

GAIN AND NOISE FIGURE PERFORMANCE OF ERBIUM DOPED FIBER AMPLIFIERS (EDFA)

A.Cem ÇOKRAK¹

Ahmet ALTUNCU²

^{1,2}Dumlupınar Üniversitesi, Mühendislik Fakültesi, Elektrik-Elektronik Mühendisliği Bölümü,
Tavşanlı Yolu 12.km 43100 KÜTAHYA

E-mail: altuncu@dumlupinar.edu.tr

ABSTRACT

Fiber loss is a fundamental limitation in realising long haul point-to-point fiber optical communication links and optical networks. One of the advanced technologies achieved in recent years is the advent of erbium doped fiber amplifiers (EDFAs) that has enabled the optical signals in an optical fiber to be amplified directly in high bit rate systems beyond Terabits. In this study, an EDFA simulation program has been written in Matlab to characterize Gain, Noise Figure and ASE power variations of a forward pumped EDFA operating in C band (1525-1565 nm) as functions of Er^{3+} fiber length, injected pump power, signal input power and Er^{3+} doping density. The program solves the rate and propagation equations numerically and shows the results graphically. Thus, Gain and Noise Figure performance of an EDFA given with its physical parameters can be graphically obtained or the required physical parameters of an EDFA with desired operating performance can easily be optimised.

Keywords: : Optical Amplifiers, EDFA, Erbium Doped Fiber, Gain, Noise Figure..

1. INTRODUCTION

One of the most important factors limiting the transmission distance in fiber optical communication systems is the optical power loss caused by scattering and absorption mechanisms in optical fiber [1]. Electrical repeaters, which require optical-electrical signal conversion, have previously been used to compensate the power losses increasing with distance. The use of such repeaters in optical communication systems have made the systems more complex and increased their installation costs. The optical amplifiers, that were developed in 1980s and came partially into the use commercially in 1990s, enable the optical signals to be directly amplified optically.

The most significant types of optical amplifiers are semiconductor laser amplifiers, Raman and Brillouin amplifiers, and rare-earth doped fiber amplifiers.

The fiber amplifiers operating at specific wavelengths from visible to infrared light region (up to 3 μ m) can be made using different rare earth ions such as Erbium (Er^{3+}), Holmium (Ho^{3+}), Neodymium (Nd^{3+}), Praseodymium (Pr^{3+}), Samarium (Sa^{3+}), Thulium (Tm^{3+}) and Ytterbium (Yb^{3+}). However, the most interesting element listed above is erbium. Because, Erbium Doped Fiber Amplifiers (EDFA) made by doping the silica fiber with erbium ions can operate in a broad range within the 1550 nm window at which the attenuation of silica fiber is minimum

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and therefore it is ideal for the optical fiber communication systems operating at this wavelength range. According to the research performed in recent years, it is known that the pumping of erbium doped fiber at 980 nm or 1480 nm is the most efficient way [3]. High gain (30~50 dB), large bandwidth (≥ 90 nm), high output power (10~20 dBm) and low noise figure (NF=3~5 dB) can be obtained using an erbium doped fiber amplifier optimised for 1.55 μ m range. The amplification that could previously be made within C band (1525-1565 nm) has now extended to L band (1570-1620 nm) by codoping the active fiber with Erbium (Er^{3+}), and Ytterbium (Yb^{3+}). On the other hand, Thulium (Tm^{3+}) doped Raman fiber amplifiers have enabled to operate within the S band (1480-1520 nm).

2.THE STRUCTURE OF EDFA AND ITS PUMPING REQUIREMENTS

The structure of a typical EDFA is shown in figure 1. EDFAs consist of optical couplers to combine pump and signal lights injected to active fiber, unidirectional optical isolators, pumping lasers, polarisation couplers to combine pump sources and optical bandpass filters to reduce out of band noise. The gain characteristics

of EDFAs depend mainly on their pumping schemes. EDFAs can be pumped at 980 nm or 1480 nm, and with different configurations: backward, forward or bi-directional. The pumping at 980 nm provides lower noise figure than pumping at 1480 nm. Therefore pre-amplifier version of EDFA chooses 980 nm for pumping wavelength. On the other hand, 1480 nm pumping has higher quantum efficiency and so provides higher output power at a lower cost and therefore it is preferred for booster amplifier operations. In forward pumping, both of the signal and pump lights propagate in the same direction through the fiber whereas in the backward pumping they propagate in the opposite direction.

The forward pumping direction provides the lowest noise figure. In fact, the noise is sensitive to the gain and the gain is the highest when the input power is the lowest. Backward pumping provides the highest saturated output power [2]. Bi-directional pumping scheme has a higher performance than the other two by combining the lowest noise figure and the highest output power advantageous although it requires two pump lasers. In addition, in this scheme the small signal gain is uniformly distributed along the whole active fiber.

