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PERFORMANCE OF TURBO CODED SIGNALS OVER PARTIAL *RESPONSE FADING CHANNELS WITH IMPERFECT PHASE REFERENCE*

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SUMMARY

In this paper, the performance of turbo coded signals are investigated over a new channel model, denoted as $1\pm D^n$ */ Partial Response Fading Channels (PRFC) with imperfect phase reference. The combined effects of the* $1\pm D^n$ */ PRFC and nonideal coherent receiver on the phase of the received amplitude and of a noisy carrier reference are considered, each modelled by the Rician and Tikhonov distributions respectively. As an example, the jitter performance of turbo coded signals are evaluated over* $1+D$ */ PRSC channel with different fading parameter K, effective signal-to-noise ratio in the carrier tracking loop* α*, iteration number and data block size N. The numerical results clearly demonstrate the error performance degradation due to both amplitude fading and phase noise process.*

KEY WORDS: *Turbo codes; Partial Response Fading Channel; Imperfect Phase Reference*

1. INTRODUCTION

Turbo codes are a new a class of error correction codes that were introduced a long with a practical decoding algorithm in [1]. The importance of turbo codes is that they enable reliable communications with power efficiencies close to the theoretical limit predicted by Claude Shannon [2] . Since their introduction, turbo codes have been proposed for low-power applications such as deep-space and satellite communications, as well as for interference limited applications such as third generation cellular and personal communication services.

For wireless applications on fading channels, channel coding is an important tool for improving communication reliability. Coding theorists have great problems in developing codes which have a sufficent performance in fading channels. The challenge to find pratical decoders for large codes has not been considered until turbo codes which was announced by Berrou and et'al in 1993 [1]. The performance of this new code was near the Shannon-limit error correction with relatively simple component codes and large interleavers.

Turbo codes represent a more recent development in the coding research field, which has risen a large interest in the coding community. They are *parallel concatenated convalutional codes* (PCCC) whose encoder is formed by two (or more) costituent systematic encoders joined through one or more interleavers. The input information bits feed the first encoder and, after having been scrambled by the interleaver , they enter the second encoder. A code word of a parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders.

On partial response channels, such as the digital magnetic recording channel, convolutional coding techniques were often proposed. Wolf and Ungerboeck[3] examined the channel with (1-D) in AWGN case. Their system uses a binary convolutional code, with a 2^n -state trellis and a good free Hamming distance, followed by a channel precoder. Mohammed Siala and Ghassan K. Kaleh[4] , concentrated on a class of convolutional codes which the maximum likelihood decoder, matched to the encoder, precoder and the channel has the same trellis structure as the encoder. Thus, doubling the number of states due to the channel memory is avoided.

In this paper, performance of turbo codes over $1 \pm D^n$ / Partial Response Fading Channels with no channel state information (CSI) are investigated. We consider turbo codes for improving the reliability of data transmission over the binary precoded noisy ($1+D^n$) Partial Response Fading Channels with imperfect phase reference. Here the combined effects of the fading, and a nonideal coherent receiver, on the phase of the received signal will be taken into account. In the presence of Partial Response Rician Fading Channel with phase jitter, the performance of turbo coded signals are evaluated for fading parameter K, effective signal-to-noise ratio in the carrier tracking loop a, iteration number, data block size N and signal-to-noise ratio SNR.

The paper is organised as; in Section 2, our proposed system model is explained, in Section 3, Turbo Decoder is given and at the last Section, performance of turbo coded signals over 1+D/PRFC with / without channel state information is simulated for different parameters. It is shown that there causes bit error performance degredation due to phase jitter at all SNR, K values.

2. SYSTEM MODEL

Our considered system is composed of a new turbo encoder structure including pre-coder and $1+D^n$ / PRSC channel equivalence, followed by a Rician fading environment without channel state information (CSI) and turbo decoder (Figure 1-4). The rapid advances in digital information processing and transmission technologies have led to an increase in studies on storage channel models. Partial Response Fading Channel (PRFC) acts in a similar way to Partial Response Signalling (PRS) with a Rician Fading probability density function.

The block diagram for a turbo coded system operating over imperfect phase reference and fading are shown in Figure 4. Then we have the channel output u_k is

$$u_k = a_k.x_k.e^{j\theta k} + n_k. \qquad (1)$$

n_k is Gaussian Noise where the noise variance $s^2=N_0/2E_s$, a_k is fading amplitude and the term $e^{j\theta k}$ is a unit vector where θ_k represents the phase noise as mentioned in[5,6] for the signaling interval k which is assumed to have the Tikhonov distribution given by

$$P(\theta_k) = \frac{\exp(\alpha\cos(\theta_k))}{2\pi I_0(\alpha)} \quad |\theta_k| < \pi \qquad (2)$$

Here α is effective signal-to-noise ratio at the receiver tracking loop in dB and $I_0(\alpha)$ is the zero order modified Bessel function.

Here, we focus on the performance of turbo coding over 1+D / PRSC, but the results can easily be enlarged to general storage channel models. In this scheme, owing to the catostrophic nature of the partial response channel structure, it is necessary to place an appropriate precoder (Figure 5). Generally we assume that at the k^{th} coding step, the information bit d_k, takes $\{0,1\}$ values with equal probability. These values are mapped to $\{\pm 1\}$ by using QPSK modulator. Furthermore at the output of 1+D / PRSC , $\{\pm 1\}$ values are mapped to $\{0,\pm 2\}$ as shown in Table III. When no precoder is used, at the coding step k, if the input of the channel model is $x'^{(0)}_k$ and alters from 0 to 1 or 1 to 0, channel output $x^{(0)}_k$ takes value of 0, and if $x'^{(0)}_{k-1}$ is 1 and $x'^{(0)}_k$ is 1, $x^{(0)}_k$ takes value +2 and finally if $x'^{(0)}_{k-1}$ is 0 and $x'^{(0)}_k$ is 0, $x^{(0)}_k$ takes value –2 and these states are shown in Table I. A suitable pre-coder is replaced to solve error propagation problems of partial response channels. The pre-coder illustrated in Figure 5 is for 1+D/PRFC case and relation between the channel output and the input bit is shown in Table II.

We prefer Recursive Systematic Convolutional (RSC) encoder model shown in Figure 6. We will give more information about RSC in the following subsection. After applying pre-coder and 1+D/PRFC channel model for all lines, the transmitter block diagram of our proposed model can be drawn as as Figure 7.

2.1. *The Recursive Systematic Convolutinal (RSC) Encoders*

In this section, we give general information about Recursive Systematic Convolutional (RSC) codes which we will be used in our paper . Consider a half-rate RSC encoder with M memory size. If the d_k is an input at time k the output X_k is equal to

$$X_k = d_k \qquad (3)$$

Remainder r(D) can be found using feedback polynomal $g^{(0)}(D)$ and feedforward polynomal is $g^{(1)}(D)$. The feedback variable is

$$r_i = d_k + \sum_{j=1}^{K} r_{k-j} g_j^{(0)} \qquad (4)$$

and RSC encoder output Y_k which called parity data [7] , is

$$Y_k = \sum_{j=0}^{K} r_{k-j} g_j^{(1)} \qquad (5)$$

RSC encoder with memory M=2 and rate R=1/2 which feedback polynomal $g^{(0)}=7$ and feedforward polynomal $g^{(1)}=5$ is illustrated in Figure 6 and it has a generator matrix

$$G(D) = \begin{bmatrix} 1 & \dfrac{1+D+D^2}{1+D^2} \end{bmatrix} \qquad (6)$$

3. TURBO DECODING

The problem of estimating the state sequence of a Markov process observed through noise has two well known trellis-based solutions- Viterbi algorithm[8] (VA) and the symbol-by-symbol maximum a posteriori (MAP) algorithm. The key difference between algorithms is that the states estimated by the VA must form a connected path through the trellis, while the states estimated by the MAP algorithm need not to be connected. When applied to digital transmission systems, the VA minimizes the frame error rate (FER), while the MAP algorithm minimizes the bit error rate (BER).

The problem of decoding turbo codes involves the joint estimation of two Markov processes, one for each constituent code. While in theory it is possible to model a turbo code as a single Markov process, such a representation is extremely complex and does not lend itself to computationally tractable decoding algorithms. Turbo decoding proceeds instead by first independently estimating the individual Markov processes. Because the two Markov processes are defined by the same set of data, the estimates can be refined by sharing information between the two decoders in an iterative fashion. More specifically, the output of one decoder can be used as a priori information by the other decoder (Figure 3). If the outputs of the individual decoders are in the form of hard-bit decisions, then there is little advantage to sharing information. However, if soft-bit decisions are produced by the individual decoders, considerable performance gains can be achieved by executing multiple iterations of decoding[6] .

3.1 Soft-Input, Soft Output (SISO) Decoding

In the symbol-by-symbol MAP decoder, the output is given by

$$\Lambda_k = \ln \frac{P[m_k = 1|\mathbf{y}]}{P[m_k = 0|\mathbf{y}]} \qquad (7)$$

m is message bit and **y** is received sequence. There are three inputs for Soft-Input,Soft-Output (SISO) decoder. $y_k^{(s)}$ is the systematic observation, $y_k^{(p)}$ is the parity information and z_k is the priori information which is derived from the other decoder's output. The log-likelihood at the output of a SISO decoder using the channel model can be factored into three terms as[6]

$$\Lambda_k = \frac{4a_k^{(s)}\cos(q_k^{(s)})E_s}{N_0} y_k^{(s)} + z_k + l_k \qquad (8)$$

where the term l_k is called the extrinsic information. a_k is fading amplitude, E_s is the energy per code symbol and N_0 is the noise power. The priori information at the input of one decoder is found by subtracting two values from its output to prevent "positive feedback problem" as shown in Figure 6 of M.C. Valenti[6] study. First of all we make the following notation;

$$L_c = \frac{4a_k\cos(q_k)E_s}{N_0} \qquad (9)$$

which is called the reliability of the channel, and

$$l_k = \Lambda_k - L_c^{(s)} y_k^{(s)} - z_k \qquad (10)$$

The converter in the receiver, changes the received bits domain from {2,0, –2} to {1, –1} with mathematical computation which is given bellow

$$y_k = -\text{abs}(u_k) + 1 \qquad (11)$$

It is clear to understand that the BER will increase by the nature of the PRFC model. All the probable observation bits **y** are lesser than 1 and there are no bits higher than 1 because of the transformation of u_k. Figure 8a. shows the observation **y** for without PRFC and Figure 8b. shows the observation **y** for PRFC.

