ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2005 (1287-1293)
VOLUME : 5
NUMBER : 1

PROPAGATION PROPERTIES AND SCATTERING PARAMETERS OF INHOMOGENEOUSLY LOADED RECTANGULAR WAVEGUIDES

Serkan ŞİMŞEK[1] Ercan TOPUZ[2] Cevdet IŞIK[3]

[1] Istanbul University, Engineering Faculty, Department of Electrical and Electronics Engineering 34320, Avcılar, Istanbul, Turkey

[2,3] Istanbul Technical University, Electrical and Electronics Faculty, Department of Electronics and Communication Engineering, 80626, Maslak, Istanbul, Turkey

[1] **E-mail:** ssimsek@istanbul.edu.tr [2] **E-mail:** topuz@ehb.itu.edu.tr
[3] **E-mail:** isikcev@ehb.itu.edu.tr

ABSTRACT

In this study mode matching technique is used to obtain scattering parameter representations of waveguides loaded inhomogeneously with lossless dielectrics in the cross-sectional direction. The accuracy of our numerical solutions are assessed by comparing our results both with measured data and also, whenever available, with the results reported in the literature. Numerical results are presented to display the dependence of the scattering parameters on the electrical properties and geometrical dimensions of such structures.

Keywords: *Propagation, scattering parameters, inhomogeneous waveguides*

I. INTRODUCTION

Propagation in waveguides which are inhomogeneously loaded in transverse direction has been of interest for many years, since they find application in a variety of microwave components, including phase changers, matching transformers and quarter wave plates [1]. The propagation characteristics in inhomogeneously loaded rectangular waveguides (ILRW) are investigated for lossless dielectric loads for match-terminated symmetric and asymmetrical structures in [2-5]. ILRW are also used for determining relative complex permeability and permittivity of a rectangular post material [6-9]. Analysis of rectangular and circular dielectric posts are presented in [10,11] and [12,13], respectively. Scattering parameters are obtained for only lossless and symmetric rectangular posts in [10,11]. Frequency behaviour of ILRW are presented for symmetric and lossless structures in [10, 13], and for asymmetrical structures in [14]. However, the dependence of scattering

Received Date : 28.07.2004
Accepted Date: 14.01.2005

parameters on several parameters has not yet been throughly discussed in the literature.

In this study we address this problem using the mode matching technique and show that this approach yields a computationally efficient representation capable of producing accurate results under quite general loading conditions.

II. FORMULATION OF THE PROBLEM

A rectangular waveguide is considered which is inhomogeneously loaded with finite length dielectric strata placed along a direction perpendicular to the larger side wall of the waveguide. It is assumed that the dielectrics extend along the entire length of the narrower wall of the guide and that unloaded waveguide sections are dimensioned in such a way as to support the propagation of the dominant TE_{10} mode only.

Geometry of problem is shown in Fig. 1. Waveguide is inhomogeneously loaded along x direction. It is assumed that waveguide is excited by the dominant TE_{10} mode from the left and terminated by matched load from on the right. High order modes are created due to discontinuities. Under these conditions it sufficies to use mode matching technique involving only the TE_{m0} type modes in both loaded II ($0 \le z \le d$) and unloaded regions I, III ($z \le 0$, $z \ge d$).

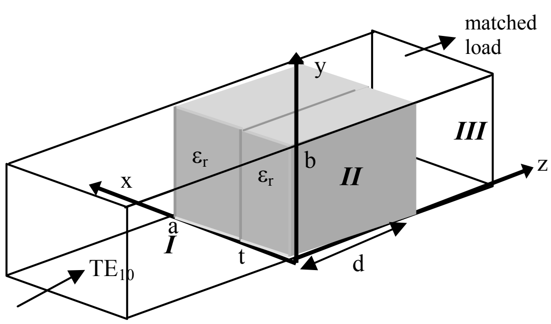


Figure 1. Inhomogeneously loaded rectangular waveguide.