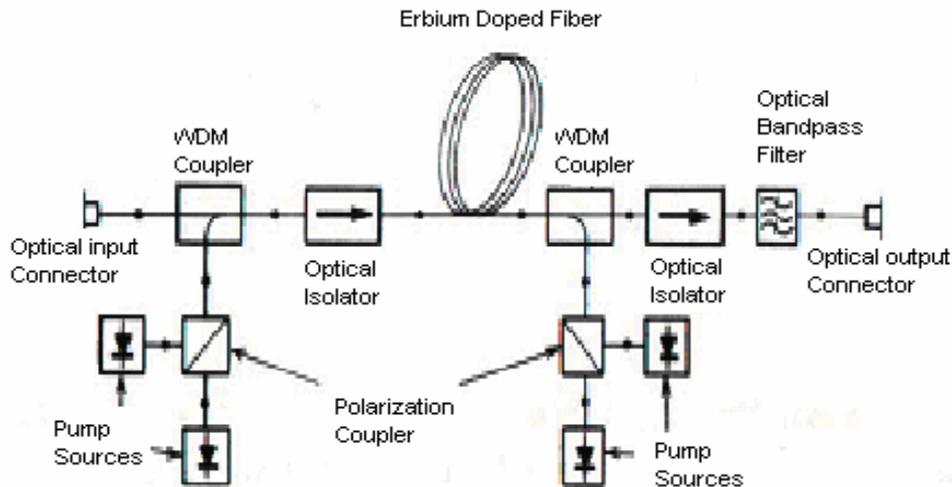


Figure 1. A bidirectionally pumped EDFA structure

3. EDFA THEORY

In this section, the rate and propagation equations characterizing an EDFA operating in C band and pumped at 1480 nm are given. The simulation of EDFA requires the simultaneous calculation of the rate equations, which define the transitions between energy levels, and the propagation equations, which characterize signal, pump and ASE power variations along the active fiber.

The performance of absorbing injected pump light and then emitting it as the signal light by stimulation are known as *absorption cross section and emission cross section* or in a different way as *loss and gain coefficients, respectively*. A typical absorption and emission spectra of an erbium doped silica fiber is shown in figure 2. In erbium, the absorption is higher at 1480 nm whereas the emission is higher at 1535 nm. The signal and pump powers along the fiber vary due to absorption, spontaneous emission and stimulated emission processes.

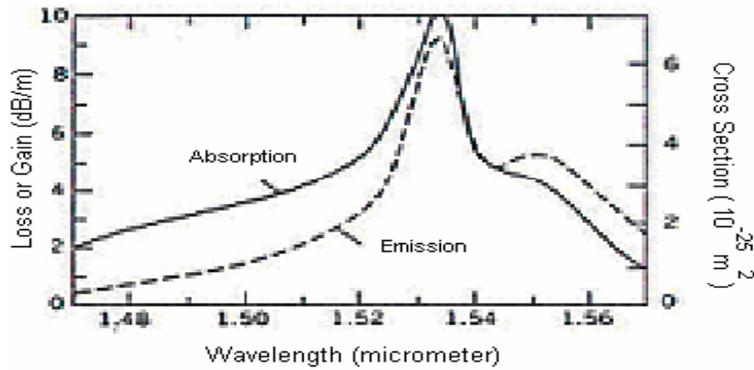


Figure 2. A typical absorption and emission spectra of erbium doped fiber (EDF) [2]

3.1 Rate Equations

In pumping at 980 nm, it is required to form a system model with three energy levels. When the excited state absorption (ESA) is considered, a fourth level should also be included. If the pumping is performed at 1480 nm, a simple system model with two energy levels can be used. In figure 3, erbium ion transitions of an EDFA pumped at 1480 nm was given. From figure 3, time dependent population rates in level 2 and 1 can be given as:

$$\frac{dn_2}{dt} = +(R_{12} + W_{12})n_1 - (R_{21} + W_{21} + A_{21})n_2 \quad \frac{dn_1}{dt} = -\frac{dn_2}{dt} \quad (1)$$