The MAP algorithm attempts to find the most likely individual state s_i given **y**

$$\hat{s}_k = \arg\left\{\max_{s_k} \; P[s_k|\mathbf{y}]\right\} \qquad (12)$$

Befor finding the a posteriori probabilities (APPs) for the messege bits, the MAP algorithm finds the probability of $P[s_k \rightarrow s_{k+1}|\mathbf{y}]$ of each valid state transition given the noisy channel obsevation **y**.

$$P[s_k \rightarrow s_{k+1}|\mathbf{y}] = \frac{P[s_k \rightarrow s_{k+1}, \mathbf{y}]}{P[\mathbf{y}]} \qquad (13)$$

3.2. Log-MAP Algorithm

The maximum a posteriori (MAP) algorithm can calculate the a posteriori probability of each bit with a perfect performance. However there are known problems as the number of calculation depending on the memory states. These problems can be solved by performing the entire algorithm in the log domain[9,10]. To illustrate how performed in the log domain, consider the *Jacobian Logarithm* :

$$\ln(e^x+e^y) = \max(x,y) + \ln(1 + \exp\{-|y-x|\}) \qquad (14)$$
$$= \max(x,y) + f_c(|x-y|)$$

this equation discribes the log-MAP algorithm with a correction function f_c.

Let $\bar{g}(s_k \rightarrow s_{k+1})$ denoted the natural logarithm of $g(s_k \rightarrow s_{k+1})$

$$\bar{g}(s_k \rightarrow s_{k+1}) = \ln g(s_k \rightarrow s_{k+1}) \qquad (15)$$
$$= \ln P[m_k] + \ln P[y_k|x_k] \qquad (16)$$

and

$$\ln P[m_k] = z_k m_k - \ln(1 + e^{z_k}) \qquad (17)$$

and the Equation 17 becomes

$$\bar{g}(s_k \rightarrow s_{k+1}) = \ln P[m_k] - \frac{1}{2}\ln(pN_0/E_s) - \frac{E_s}{N_0}\sum_{q=0}^{n-1}\left[y_k^{(q)} - (-abs(a_k^{(q)}x_k^{(q)}\cos(q_k^{(q)})+1)\right]^2$$

$$= l(s_k \rightarrow s_{k+1}) \qquad (18)$$

Now let $\bar{a}(s_k)$ be the natural logarithm of $a(s_k)$,

$$\bar{a}(s_k) = \ln a(s_k)$$

$$= \ln\left\{\sum_{s_{k-1}\in A} \exp[\bar{a}(s_{k-1}) + \bar{g}(s_{k-1} \rightarrow s_k)]\right\} \qquad (20)$$

$$= \max_{s_{k-1}\in A} * [\bar{a}(s_{k-1}) + \bar{g}(s_{k-1} \rightarrow s_k)] \qquad (21)$$

where A is the set of states s_{k-1} that are connected to the state s_k.

Now let $\bar{b}(s_k)$ be the natural logarithm of $b(s_k)$,

$$\bar{b}(s_k) = \ln\, b(s_k) \qquad (22)$$

$$= \ln\left\{ \sum_{s_{k+1}\in B} \exp\left[\bar{b}(s_{k+1}) + \bar{g}(s_k \rightarrow s_{k+1})\right]\right\} \qquad (23)$$

$$= \max_{s_{k+1}\in B} * \left[\bar{b}(s_{k+1}) + \bar{g}(s_k \rightarrow s_{k+1})\right] \qquad (24)$$

where *B* is the set of statets s_{k+1} that are connected to state s_k , and we can calculate the Log Likelihood Ratio (LLR) by using

$$\Lambda_k = \ln \frac{\sum_{S_1} \exp\left[\bar{a}(s_k) + \bar{g}(s_k \rightarrow s_{k+1}) + \bar{b}(s_{k+1})\right]}{\sum_{S_0} \exp\left[\bar{a}(s_k) + \bar{g}(s_k \rightarrow s_{k+1}) + \bar{b}(s_{k+1})\right]}$$

(25)

where $S_1 = \{s_k \rightarrow s_{k+1} : m_k = 1\}$ is the set of all state transitions associated with a message bit of 1, and $S_0 = \{s_k \rightarrow s_{k+1} : m_k = 0\}$ is the set of all state transitions associated with a message bit of 0.
At the last iteration we make the hard decision by using the second decoder output $\Lambda^{(2)}$,

$$\hat{m}_k = \begin{cases} 1 & if\ \Lambda^{(2)} \geq 0 \\ 0 & if\ \Lambda^{(2)} < 0 \end{cases} \qquad (26)$$

4. PERFORMANCE OF TURBO CODED SIGNALS OVER 1+D / PRFC WITH IMPERFECT PHASE REFERENCE

In this section, the performance of turbo coded signals are evaluated over $1+D$ / PRSC channel with different fading parameter K, effective signal-to-noise ratio in the carrier tracking loop a, iteration number and data block size N.

In our example, ½ rate turbo encoder with channel model is investigated as shown in Figure 7. Here the generator matrix is g=[111:101], a random interleaver is used and the frame size N=400. The bit error performance in $1+D$ / PRSC with ideal CSI for K=∞, 20, 10 and 0 [dB] are compared for various iteration numbers (Figure 9a-9e). In Figure 9e, for the 5th iteration, with ideal CSI, the performance is simulated for K values. It is clear that for a constant iteration number, as K increases performance improves for the same SNR values.

Jitter effect of the channel is simulated for various numbers of iterations, different SNR and K values (Figure 10a-10e). In these curves SNR and effective signal-to-noise ratio in the carrier tracking loop a varies simultanously. The x-axis is showing the simultanous change of E_b/N_o (SNR) with a both in dB, while y-axis is the bit error performance (BER). In Figure 10e, for the 5th iteration, with no CSI, the performance is simulated for K values. It is clear that for a constant iteration number, as K increases performance improves for the same SNR and a values. When the performance results obtained in Figures 9 (the ideal CSI) with Figures 10 (no CSI), the degradation of error performance due to phase distortion can easily be seen for all SNR , K values.

To emphasize the importance of imperfect phase effect, the performance of the considered scheme is simulated with various a values and K as shown in Figures (11a-11b) . For a=10 dB and K=∞, for the same SNR value, as iteration number increases, performance gets better (Figure 11a). The similar results are obtained for a=20 dB, K= ∞ case (Figure 11b). The bit performance of the one with greater a (Figure 11b) is clearly better than the other case (Figure 11a) at all SNR and iteration number, showing the effect of jitter. The similar results are obtained for K= 20 dB and K=0 dB values (Figure 12-14). It is clear that phase jitter distortion is effective for severe fading (K=0 dB) and also Rician fading.

In the last group of Figures (15a-15d), for constant K values, and for the 5th iterations, we only change the a parameter (From 0 dB to ∞ dB) for different SNR values. The interesting point, here is the effect of a parameter on bit error performance. Jitter effect becomes more effective at higher SNR values for all K values.

5. CONCLUSION

In this paper we have shown how turbo codes can be adapted to $1+D^n$ / Partial Response Fading Channels with no channel state information (CSI). As an example, the jitter performance of turbo coded signals are simulated over $1+D/$PRSC channel with different fading parameter K, effective signal-to-noise ratio in the carrier tracking loop α, iteration number and data block size N.

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LIST OF TABLES and FIGURES:

Table I. Channel outputs without pre-coder

$x'^{(0)}_{k-1}$ to $x'^{(0)}_k$	1 to 0 or 0 to 1	1 to 1	0 to 0
$x_k^{(0)}$	0	+2	-2