E_y field components for regions I, III and II are written as

$$f_m(x) = A_1 \sin\frac{m\pi}{a}x \tag{1}$$

$$\bar{f}_m(x) = \bar{A}_1 \begin{cases} \sin k_1 x & 0 \le x \le t \\ \dfrac{\sin k_1 t}{\sin k_2(a-t)} \sin k_2(a-x) & t \le x \le a \end{cases} \tag{2}$$

where

A_1 and $\bar{A}_1$ coefficients are found invoking orthonormality,

$$\langle f_m, f_n \rangle = \langle \bar{f}_m, \bar{f}_n \rangle = \delta_{m,n} = \begin{cases} 1 & m = n \\ 0 & m \ne n \end{cases} \tag{3}$$

k_1 and k_2 are separation constants which are evaluated numerically by solving the transcendental eigenvalue equation for region II,

$$\det(M) = 0 \tag{4}$$

with

$$M = \begin{bmatrix} -\dfrac{\sin k_1 t}{k_1} & -\dfrac{\sin k_2(a-t)}{k_2} \\ \cos k_1 t & -\cos k_2(a-t) \end{bmatrix} \tag{5}$$

$$k_i = \sqrt{k_0^{\,2}\varepsilon_{ri} - \bar{\beta}^2} \quad i = 1,2 \tag{6}$$

$\bar{\beta}$ is propagation constant and ε_{ri} $i = 1,2$ relative dielectric constants of the dielectric layers ($0 \le x \le t$, $t \le x \le a$) in region II. As an example variation of $\bar{\beta}/k_0$ in WR90 the X-band waveguide is shown in Fig. 2 for $\varepsilon_{r1} = 2.56$, $\varepsilon_{r2} = 1$ and for various values of t (see Fig. 2).

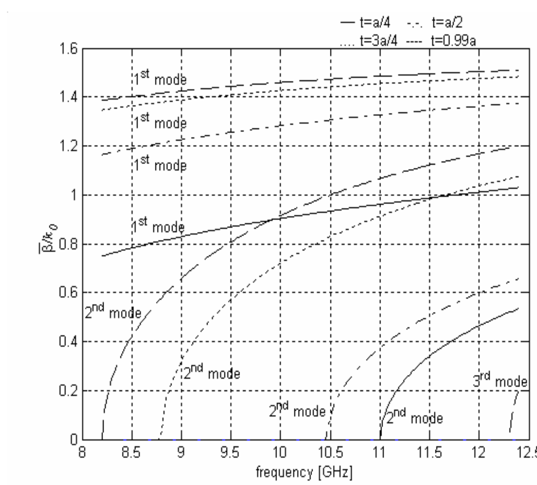


Figure 2. Variation of $\bar{\beta}/k_0$ with frequency.

It is clear from Fig. 2 that the waveguide supports only two modes for $t = a/4$, $t = a/2$, $t = 3a/4$, and three modes for $t = 0.99a$ in the frequency band 8.2-12.4 GHz.

In each region E_y and H_x fields are expressed in terms of modal components as

Region I: ($z \le 0$)

$$E_{y1} = f_1 e^{-j\beta z} + R f_1 e^{j\beta z} + \sum_{m=2}^{\infty} E_m^- f_m e^{\alpha_m z} \quad (7)$$

$$H_{x1} = f_1 Y_1 e^{-j\beta z} - R Y_1 f_1 e^{j\beta z} - \sum_{m=2}^{\infty} E_m^- f_m Y_m e^{\alpha_m z} \quad (8)$$

Region II: ($0 \le z \le d$)

$$E_{y2} = \sum_{m=1}^{\infty} \overline{E}_m^+ \bar{f}_m e^{-\overline{\alpha}_m z} + \sum_{m=1}^{\infty} \overline{E}_m^- \bar{f}_m e^{\overline{\alpha}_m z} \quad (9)$$

$$H_{x2} = \sum_{m=1}^{\infty} \overline{E}_m^+ \bar{f}_m \overline{Y}_m e^{-\overline{\alpha}_m z} - \sum_{m=1}^{\infty} \overline{E}_m^- \bar{f}_m \overline{Y}_m e^{\overline{\alpha}_m z} \quad (10)$$