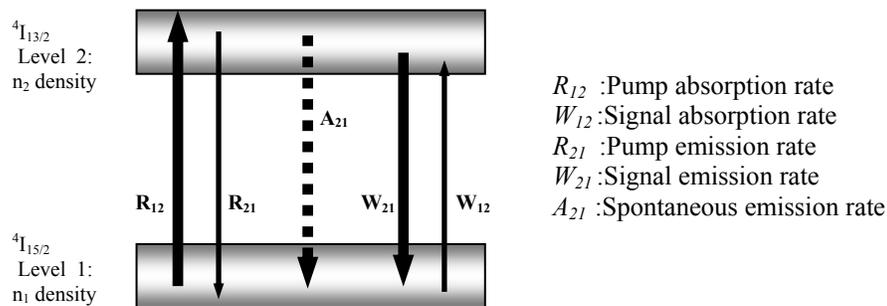


Figure 3. Erbium ion levels

Here, n_t (ion/m³) is called total ionic population and is equal to the total of populations in two energy levels ($n_t = n_1 + n_2$). In steady state, ($dn_2 / dt = 0$), the ratio of level 1 and level 2 population to the total population can be written as:

$$\frac{n_1}{n_t} = \frac{(R_{21} + W_{21} + A_{21})}{(R_{21} + R_{12} + W_{21} + W_{12} + A_{21})} \quad \frac{n_2}{n_t} = \frac{(R_{12} + W_{12})}{(R_{21} + R_{12} + W_{21} + W_{12} + A_{21})} \quad (2)$$

Expressing the absorption and emission rates in terms of pump and signal powers makes the population equations more meaningful.

$$R_{ij} = \frac{P_p \sigma_{ij}^p \Psi_p(r)}{h\nu_p} ; i, j=1,2 \text{ (pump)} \quad W_{ij} = \frac{P_s \sigma_{ij}^s \Psi_s(r)}{h\nu_s} ; i, j=1,2 \text{ (signal)} \quad (3)$$

Here, $\sigma_{ij}^{p,s}(\nu)$ is absorption and emission cross-section for pump and signal (absorption for $ij = 12$, emission for $ij = 21$), $h\nu_{p,s}$ is photon energy and $\Psi_{p,s}(r, \theta)$ is normalized line shape function. The threshold power at pumping wavelength and saturation power at signal wavelength is given by:

$$P_{qu} = \frac{h\nu_p \pi \omega_p^2 A_{21}}{\sigma_{12}^p} \quad P_{sat} = \frac{h\nu_s \pi \omega_s^2 A_{21}}{\sigma_{12}^s} \quad (4)$$

The pumping of erbium doped fiber causes to amplify not only signal but also spontaneous emission. Spontaneous emission (SE) is a noncoherent, randomly polarized optical wave having a wide bandwidth and is propagated in both directions in the fiber. The spontaneous emission amplified simultaneously with the signal consists of noise produced in the amplifier and is therefore called Amplified Spontaneous Emission (ASE). ASE simply causes decrease in amplifier gain and increase in noise figure. Therefore, forward and backward ASE powers should also be included into the rate equations. Thus, the rate equations can be rewritten as :

$$n_1 = \frac{I + \left(\frac{\sigma_{21}^p}{\sigma_{12}^p} \right) \left(\frac{P_p \Psi_p}{P_{qu}} \right) + \left(\frac{(P_s + P_a^+ + P_a^-) \Psi_s}{P_{sat}} \right)}{I + \left(\frac{\sigma_{21}^p}{\sigma_{12}^p} \right) \left(\frac{P_p \Psi_p}{P_{qu}} \right) + \left(\frac{\sigma_{12}^s}{\sigma_{21}^s} \right) \left(\frac{(P_s + P_a^+ + P_a^-) \Psi_s}{P_{sat}} \right)} n_t \quad n_2 = n_t - n_1 \quad (5)$$

3.2 Propagation Equations :

The amplification of optical signals in EDFA is also defined with propagation equations which is directly related to Er doped fiber characteristics. The propagation equations, which give the signal, pump and ASE variations along the fiber, depend also on the pumping scheme. For forward pumping, the pump, signal and bi-directional propagating ASE noise power can be defined as [3]:

$$\frac{dP_p^\pm}{dz} = +2\pi \int_{r=0}^a [\sigma_{21}^p n_2 - \sigma_{12}^p n_1] P_p^\pm \Psi_p(r) r dr - \alpha_p P_p^\pm \quad (6)$$

$$\frac{dP_s^+}{dz} = +2\pi \int_{r=0}^a [\sigma_{21}^s n_2 - \sigma_{12}^s n_1] P_s^+ \Psi_s(r) r dr - \alpha_s P_s^+ \quad (7)$$

$$\frac{dP_a^\pm}{dz} = \pm 2\pi \int_{r=0}^a [\sigma_{21}^s n_2 (P_a^\pm \Psi_s + 2P_o) - \sigma_{12}^s n_1 P_a^\pm \Psi_s(r)] r dr \mp \alpha_s P_a^\pm \quad (8)$$