Table II. Channel outputs with pre-coder

d_k	0	1
$x_k^{(0)}$	+2 or –2	0

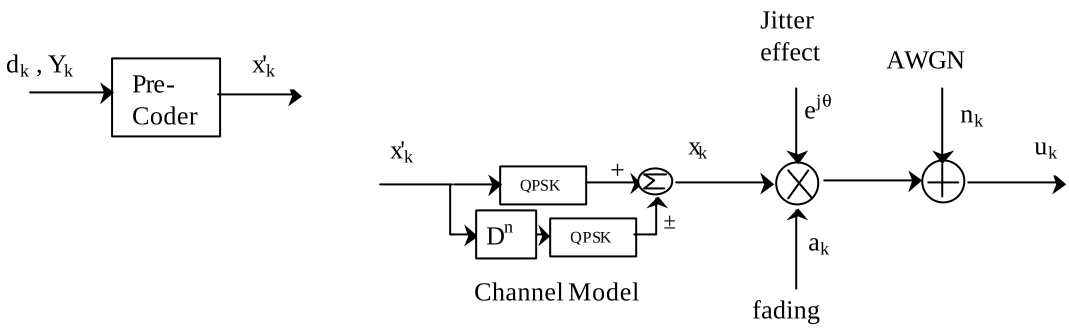

Figure 1. Pre-coder and $1 \pm D^n$ / PRFC Model

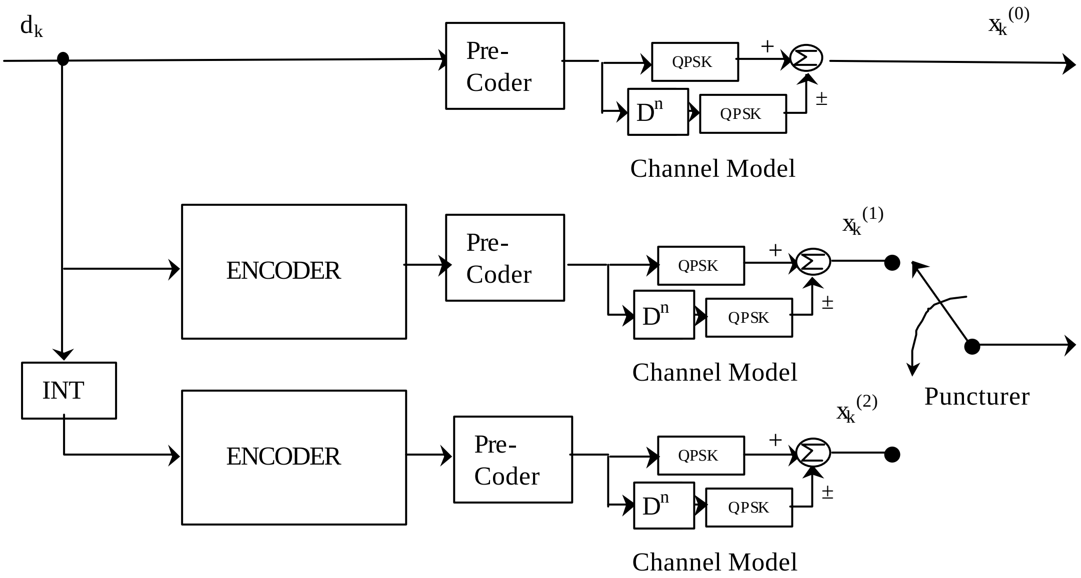

Figure 2. Turbo Encoder for $1 \pm D^n$ / PRFC Model

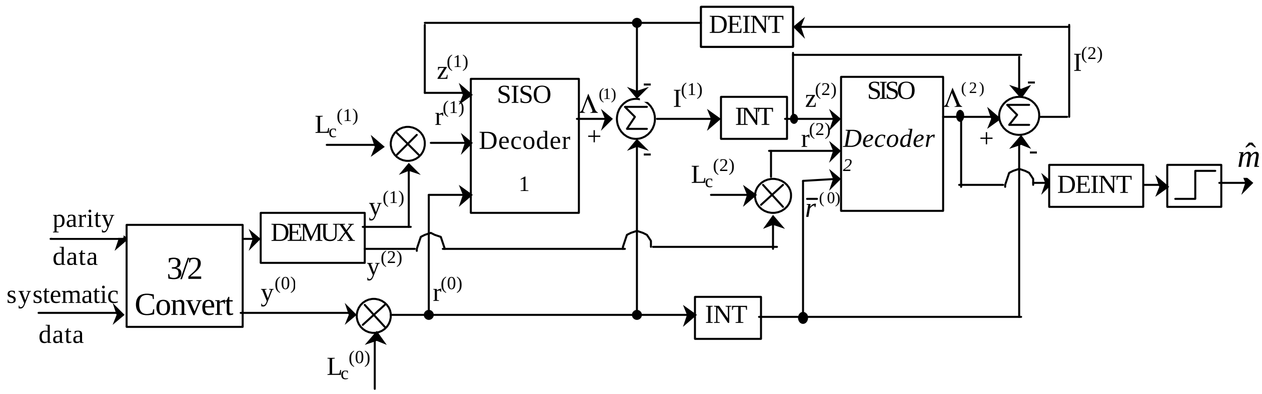

Figure 3. Turbo Decoder for $1 \pm D^n$ / PRFC Model

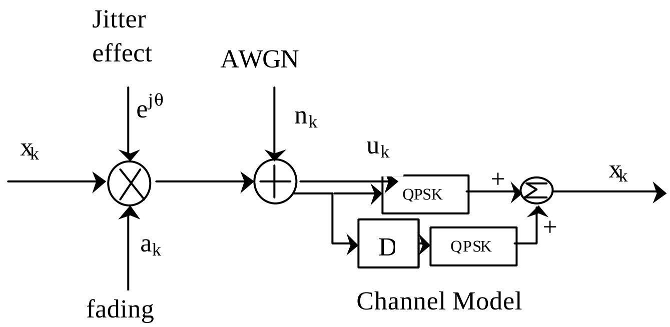

Figure 4. Fading channel with imperfect phase reference

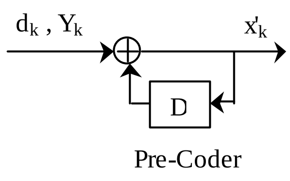

Figure 5. Pre-coder and 1+D/PRFC Model

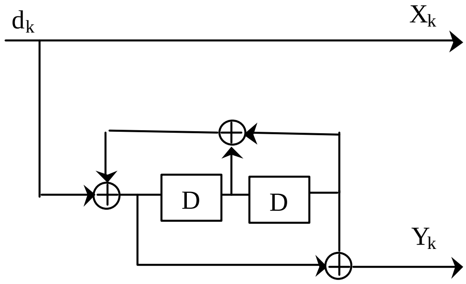

Figure 6. Recursive Systematic Convolutional (RSC) Encoder

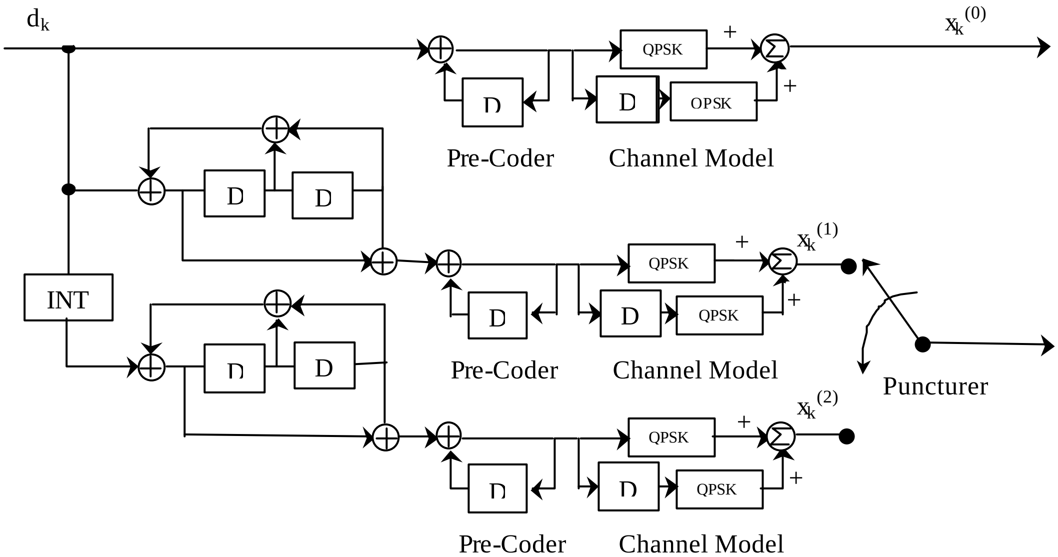

Figure 7. 1/2 Rate Turbo Encoder with 1+D / PRFC Model

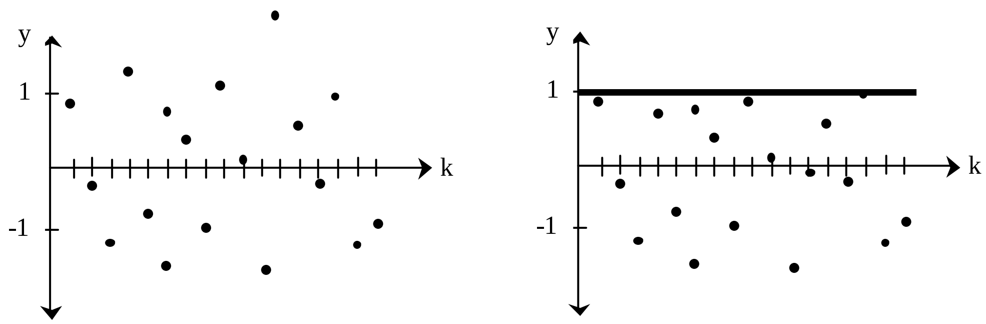

Figure 8a. Observation of y without PRFC **Figure 8b. Observation of y with PRFC**