Region III: ($z \ge d$)

$$E_{y3} = E_1^+ f_1 e^{-j\beta(z-d)} + \sum_{m=2}^{\infty} E_m^+ f_m e^{-\alpha_m (z-d)} \quad (11)$$

$$H_{x3} = E_1^+ Y_1 f_1 e^{-j\beta(z-d)} + \sum_{m=2}^{\infty} E_m^+ f_m Y_m e^{-\alpha_m (z-d)} \quad (12)$$

Here, R, E_m^-, $\overline{E}_m^{\pm}$ ve E_m^+ are expansion coefficients, f and $\bar{f}$ are E_y component in regions I, III and II, respectively. m mode index, α, Y and $\overline{\alpha}$, $\overline{Y}$ are attenuation constants and wave admittances for regions I, III and II respectively. β, α and Y are defined as

$$\beta = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2} \quad (13)$$

$$Y_1 = \frac{\beta}{\omega\mu_0} \quad (14)$$

$$\alpha_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - k_0^2} \qquad m > 1 \quad (15)$$

$$Y_m = -j\frac{\alpha_m}{\omega\mu_0} \qquad m > 1 \quad (16)$$

$\overline{\alpha}$ are $\overline{Y}$ found from $\overline{\beta}$ via as

$$\overline{\alpha}_m = j\overline{\beta}_m \quad (17)$$

$$\overline{Y}_m = -j\frac{\overline{\alpha}_m}{\omega\mu} \quad (18)$$

We now expand the total electric fields at z=0 (E_{a1}) and z=d (E_{a2}) planes in terms of a superposition over the modes in the dielectric loaded region; involving N modes,

$$E_{a1} \cong \sum_{t=1}^{N} C_t \bar{f}_t \quad (19)$$

$$E_{a2} \cong \sum_{t=1}^{N} D_t \bar{f}_t \quad (20)$$

Invoking continuity of tangential field components at z=0 and z=d planes one can obtain the expansion coefficients C and D in (19), (20) as

$$[C] = \{[Q] - [S][Q]^{-1}[S]\}^{-1}[U] \quad (21)$$

$$[D] = [Q]^{-1}[S][C] \quad (22)$$

The elements of the matrices in the above equations are defined as,

$$U_\gamma = 2Y_1 \langle f_1, \bar{f}_\gamma \rangle \quad (23)$$

$$Q_{\gamma,t} = \left[\sum_{m=1}^{P} Y_m \langle f_m, \bar{f}_\gamma \rangle \langle f_m, \bar{f}_t \rangle\right] + \overline{Y}_t \coth \overline{\alpha}_t d \, \delta_{\gamma,t} \quad (24)$$

$$S_{\gamma,t} = \frac{\overline{Y}_t \delta_{\gamma,t}}{\sinh \overline{\alpha}_t d} \quad (25)$$

where $\langle f, \bar{f} \rangle$ denotes integration over the cross section of the guide, and P in (24) stands for a suitably chosen upper limit. The sought after S parameter representation for the structure referred to planes z=0 and z=d are then obtained as

$$S_{11} \cong \sum_{t=1}^{N} C_t \langle \bar{f}_t, f_1 \rangle - 1 = S_{22} \quad (26)$$

$$S_{21} \cong \sum_{t=1}^{N} D_t \langle \bar{f}_t, f_1 \rangle = S_{12} \quad (27)$$

III. NUMERICAL CALCULATIONS

It should be noted that the S parameter representations given above are exact except for the truncation of the series representations of suitably chosen upper limits N and P. Clearly increasing the number of modes included into the representations will improve the accuracy of the calculated results at the expense of increasing the required computational time and resources. In Table 1 we have listed the magnitudes of S parameters using different number of modes (N=P) in the calculations. As can be seen from this table, 10 modes are sufficient to obtain excellent accuracy.