Here, α_p and α_s are the fiber loss for the pump and signal, respectively. In short fibers, these losses are negligible. However, they should be taken into account for long fibers specifically distributed erbium doped fibers (DEDF) [4]. The propagation equations given (6-8) are the non-linear differential equations and their solution can only be obtained using numerical techniques. In (6-8), 2π values is a result of integration by θ . The integration in (6-8) characterizes the light intensity variations in optical fiber with coordinate r . Rather than performing integration for radius r , the pump and signal overlap ratio with fiber core can be used. Thus, equations (7-9) can be given using the overlap ratio (Γ) as [5] :

$$\frac{dP_p^+(z,t)}{dz} = -P_p^+ \Gamma_p (\sigma_{12}^p n_1 - \sigma_{21}^p n_2) - \alpha_p P_p^+ \quad (9)$$

$$\frac{dP_s(z,t)}{dz} = P_s \Gamma_s (\sigma_{21}^s n_2 - \sigma_{12}^s n_1) - \alpha_s P_s \quad (10)$$

$$\frac{dP_a^\pm(z,t)}{dz} = \pm P_a^\pm \Gamma_s (\sigma_{21}^s n_2 - \sigma_{12}^s n_1) \pm 2\sigma_{21}^s n_2 \Gamma_s P_o \mp \alpha_s P_a^\pm \quad (11)$$

3.3 Gain and Noise Figure

Gain of an erbium-doped fiber with a length of L is the ratio of the signal power at the fiber output to the signal power injected at the fiber input as:

$$G = \frac{P_s(L)}{P_s(0)} \quad (12)$$

ASE noise generated during amplification process is added to the signal leading to decrease in signal to noise ratio (SNR) at the amplifier output. SNR reduction ratio from input to output of the amplifier is defined as Noise Figure (NF), which is also used for electronic amplifiers:

$$NF = \frac{(SNR)_{in}}{(SNR)_{out}} \quad (13)$$

Noise Figure can also be expressed in terms of gain and spontaneous emission factor (n_{sp}) (or population inversion factor) [3]:

$$NF = 2n_{sp} \frac{(G-1)}{G} \approx 2n_{sp} \quad (14.a)$$

$$n_{sp} = \frac{n_2}{n_2 - n_1} \quad (14.b)$$

The power spectral density of spontaneous emission induced noise $S_{sp}(\nu)$ is a function of frequency and follows the emission spectrum of Er^{3+} ions :

$$S_{sp}(\nu) = (G-1)n_{sp} h\nu \quad S_{sp}(\nu) = \frac{P_a^+}{\Delta\nu} \quad (15)$$

Using equation (15), EDFA noise figure can be expressed in terms of forward propagating ASE power (P_a^+) [5]:

$$NF = \frac{2P_a^+}{Gh\nu\Delta\nu} \tag{16}$$

Here, $\Delta\nu$ is the bandwidth of the optical bandpass filter and $h\nu$ is the photon energy. As it can be seen from (16), EDFA noise figure depends directly on forward ASE power and gain. Noise Figure increases with increasing ASE power, on the other hand, decreases with increasing gain.

4. EDFA SIMULATION PROGRAM

In this study, the rate and propagation equations characterizing a forward pumped C band EDFA were numerically solved in Matlab environment and the results were graphically simulated. After entering the required parameters for a desired amplifier in main menu and sub menus of the program, gain, noise figure and ASE power variations can be obtained as a function of four fundamental fiber parameters namely : fiber length, pump power, signal input power and erbium doping density. Thus, gain-NF performance for typical parameters of a given EDFA can be simulated or the required fiber parameters and signal/pump power values can be optimized for a desired EDFA gain-NF performance. The main menu and some of the submenus of the simulation program are shown in figure 4.



Figure 4. The main and sub menus of the simulation program.

5. TYPICAL EDFA CHARACTERISTICS OBTAINED WITH SIMULATION

Table 1 shows the typical EDFA parameters used in the simulation program.

Table 1. The typical EDFA parameters used in the simulation program.

Parameter	Sembol	Value	Unit
Fiber radius	a	2	μm
Pump-fiber core overlap ratio	Γ_p	0.4	-
Signal-fiber core overlap ratio	Γ_s	0.4	-
Power for effective bandwidth	P_0	1	μW
Pump absorption cross section	σ_{12}^p	$0.75 \cdot 10^{-25}$	m^2
Signal absorption cross section	σ_{12}^s	$2.40 \cdot 10^{-25}$	m^2
Pump emission cross section	σ_{21}^p	$0.19 \cdot 10^{-25}$	m^2
Signal emission cross section	σ_{21}^s	$3.80 \cdot 10^{-25}$	m^2
Pump wavelength	λ_p	1480	nm
Signal wavelength	λ_s	1550	nm
Cut-off wavelength	λ_c	1400	nm
Background loss at pump wavelength	α_p	0.25	dB/km
Background loss at signal wavelength	α_s	0.25	dB/km