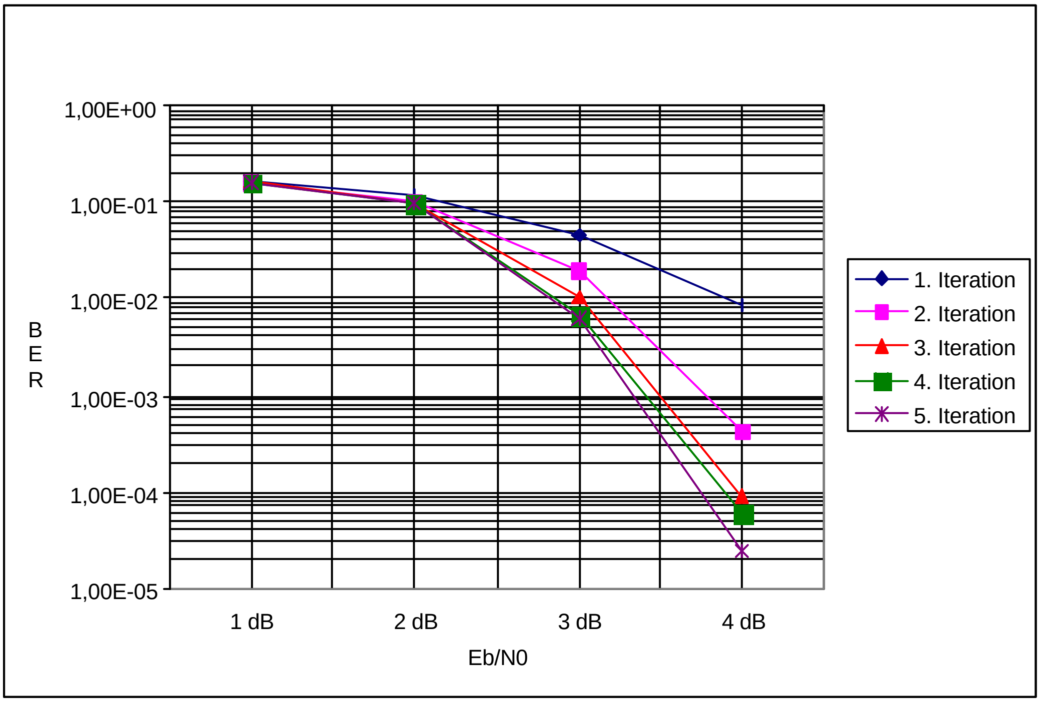

Figure 9a. The performance in 1+D / PRFC with ideal CSI for K= ∞ , N=400

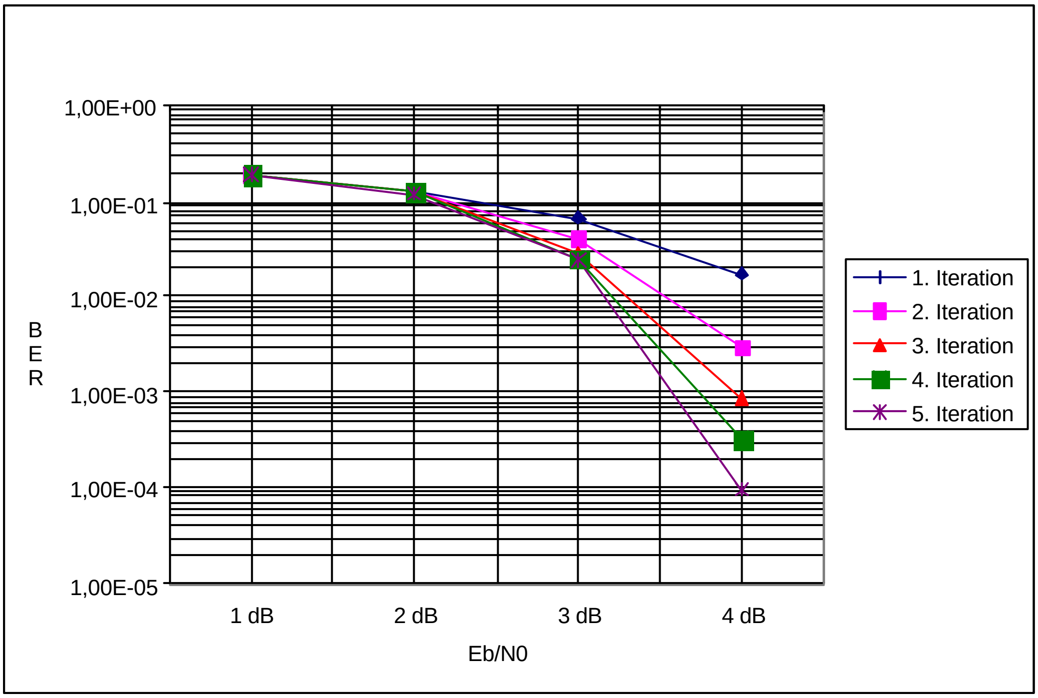

Figure 9b. The performance in 1+D / PRFC with ideal CSI for K=20 dB , N=400

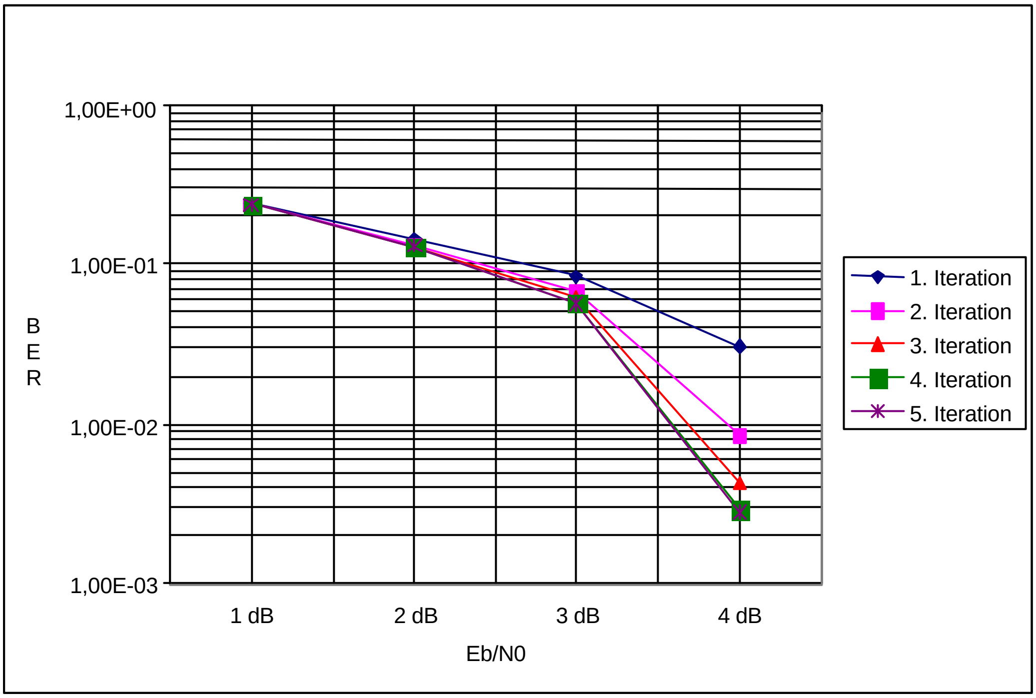

Figure 9c. The performance in 1+D / PRFC with ideal CSI for K=10dB , N=400

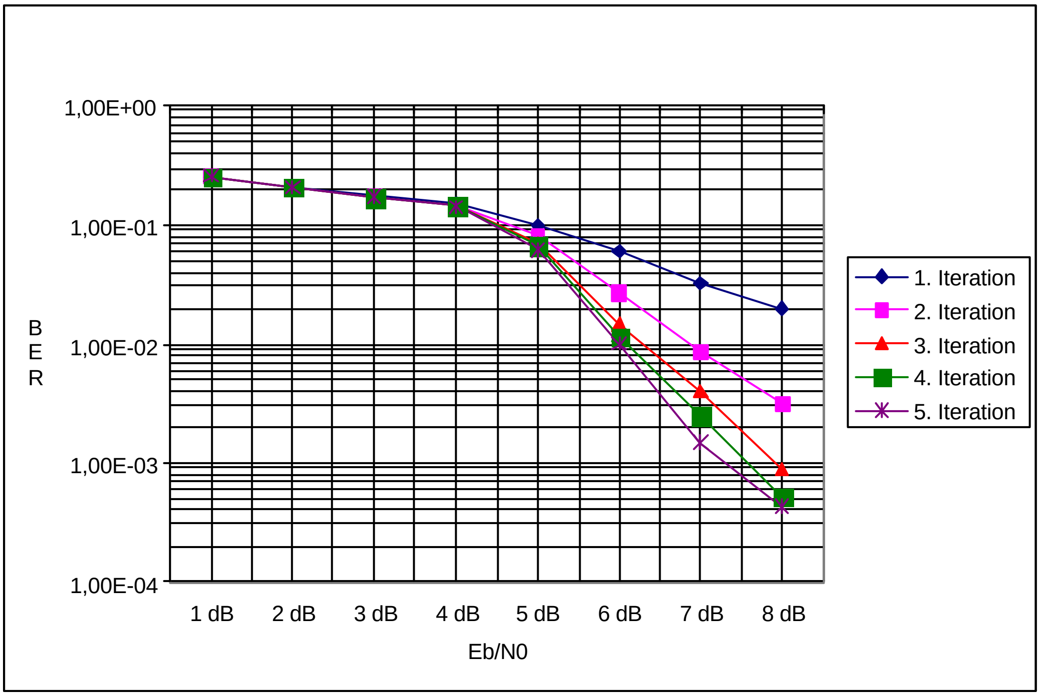

Figure 9d. The performance in 1+D / PRFC with ideal CSI for K=0 dB , N=400

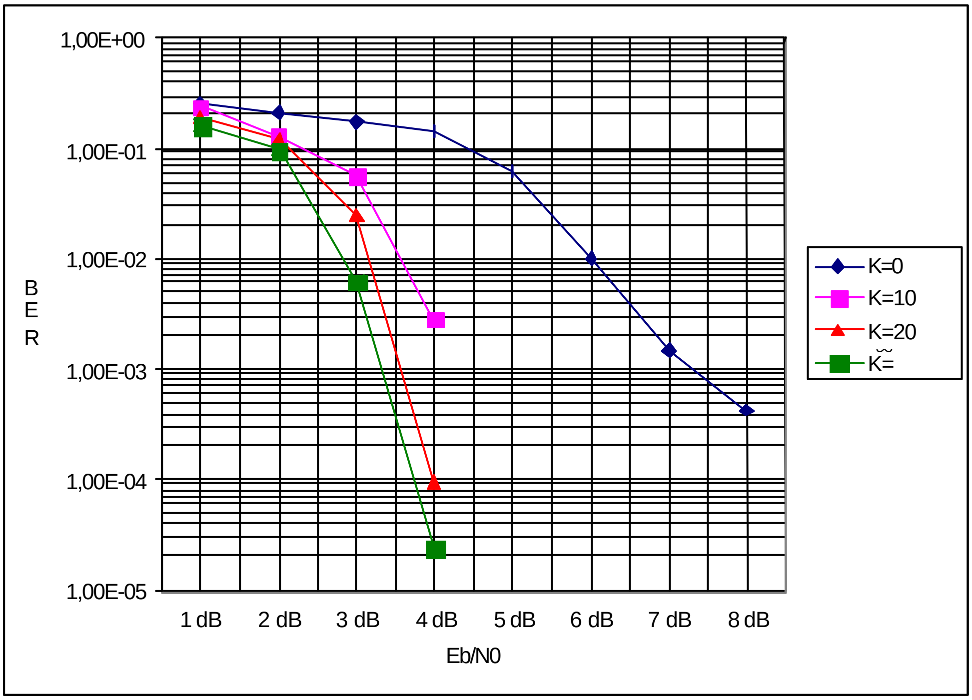

Figure 9e. The performance in 1+D / PRFC for K=0,10,20,∞ (dB), with ideal CSI, N=400, for 5th iteration