Table 1. Convergence of S parameters (t=a/4, $\varepsilon_{r1} = 2.56$, $\varepsilon_{r2} = 1$, f=12.1 GHz, d=1 cm).

Mode number	$\lvert S_{11}\rvert$	$\lvert S_{21}\rvert$
1	0.2289	0.9375
2	0.8576	0.5144
3	0.9859	0.1676
4	0.9963	0.0865
5	0.9972	0.0753
6	0.9972	0.0752
7	0.9972	0.0747
8	0.9973	0.0740
9	0.9973	0.0737
10	0.9973	0.0735
20	0.9973	0.0735
30	0.9973	0.0735

Our numerical results are then compared with the data given in [11] for the problem defined in the caption of Fig. 3. The variation of the amplitudes of S_{11} and S_{21} with ε_r as calculated from our model are shown in Fig. 3. They are in perfect agreement with the results reported in [11], so that the differences can not be discerned in the scale of the figure.

Frequency behaviour of ILRW and dependence of this behaviour on several parameters are given in Fig. 4, and Fig. 5.

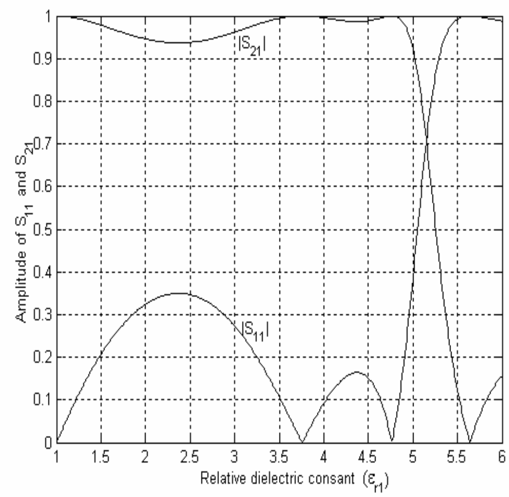


Figure 3. $\lvert S_{11}\rvert$ and $\lvert S_{21}\rvert$ for a rectangular post placed adjacent to a side wall ($t = a/2$, $d = a/2$, $\varepsilon_{r2} = 1$, $1 < \varepsilon_{r1} < 6$, $\lambda = 1.4a$).

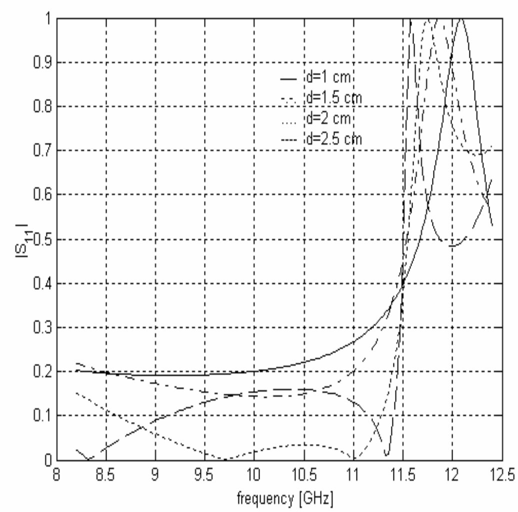


Figure 4. Amplitude of S_{11} for a rectangular post placed adjacent to a side wall for $t = a/4$, $\varepsilon_{r1} = 2.56$, $\varepsilon_{r2} = 1$.

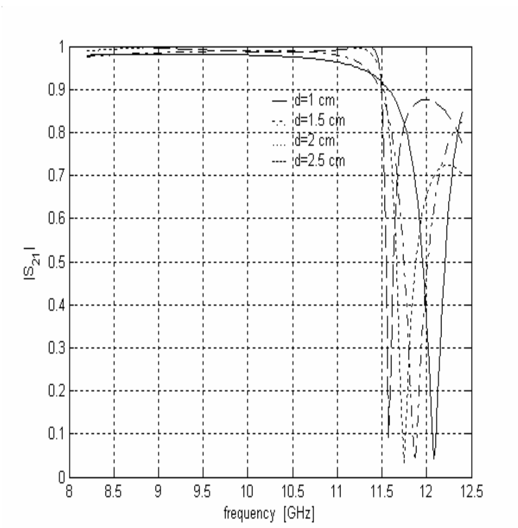


Figure 5. Amplitude of S_{21} for a rectangular post placed adjacent to a side wall for $t = a/4$, $\varepsilon_{r1} = 2.56$, $\varepsilon_{r2} = 1$.

