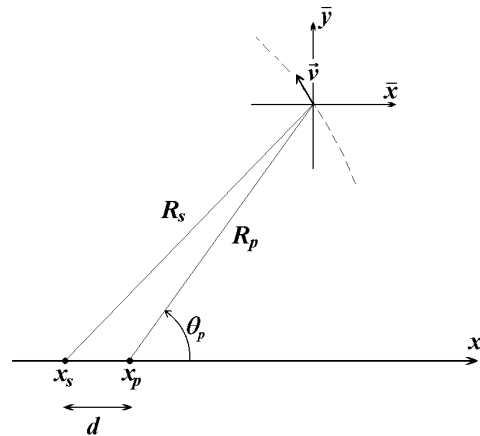
An algorithm has been presented for moving target imaging by using the target angular trajectory computation. To solve the angular trajectory, one transmitter-two receiver local SAR system is considered. Angular position of the target was calculated using the phase difference of the received signals. Two dimensional image of the target was obtained using the Fourier-Slice theorem based on the calculated angular position. As a result, we conclude that, phase compensation and increasing the bandwidth of the transmitted signal improve the obtained image quality of moving targets.

#### References

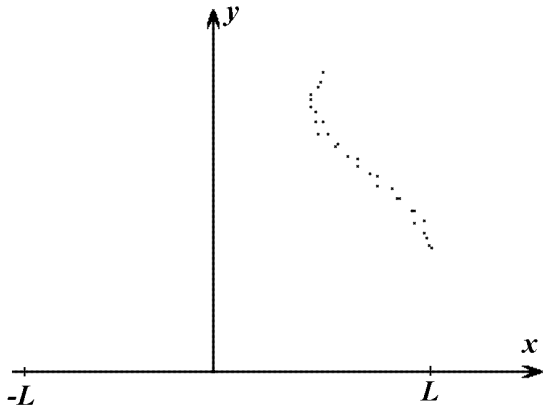
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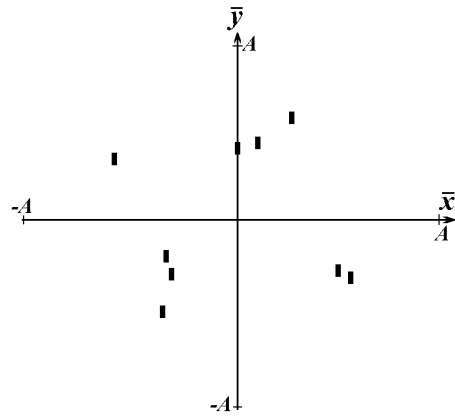
Figure\_1 The geometry of the problem.



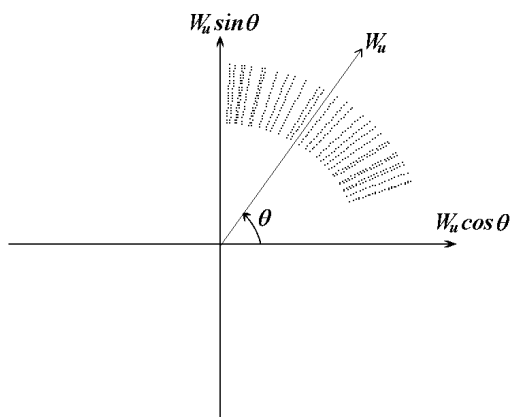
Figure\_2 The local geometry of the SAR platform



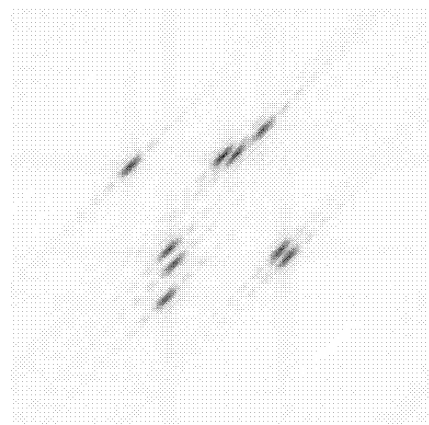
Figure\_3 Target coordinate system (x,y) locations.



Figure\_4 9-point scatterers A=15m



Figure\_5 Frequency space data of the processed data.



Figure\_6 SAR image