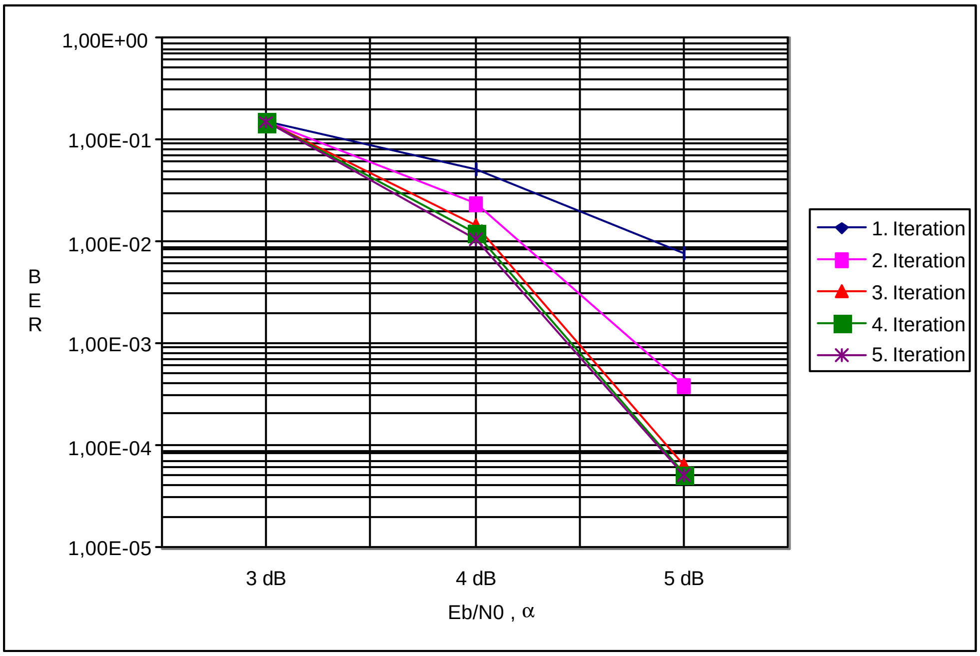

Figure 10a. The performance in 1+D / PRFC with imperfect phase reference for K= ∞ dB, N=400

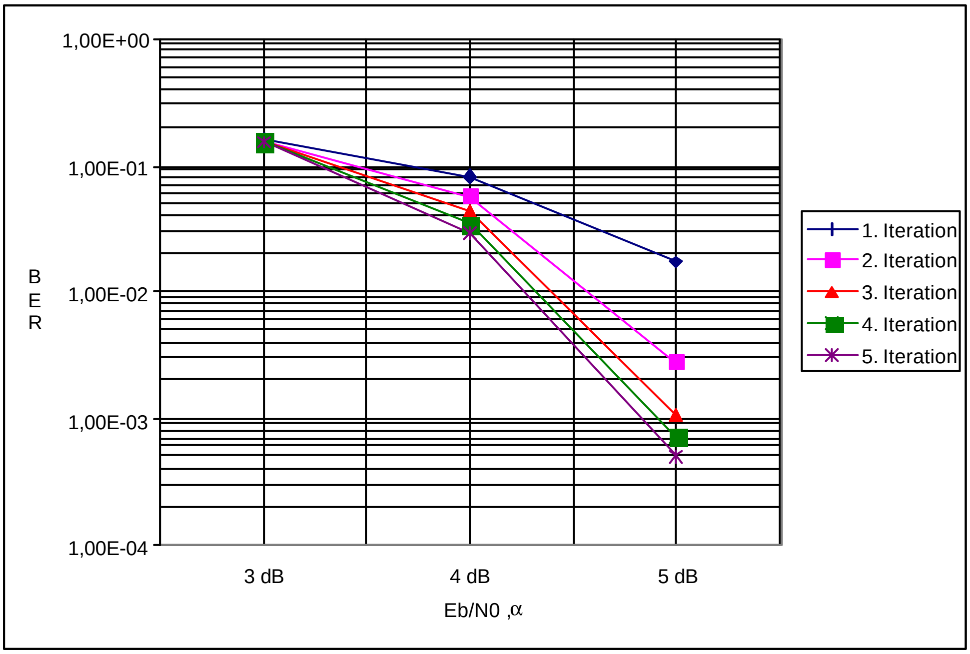

Figure 10b. The performance in 1+D / PRFC with imperfect phase reference for K =20 dB , N=400

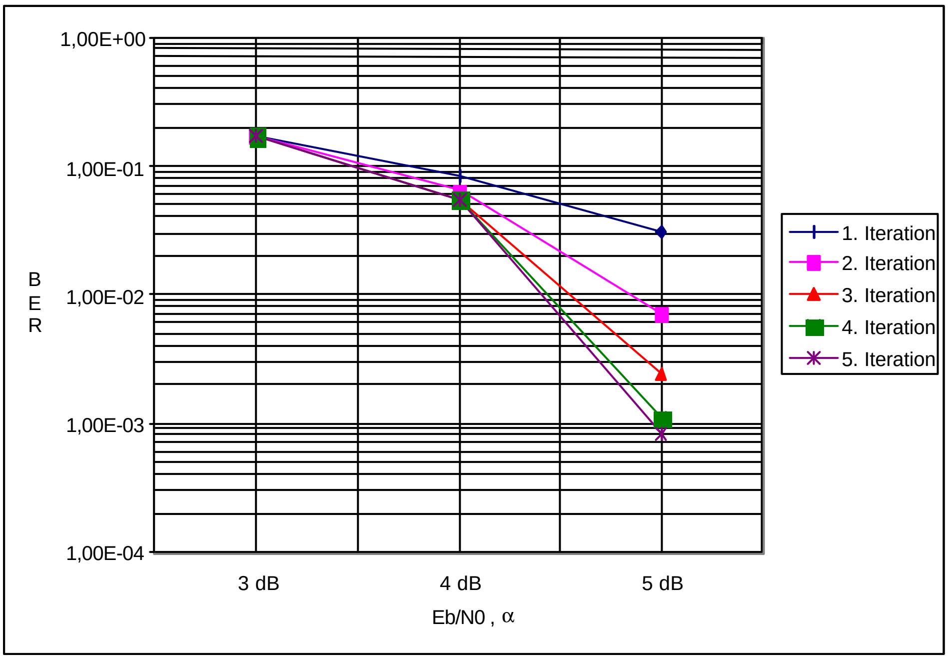

Figure 10c. The performance in 1+D / PRFC with imperfect phase reference, K=10 dB , N=400

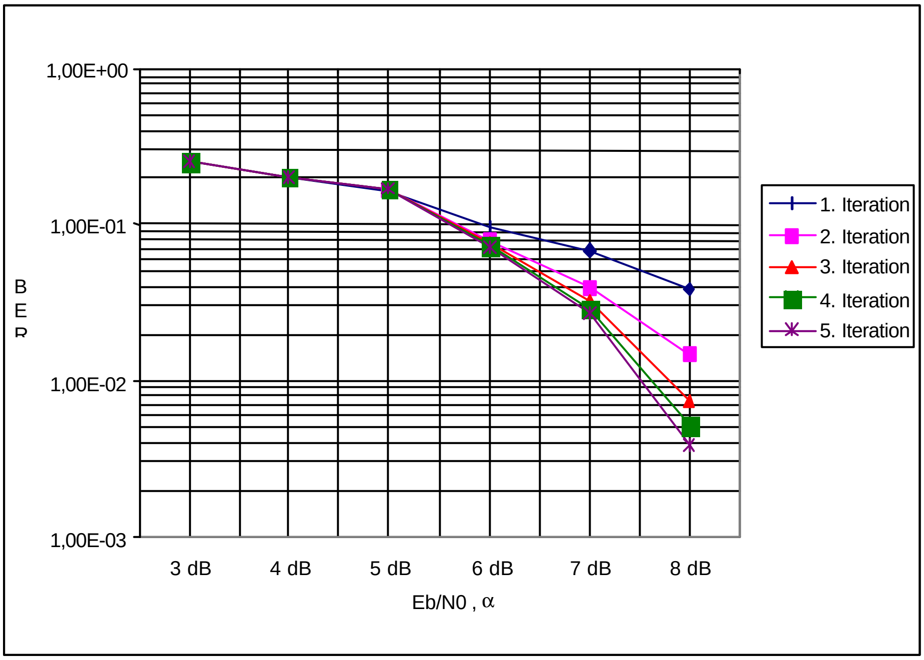

Figure 10d. The performance in 1+D / PRFC with imperfect phase reference for K=0 dB , N=400

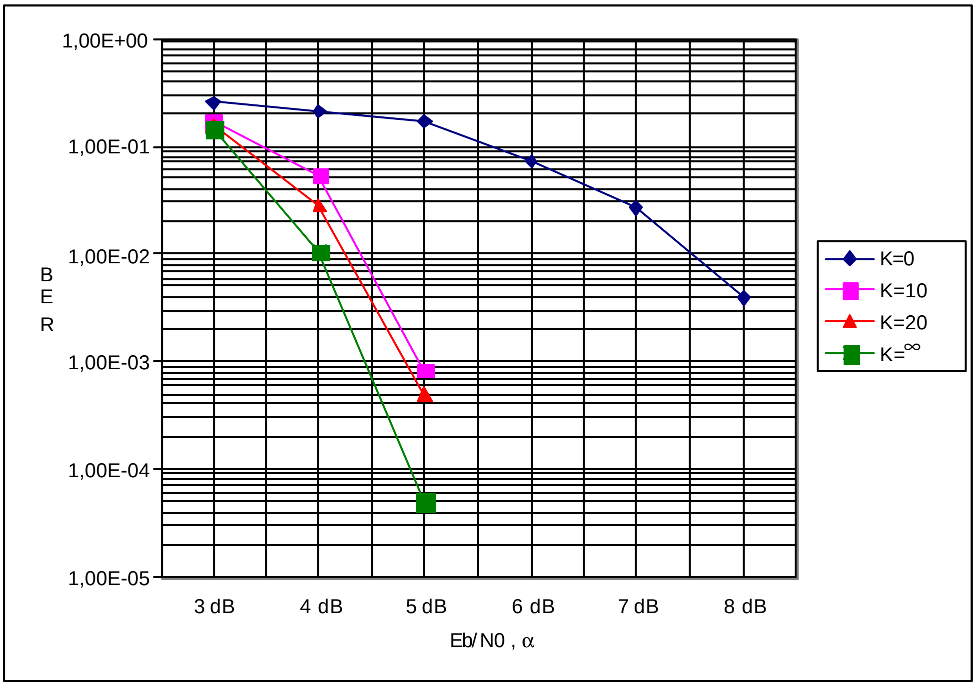

Figure 10e. The performance in 1+D / PRFC with imperfect phase reference for K=0,10,20,∞ (dB), N=400, for 5th iteration