5.1 Pump power variation

The attenuation of different pump powers applied to the input of erbium doped fiber is given in figure 5 for a certain length of fiber, a constant erbium doping density and signal input power. 10, 20, 30, 40 and 50 mW pump powers were applied to a 50 m Er-doped fiber with a 140 ppm doping concentration for a signal input power of -30 dBm and the pump power variations along the active fiber were obtained. As it can be seen from figure 5, the pump power rapidly attenuates with fiber length which results mainly from two mechanisms namely erbium absorption and background loss of silica fiber. The fiber background loss, which is less effective in a short distance, causes much higher pump power depletion in a longer fiber. Due to excessive pump depletion in longer distances, the gain obtained from an amplifier begins to decrease after a maximum level.

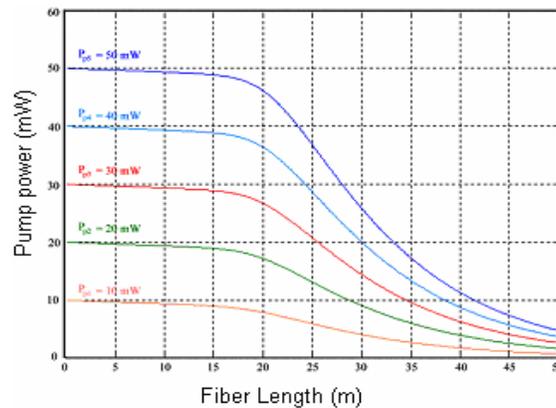


Figure 5. The attenuation of pump power along an erbium doped fiber

5.2 Gain Characteristics

The variation of gain with fiber length is shown in figure 6.a for different pump powers having a constant signal input power and erbium doping density. In this figure, the gain obtained from an amplifier for eight different pump power levels were given for a 100 m long EDF with 315 ppm doping density when a signal input power of 100 μ W was applied to the active fiber. As it is shown, the gain increases up to a certain length of fiber, and then begins to decrease after a maximum point. The reason for the decrease in gain is insufficient population inversion due to excessive pump depletion and getting higher losses than the provided gain at the signal wavelength due to high total loss of Erbium doped fiber (fiber background loss+Er absorption loss).

Figure 6.b shows the variation of gain with pump power for different fiber lengths, a constant signal input power and Er doping density. In this simulation, a 100 μ W signal power was applied at the input of an EDFA with 70 ppm doping density for six different fiber lengths and the pump power supplied was increased from 0 mW to 100 mW. It is seen that the gain of the EDFA increases with the increasing pump power and then goes to saturation after a certain level of pump power. From the figure, it is shown that, the gain of the EDFA sharply increases with increasing pump power; after a certain level of gain, the increase in gain becomes smaller when the population inversion is provided for all the erbium ions in the fiber and therefore amplifier goes to saturation. As a result, the gain efficiency defined in terms of dB gain per unit mW pump power reduces for high pump powers. In addition, a higher gain can be obtained if a longer erbium doped fiber is used with sufficient pumping.

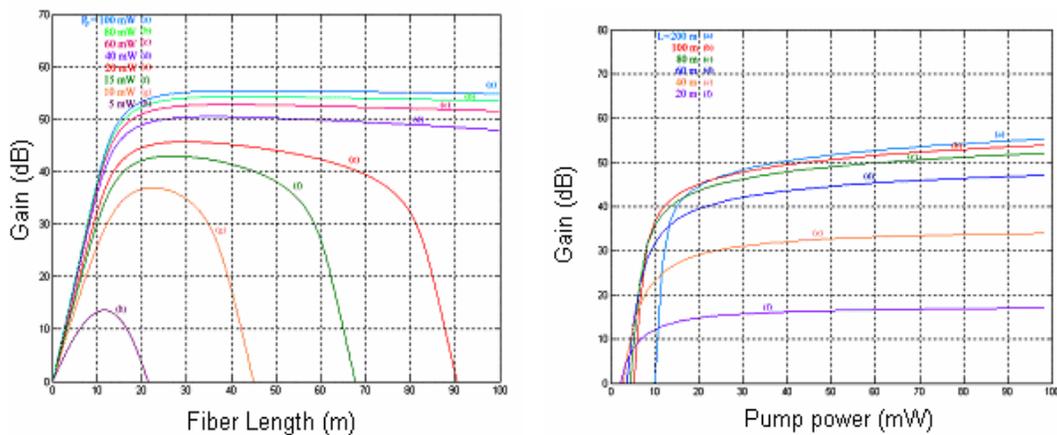


Figure 6. The variation of gain with a) Fiber length and b) Pump power

Figure 7.a shows how the gain varies as a function of signal input power for different pumping powers at a constant fiber length and erbium doping density. In this work, six different pump powers were applied to a 50 m long EDFA with a doping density of 140 ppm and the signal power was increased from -30 dBm to 10 dBm. From the figure, it is seen that EDFA gain decreases with increasing signal input power. The reason of this is the easier saturation of the EDFA at higher signal powers for a constant pump power.