IV. EXPERIMENTAL RESULTS

Relative dielectric constant of sample material (polypropylene) which completely fills the cross-section and extends over a length l_ε along the guide axis is measured using the setup given in Fig. 6.

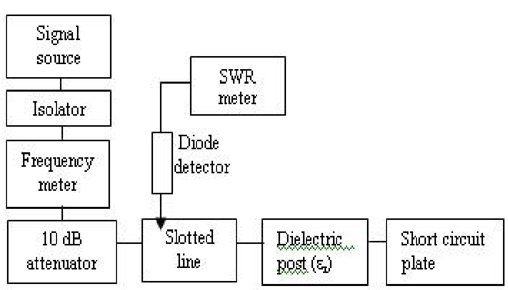


Figure 6. Dielectric measurement setup

Using two points method [15] ε_r is measured as 2.21 at f=11 GHz. We then measured the S parameters for two rectangular posts ($t = a/4$, $d = 1cm$) and ($t = a/4$, $d = 2cm$) over the entire X band region (8.2-12.4 GHz). The measured results are shown in Fig. 7, Fig. 8, Fig. 9 and Fig. 10 together with the theoretical values obtained via (26) and (27) using 10 modes in the expansions and the measured value of ε_r as input.

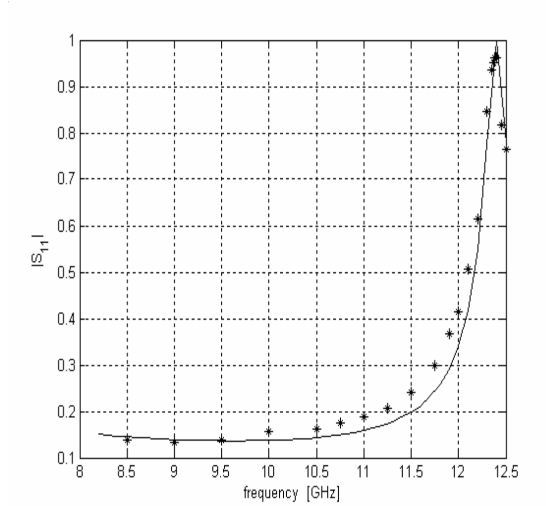


Figure 7. Measured (stars) and calculated (continuous curve) values of $|S_{11}|$ for $t = a/4$, $\varepsilon_{r2} = 1$, $\varepsilon_{r1} = 2.21$, $d = 1cm$.

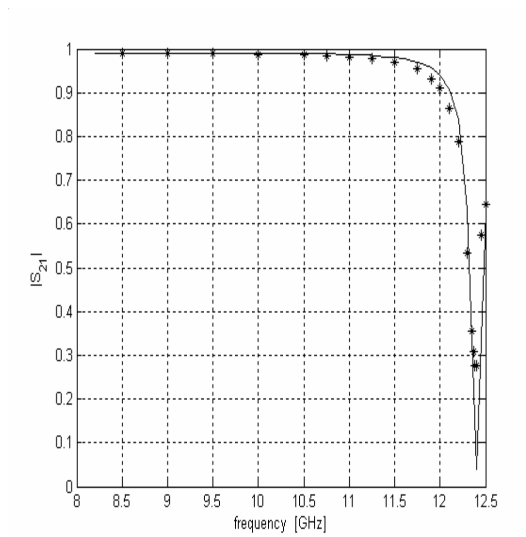


Figure 8. Measured (stars) and calculated (continuous curve) values of $|S_{21}|$ for $t = a/4$, $\varepsilon_{r2} = 1$, $\varepsilon_{r1} = 2.21$, $d = 1cm$.