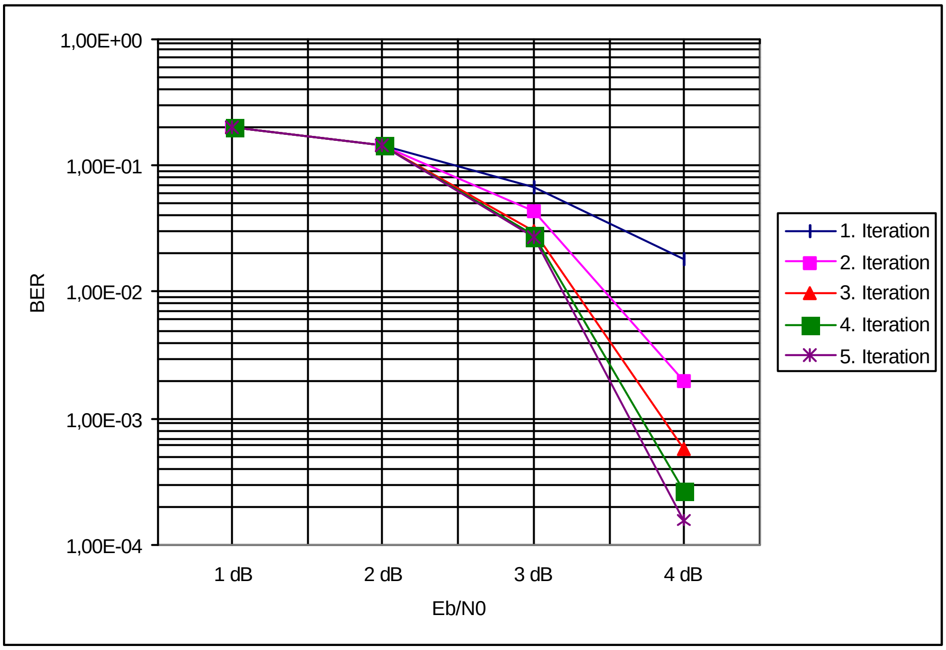

Figure 11a. The performance in 1+D / PRFC with imperfect phase reference for a =10 dB , K=∞ dB, N=400

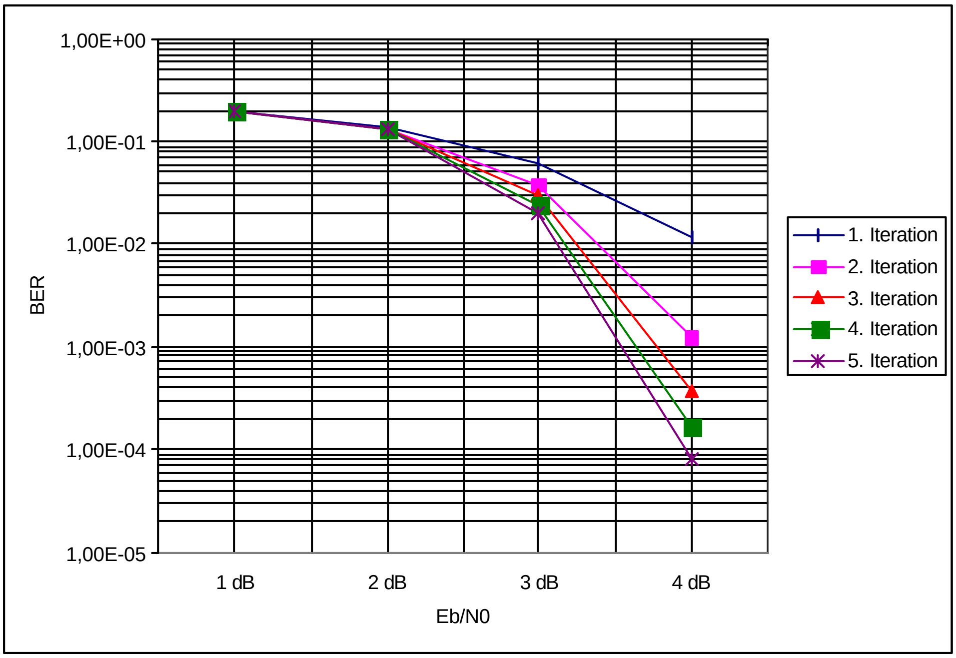

Figure 11b. The performance in 1+D / PRFC with imperfect phase reference for a =20 dB , K=∞ dB , N=400

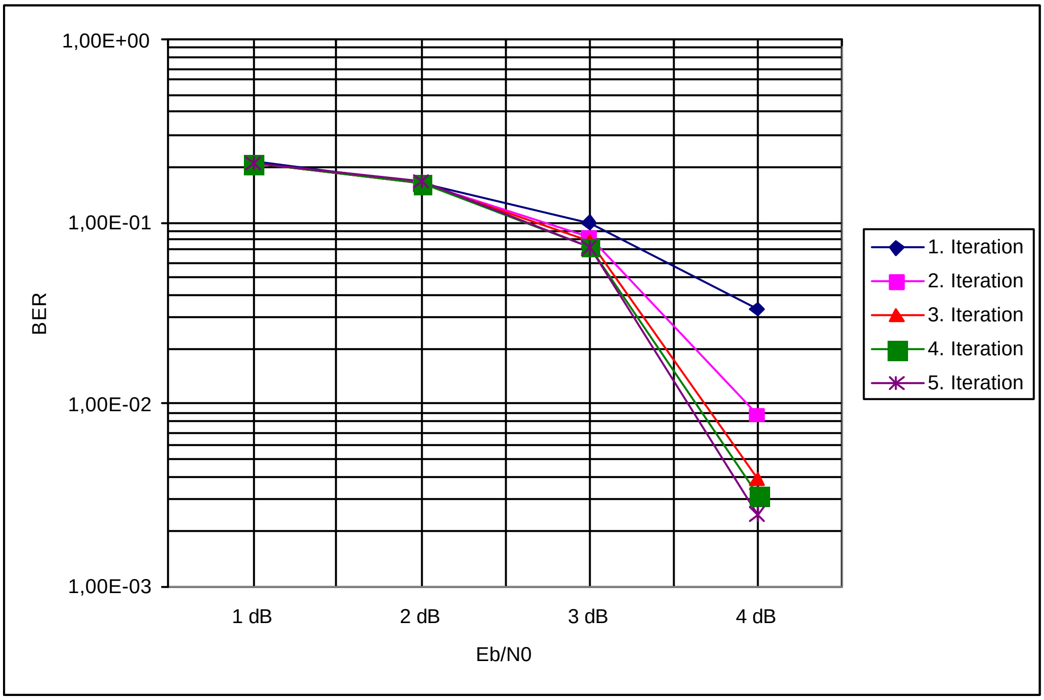

Figure 12a. The performance in 1+D / PRFC with imperfect phase reference for a =10 dB , K=20 dB , N=400

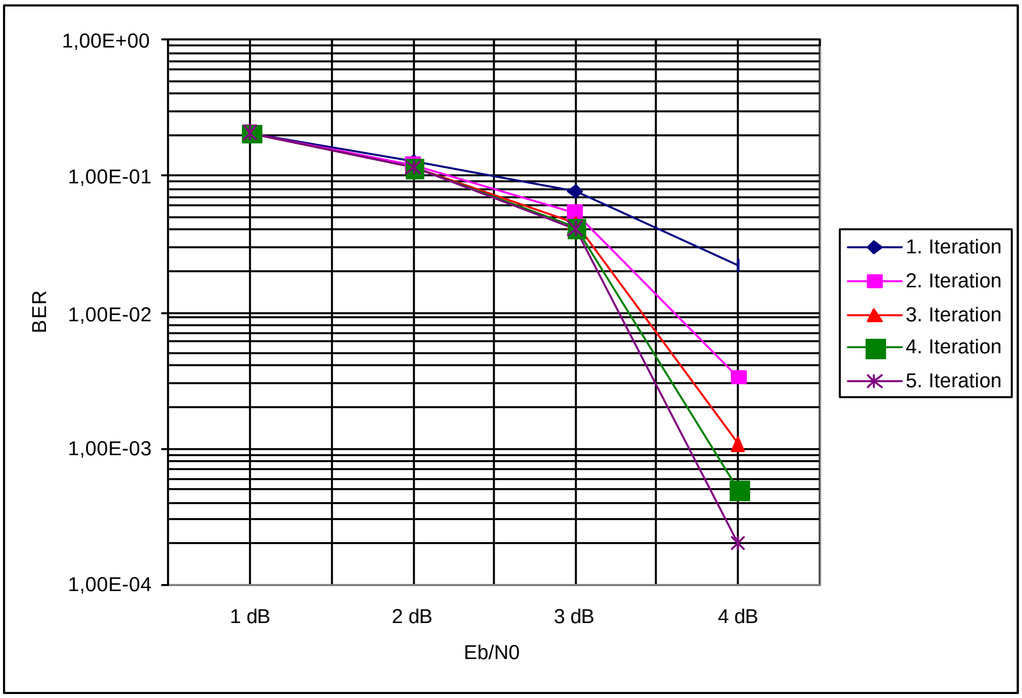

Figure 12b. The performance in 1+D / PRFC with imperfect phase reference for a =20 dB , K=20 dB , N=400