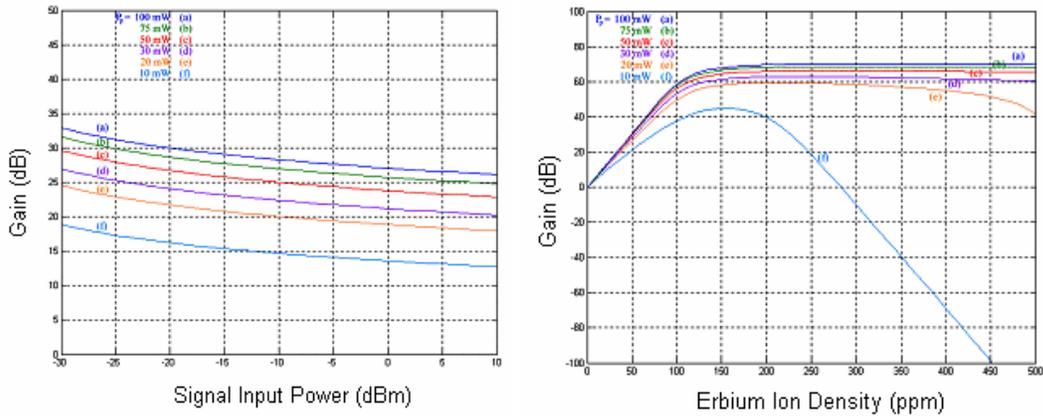


Figure 7. The variation of gain with a) Signal input power and b) Erbium ion density

The gain variation as a function of erbium doping density is shown in figure 7.b for a 50 m long fiber and a constant signal input power. The simulation was realised for the pump powers varying from 10 mW to 100 mW. It is seen that for a sufficiently large pump power, the gain linearly increases with increasing erbium ion density and stays constant after a certain level. Since the amplifier reaches the population inversion, the variation in maximum gain is small despite occurring a high increase in pump power. In the trace obtained for 10 mW pump power, the gain reduces sharply in highly doped fiber due to insufficient pumping. At high pump levels, invariance of the gain with erbium doping density can be explained such that the extra gain provided in the EDFA spends by the increasing erbium absorption.

5.3 Noise Figure Characteristics

The variation of noise figure as a function of fiber length is shown in figure 8.a for different pumping powers at a constant signal input power and erbium ion density. This graph was obtained in a 100 m long fiber with 350 ppm doping density and using an input signal power of 100 μ W. For a pump power of 15 mW, the increase in noise figure from 70 m can be clearly noticed. The reason for this increase is the decreasing gain with sharp pump depletion.

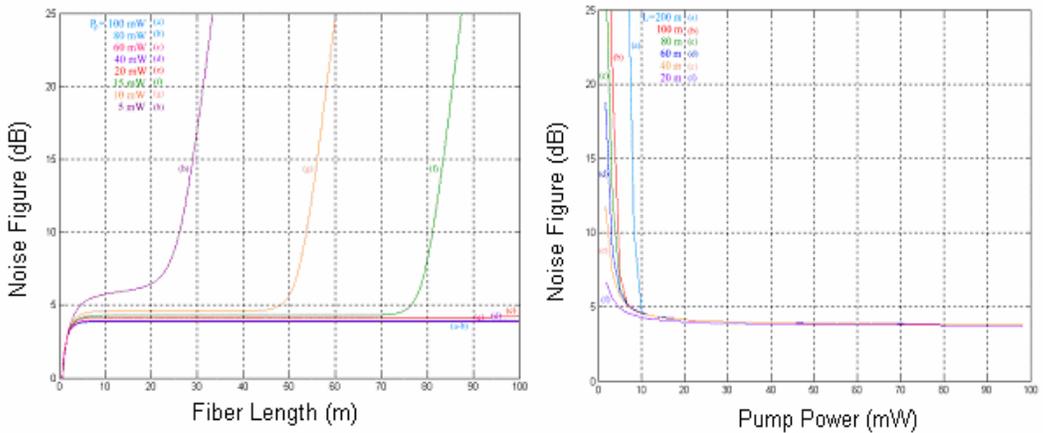


Figure 8. The variation of noise figure with a) Fiber length and b) Pump power

Figure shows the noise figure variations as a function of pump power for different fiber lengths at a constant signal input power and doping density. In the simulation performed for six different fiber length, a $100 \mu\text{W}$ signal input power was applied to an EDFA with an erbium doping rate of 70 ppm and the pump power was increased from 0 mW to 100 mW. In an amplifier having these parameters, it can be seen from the graph that the noise figure decreases with increasing pump power. The high gain in an active fiber with the total population inversion provided causes the spontaneous emission to stay in low levels. The noise figure of the EDFA varies linearly with ASE power and inversely with the amplifier gain. Therefore, the noise figure of an EDFA can be reduced to a minimum level by increasing the gain.