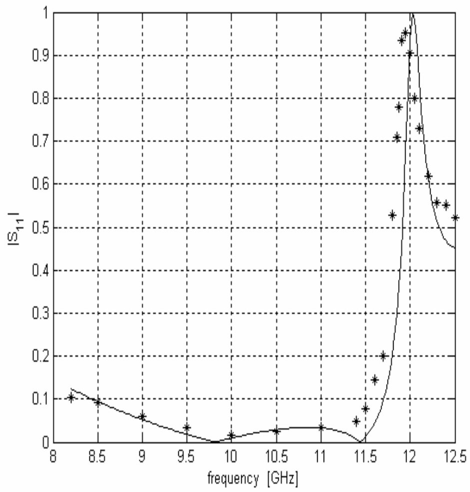


Figure 9. Measured (stars) and calculated (continuous curve) values of $|S_{11}|$ for $t = a/4$, $\varepsilon_{r2} = 1$, $\varepsilon_{r1} = 2.21$, $d = 2cm$.

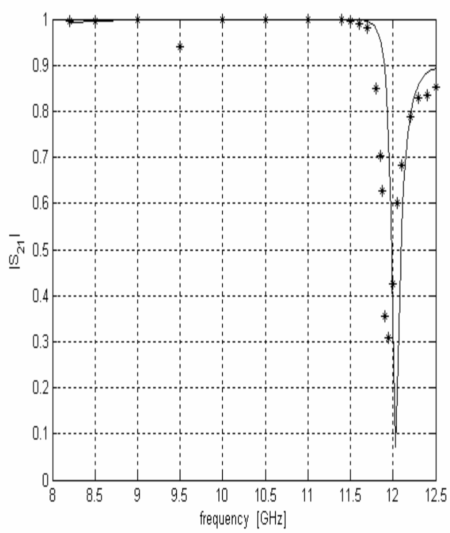


Figure 10. Measured (stars) and calculated (continuous curve) values of $|S_{21}|$ for $t = a/4$, $\varepsilon_{r2} = 1$, $\varepsilon_{r1} = 2.21$, $d = 2cm$.

As can be seen from these Figures the aggreement between measured and computed results are quite good and our error analysis reveals that the minor differences between them can be attributed to measurement errors.

CONCLUSIONS

The numerical solutions have shown that as the signal frequency is increased, more than one mode starts to propagate in the loaded waveguide and hence due to interference between these modes the field distribution at any cross-section exhibits a rapidly oscillating character as one proceeds along axial direction. Recalling that only dominant TE_{10} mode can propagate in the unloaded waveguide sections we deduce that the power transfer function of this device should also be a rapidly varying function of frequency. Our numerical results confirm these expectations.

When the frequency behaviour of S parameters of dielectric posts loaded waveguide is investigated as a function of the parameters ($t, \varepsilon_{r1}, \varepsilon_{r2}, d$) following general comments can be made:

- Increasing post dimensions both in transverse directions and also in the axial directions tends to increase the reflection coefficient.
- When the length (d) of the post along z direction is increased transmission bandwidth and resonance frequency decreases and the insertion loss increases.

The mode-matching approach reported in this work can be used to investigate the propagation characteristics of rectangular waveguides loaded with dielectric posts as a function of frequency. Moreover, the proposed method is quite versatile and yields accurate results within few CPU minutes on modest PC platforms. We believe that this method may effectively be used in aiding the design of microwave devices involving dielectric post loaded waveguides.

REFERENCES

[1] R. E. Collin, *Field Theory of Guided Waves*, The Institute of Electrical and Electronics Engineers, Inc., New York, 1991.

[2] P. H. Vartanian, W. P. Ayres, A. L. Helgesson, "Propagation in dielectric slab loaded rectangular waveguides," *IRE Trans. Microwave Theory Tech.*, 1957.