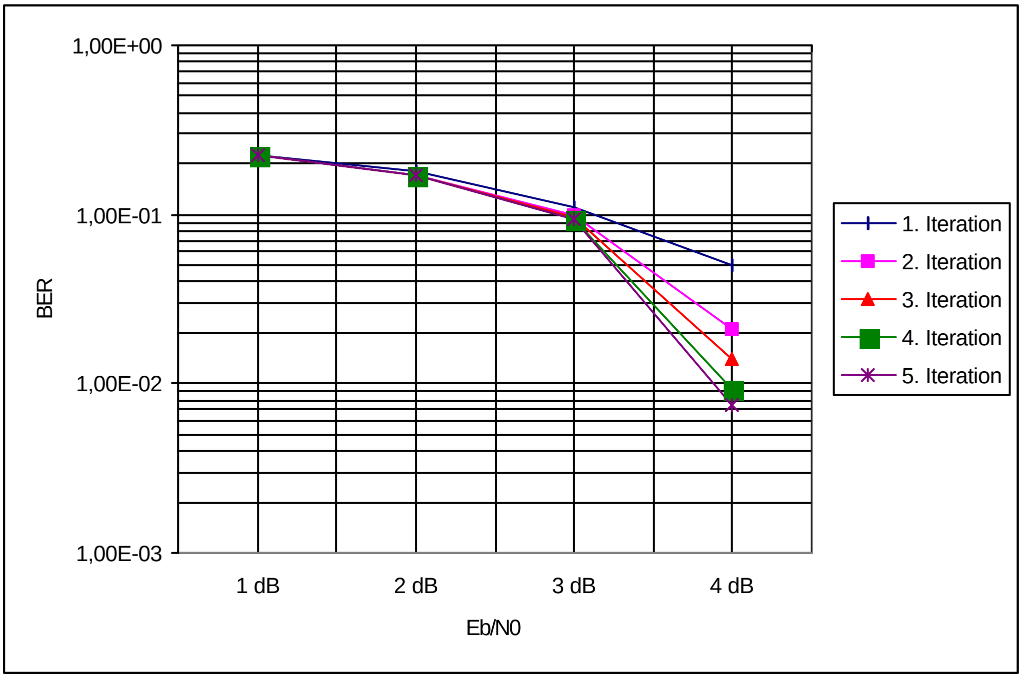

Figure 13a. The performance in 1+D / PRFC with imperfect phase reference for a =10 dB , K=10 dB , N=400

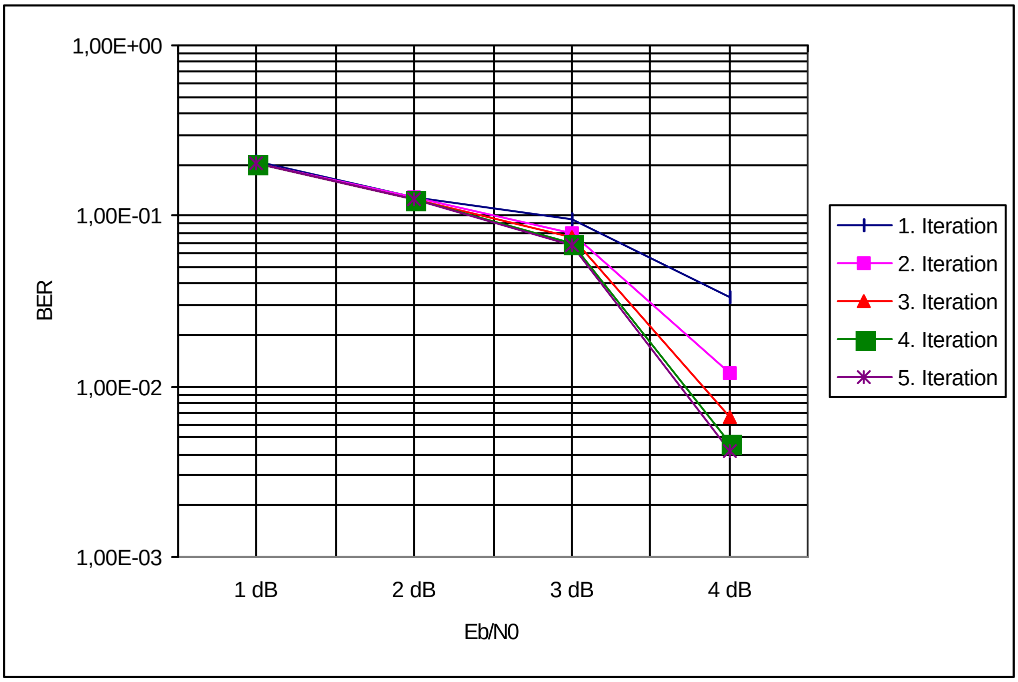

Figure 13b. The performance in 1+D / PRFC with imperfect phase reference for a =20 dB , K=10 dB , N=400

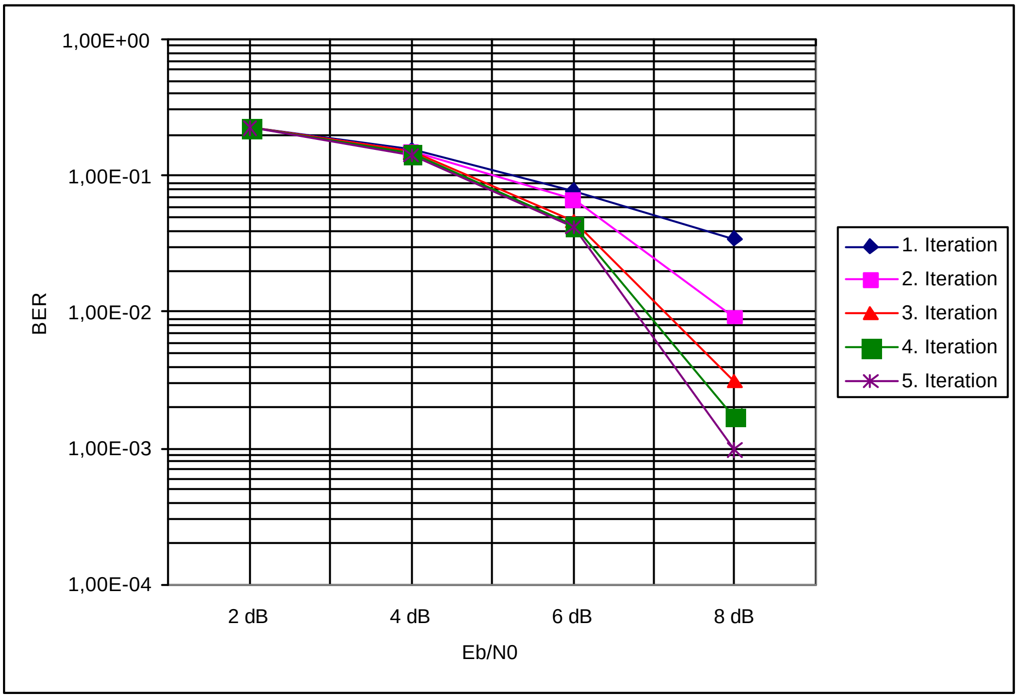

Figure 14a. The performance in 1+D / PRFC with imperfect phase reference for a =10 dB , K=0 dB, N=400

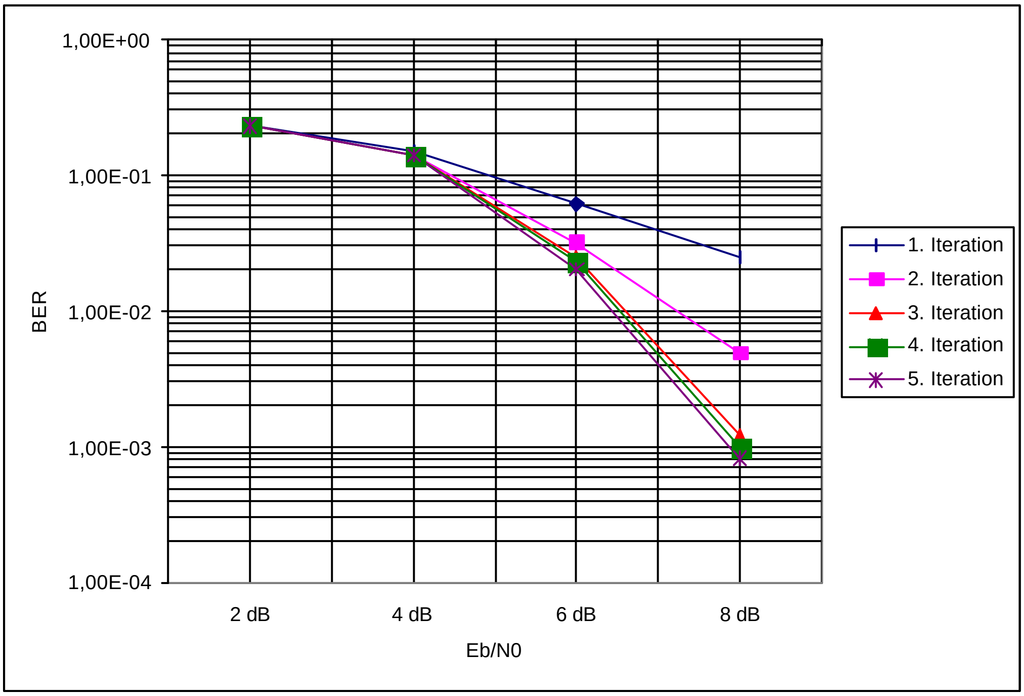

Figure 14b. The performance in 1+D / PRFC with imperfect phase reference for a =20 dB , K=0 dB , N=400