In figure 9.a, the variation of noise figure is given as a function of signal input power for a constant fiber length and erbium ion density. In this simulation, a 50 m long EDFA with an erbium ion density of 140 ppm was used. The graph shows that the NF of the EDFA increases with increasing signal input power. The variation of noise figure as a function of erbium ion density is given in figure 9.b for a constant fiber length and signal input power. These graphs were obtained using a 50 m long erbium doped fiber and $1 \mu\text{W}$ signal input power. From figure 9.b, it is seen that, NF remains constant from approximately 30 ppm even if the pumping power would be increased. The noise figure at 0 ppm density has not started from 0 dB due to some inability of the EDFA model used here. Beyond 30 ppm and for a 10 mW pumping power, insufficient pumping occurs and the noise figure sharply increases due to not to have population inversion. The pump powers of 20 mW or beyond is sufficient to obtain $n_{sp}=1$.

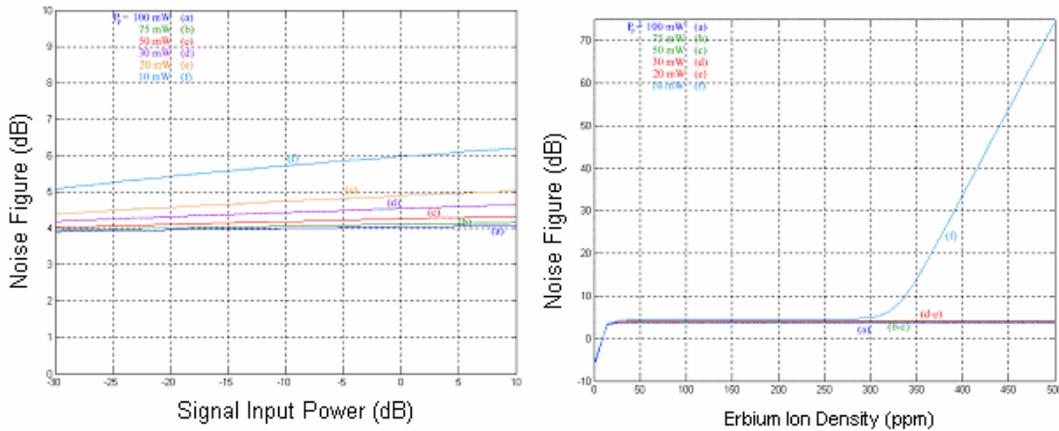


Figure 9. The variation of noise figure with a) Signal input power and b) Erbium ion density

5.4 ASE Power Characteristics

The variation of ASE power in EDFA with fiber length is shown in figure 10.a for the pump powers from 10 mW to 50 mW and for a constant signal input power and erbium doping density. This graph was obtained by applying $1 \mu\text{W}$ signal input power to a 50 m long EDF with a doping density of 140 ppm. The ASE power propagates in an EDFA in forward and backward directions. The one shown in the graph is the forward ASE which is the parameter taken into account in the NF calculations. As can be shown, the ASE power increases with increasing fiber length due to the gain provided inside EDFA and reaches to larger values for high pumping powers.