[3] R. Seckelmann, " Propagation of TE modes in dielectric loaded waveguides", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, No. 11, pp. 518-527 , Nov. 1966.

[4] K. F. Casey, " On inhomegeneously filled rectangular waveguides", *IEEE Trans. Microwave Theory Tech*., pp. 566-567 , Aug. 1973.

[5] F. E. Gardiol, " Higher-order modes in dielectrically loaded rectangular waveguides", *IEEE Trans. Microwave Theory Tech*., vol. MTT-16, No. 11, pp. 919-924 , Nov. 1968.

[6] S. Yoshikado, I. Taniguchi, " Microwave complex conductivity of a square post in rectangular waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-37, No. 6, pp. 984-992, June, 1989.

[7] J. Baker-Jarvis., E. J. Vanzura, W. A. Kissick, " Improved technique for determining complex permittivity with the transmission/ reflection method", *IEEE Trans. Microwave Theory Tech*., vol. MTT-38, No. 8, pp. 1096-1103, Aug. 1990.

[8] J. M. Jarem, J. B Johnson, W. S. Albritton, "Measuring the permittivity and permeability of a sample at K_a band using a partially filled waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-43, No. 12, pp. 2654-2667, Dec. 1995.

[9] J. M. Catala-Civera, A. J. Canos, F. L. Peneranda-Foix, E. L. Davo, " Accurate determination of the complex permittivity of materials with transmission reflection measurements in partially filled rectangular waveguides", *IEEE Trans. Microwave Theory Tech*., vol. MTT-51, No. 1, pp. 16-24, Jan. 2003.
[10] K. Siakavara, J. N. Sahalos, " The discontinuity problem of a rectangular dielectric post in rectangular waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-39, No. 9, pp. 1617-1622, Sep. 1991.

[11] J. Abdulnour, L. Marchildon, " Scattering by dielectric obstacle in rectangular waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-41, No. 11, pp. 1988-1994, Nov. 1993.

[12] Y. Leviatan, P. G. Li, A. T. Adams, J. Perini, "Single post inductive obstacle in rectangular waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-31, No. 10, pp. 806-812, Oct. 1983.

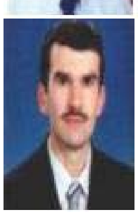
[13] P. G. Li, A. T. Adams, Y. Leviatan, J. Perini,"Multiple post inductive obstacles in rectangular waveguide", *IEEE Trans. Microwave Theory Tech*., vol. MTT-32, No. 4, pp. 365-373, April 1984.

[14] S. Şimşek, "Propagation properties of inhomogeneously loaded metallic waveguides", MSc. Thesis., Istanbul Technical University, 2003.

[15] M. Sucher, J. Fox, "Handbook of microwave measurements", Vol. II., Polytechnic Press of the Polytechnic Institute of Brooklyn, 1963.

Serkan Simsek *was born in Amasya, Turkey in 1979. He received his B.Sc. degree from the Istanbul University Department of Electrical and Electronics Engineering in 2001 and his M.Sc.degree from Istanbul Technical University, Department of Electronics and Communication Engineering in 2003. Since 2001 he has been working as a research assistant in the Department of Electrical and Electronics Engineering in I.U. His current research interests are electromagnetic theory, antennas and microwave techniques.*

Ercan Topuz *received MS. (1965) and PhD. (1973) from Istanbul Technical University, Electronics Dept. and is currently the head of the same Department. His research interests are EM theory, Microwave/Optical devices and systems.*



Cevdet Işık *was born in Adana, Turkey in 1951. He received his B.Sc., M.Sc. degrees and PhD. from the Istanbul Technical University, Faculty of Electrical and Electronics Engineering in 1974, 1977 and 1984 respectively. Since 1994 he has been working as an Assoc. Prof. in the Department of Electronics and Communication Engineering in I.T.U. His current research interests are emc, microstrip antennas and microwave measurement techniques.*