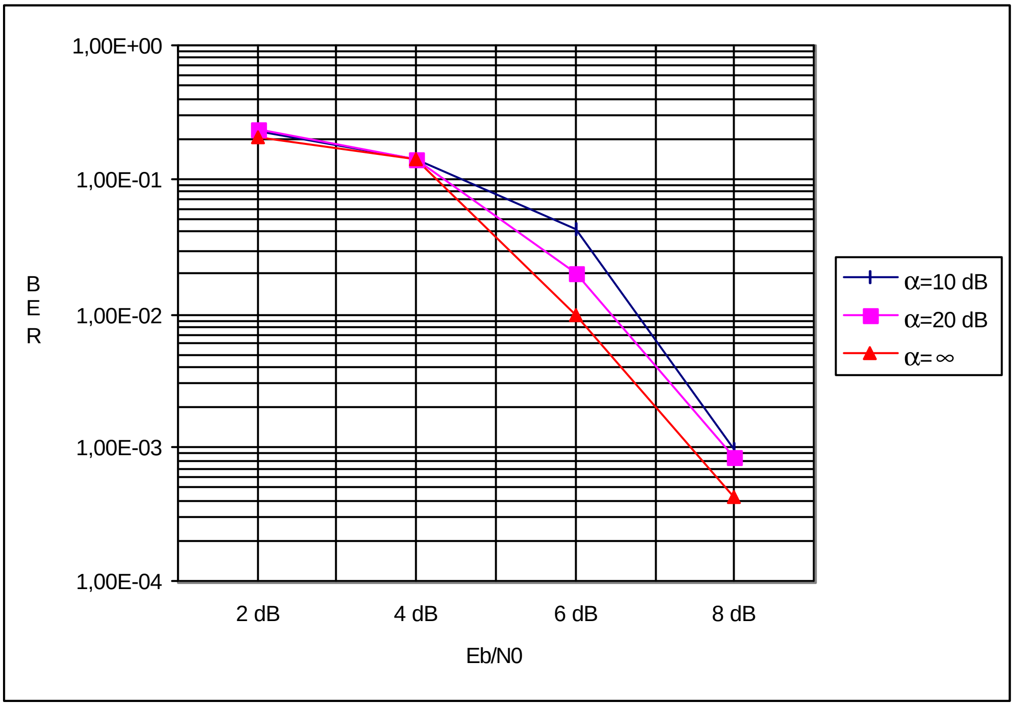

Figure 15a. The performance in 1+D / PRFC with imperfect phase reference , K=0 dB , N=400

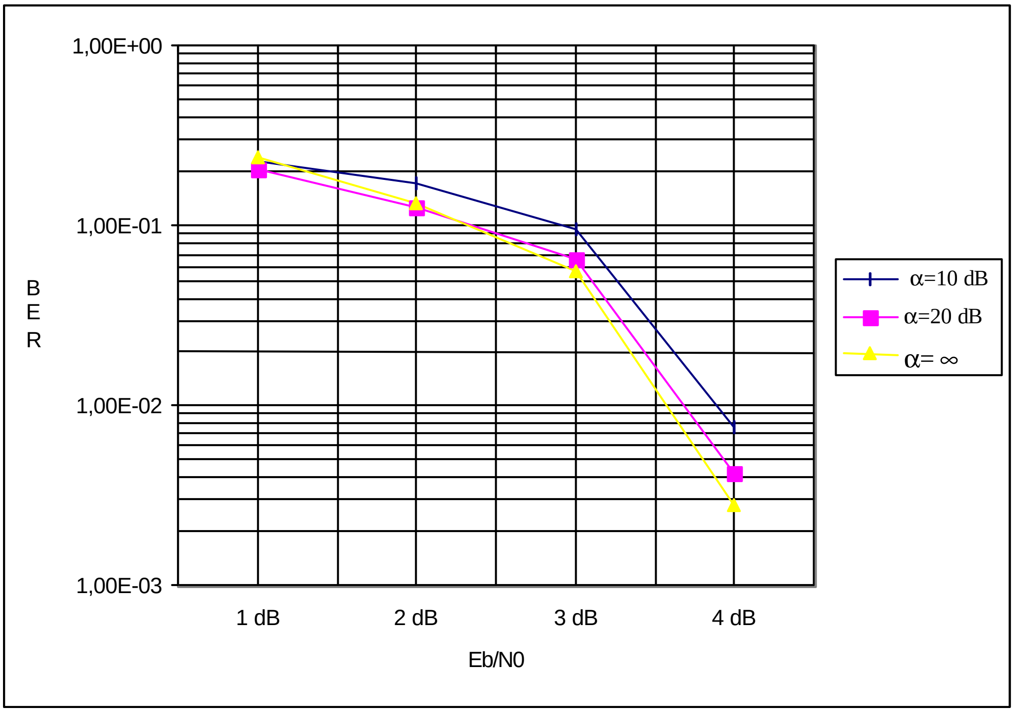

Figure 15b. The performance in 1+D / PRFC with imperfect phase reference , K=10 dB , N=400

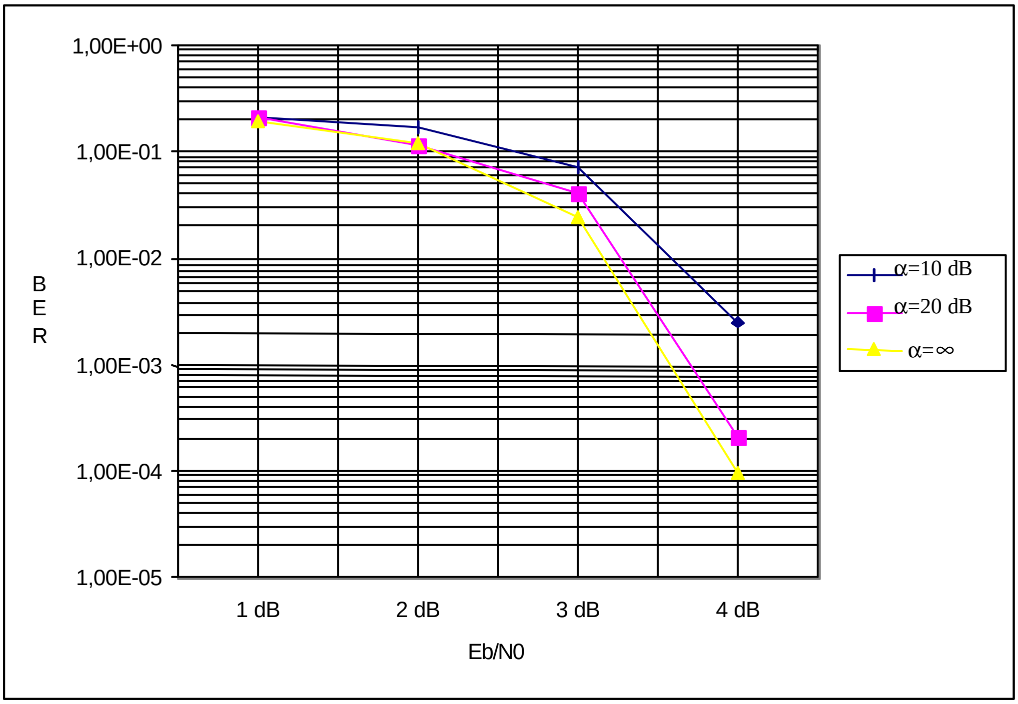

Figure 15c. The performance in 1+D / PRFC with imperfect phase reference , K=20 dB , N=400

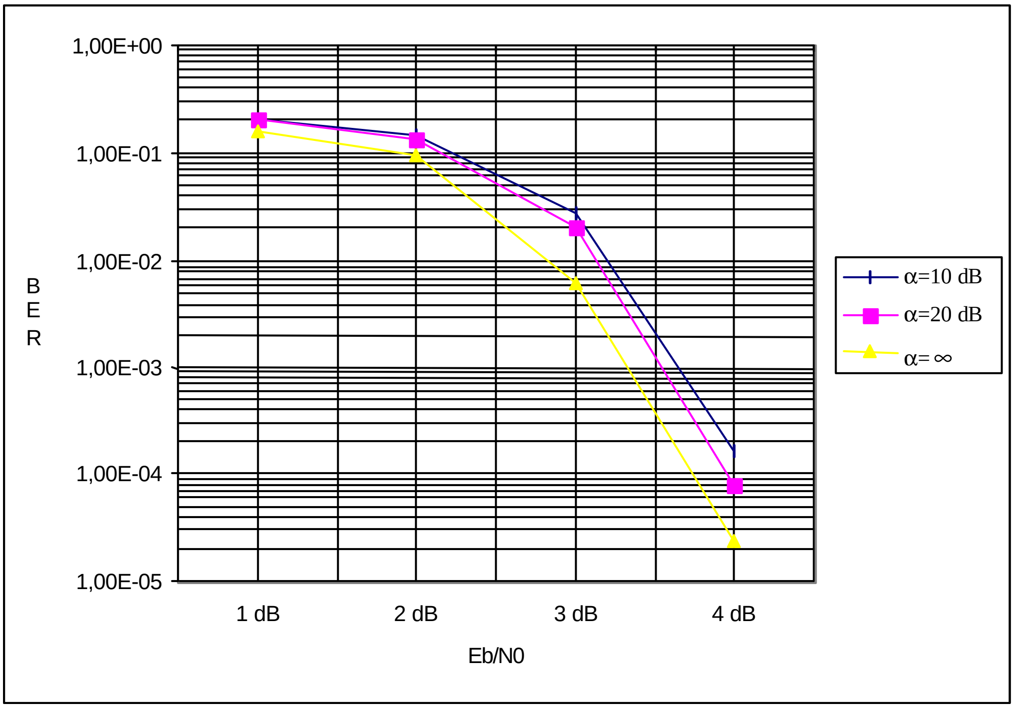

Figure 15d. The performance in 1+D / PRFC with imperfect phase reference ,
K= ∞ dB , N=400