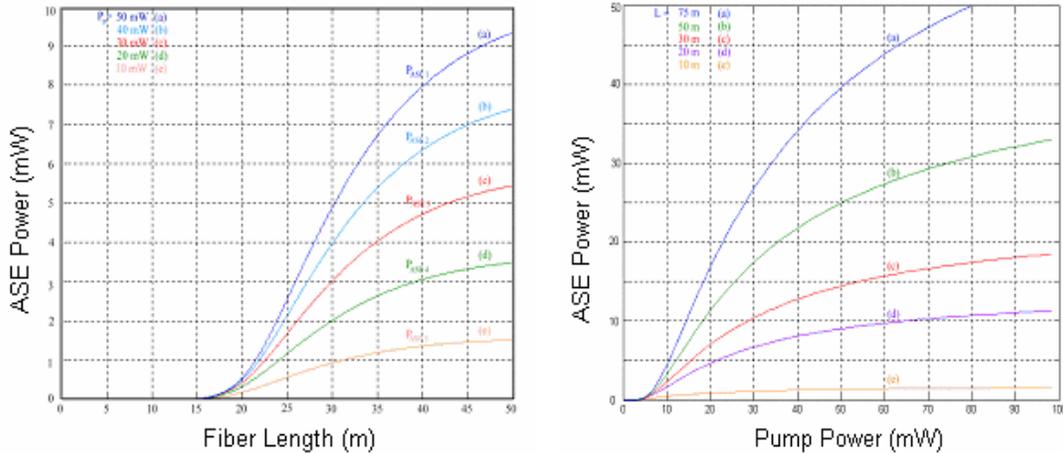


Figure 10. The variation of ASE power with a) Fiber length and b) Pumping power.

Figure 10.b shows the ASE power variation as a function of pump power for different fiber lengths, at a constant signal input power and erbium doping density. This graph was obtained for a five different fiber lengths having 245 ppm doping density and by applying a signal input power of $1 \mu\text{W}$ and increasing the pump power from 0 mW to 100 mW. As it can be shown, ASE power increases significantly with the increasing pump power. The main reason of that is to amplify not only the signal but also the spontaneous emission. The saturation of gain by the increasing pump power leads to reach the ASE power to a maximum level. This level is limited with erbium doping density.

The ASE power variations as a function of signal input power is shown in figure 11.a for a constant fiber length and erbium ion density. In this simulation, a 50 m long EDF with an erbium density of 140 ppm was used and the pump power was increased from 10 mW to 100 mW. As it is expected, higher stimulated emission and less spontaneous emission occur with increasing signal input power and therefore ASE power decreases.

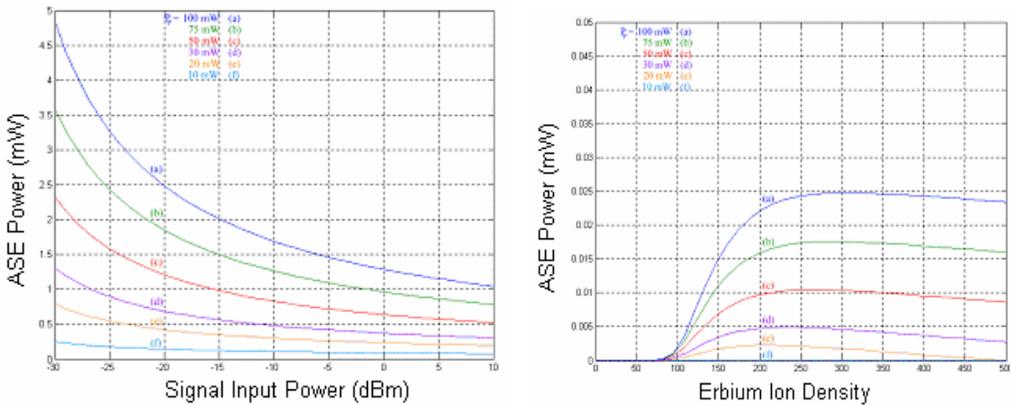


Figure 11. The variation of ASE power with a) Signal input power b) Erbium ion density

In figure 11.b, the ASE power variation is shown as a function of erbium ion density for a constant fiber length and signal input power. The simulation was realised for six different pump powers by applying $1 \mu\text{W}$ signal input power to a 50 m long EDFA. As it can be seen, ASE power sharply increases from a doping density of ~ 100 ppm and remains nearly constant at beyond ~ 250 ppm. Higher gains provided by injecting higher

pumping powers cause the ASE generated in the EDFA to be high.

6.RESULTS

In this study, the rate and propagation equations characterizing an EDFA operating in C band and pumped at 1480 nm in forward direction was numerically solved and the results were

graphically displayed. By entering the necessary parameters of an EDFA to be simulated into the main and sub menus of the simulation program; gain, noise figure and ASE power variations were obtained as functions of fiber length, pump power, signal input power and erbium doping density. In this way, the gain and NF performance could be simulated for the given EDFA parameters or the required fiber parameters and signal/pump power values could be optimised for a desired EDFA Gain-NF performance.

According to our results, it was seen that the pump power applied to EDFA sharply reduces due to absorption in erbium doped fiber; in addition, gain and NF is strongly dependent on the fiber length, pumping power, signal input power and erbium ion density. When the EDFA is supplied with sufficient pump power, it was shown that EDFA could be operated in saturation regimes leading to maximum gain and minimum NF. Due to the flexibility of the simulation program, it is possible to simulate EDFAs operating at 980 nm by using a few different parameters. These simulations can also be performed for distributed erbium doped fiber amplifiers (DEDFA) by activating the background loss coefficients.

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