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# SLIDING MODE APPLICATION IN SPEED AND FLUX CONTROL OF AN INDUCTION MACHINE

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#### **ABSTRACT**

In this paper, a nonlinear control of squirrel induction motor is designed. We propose a sliding mode technique to control the field-oriented induction machine. Our aim is to make the speed and the flux control robust to parameter variations. The variation of motor parameters during operation degrades the performance of the controllers. The use of the nonlinear sliding mode method provides very good performance for motor operation and robustness of the control law despite the external/internal perturbation. Simulation results are given to highlight the performance of the proposed control technique under load disturbances and parameter uncertainties.

Keywords: Induction machine, Motor drives, robustness, sliding mode control

#### 1. INTRODUCTION

The induction machine is largely used in industry, mainly due to its reliability and relatively low cost. The control of the induction machine (IM) must take into account machine specificities: the high order of the model, the nonlinear functioning as well as the coupling between the different variables of control. Furthermore, the machine parameters depend generally on the operating point and vary either on the temperature (resistance), or with the magnetic state of the induction machine, without taking into account the variation. These parametric variations modify the performances of the control system when we use a regulator or a control law with fixed parameters. The new industrial applications necessitate speed variators having high dynamic performances, a good precision in permanent regime, and a high capacity of overload on all the range of speed and a robustness to the different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories. The variable structure control (VSC) possesses this robustness using the sliding mode control that can offer many good properties such good performance against unmodelled dynamics, insensitivity to parameter variation, external disturbance rejection and fast dynamic [1-6]. These advantages of sliding mode control can be employed in the position and speed control of an alternative current servo system. In the field Oriented Control (FOC) of induction machine, the knowledge of rotor speed and flux

is necessary. In this work the flux is obtained by

**Received Date :** 23.03.2006 **Accepted Date:** 27.05.2006 the measurement of stator voltages and currents. However the estimation is depended on machine parameters. Therefore, although sensorless vector-controlled drives are commercially available at this time, the parameter uncertainties impose a challenge in the control performances. The speed is measured.

In this paper, we begin with the IM oriented model in view of the vector control, next the rotor flux  $\Phi_r$ , is estimated. We, then, present the sliding mode theory and design the sliding mode controllers of rotor flux and motor speed. Finally, we give some conclusion remarks on the control proposed of IM using sliding mode.

## 2. INDUCTION MOTOR ORIENTED MODEL

An induction machine model can be described by the following state equations in the synchronous reference frame whose axis d is aligned with the rotor flux vector, ( $\Phi_{rd} = \Phi_r$  and  $\Phi_{rq} = 0$ ) [8-10]:

$$\dot{i}_{sd} = -\gamma i_{sd} + \omega_s i_{sq} + \frac{K}{T_r} \Phi_{rd} + \frac{1}{\delta L_s} u_{sd} \quad (1)$$

$$\dot{i}_{sq} = -\omega_s i_{sd} - \gamma i_{sq} - P\Omega K \Phi_{rd} + \frac{1}{\delta L} u_{sq} (2)$$

$$\dot{\boldsymbol{\Phi}}_{rd} = \frac{M_{sr}}{T_r} i_{sd} - \frac{1}{T_r} \boldsymbol{\Phi}_{rd} \tag{3}$$

$$\dot{\Phi}_{rq} = \frac{M_{sr}}{T_{.}} i_{sq} - (\omega_s - P\Omega) \Phi_{rd}$$
 (4)

$$\dot{\Omega} = \frac{PM_{sr}}{JL_r} (\Phi_{rd} i_{sq}) - \frac{T_l}{J} - \frac{f}{J} \Omega$$
 (5)

With:

$$T_{r} = \frac{L_{r}}{R} ; \delta = 1 - \frac{M_{sr}^{2}}{L_{s}L_{r}} ; K = \frac{M_{sr}}{\delta L_{s}L_{r}} ;$$

$$\gamma = \frac{R_s}{\delta L_s} + \frac{R_r M_{sr}^2}{\delta L_s L_r^2}$$

Where  $\Phi_{rd}$ ,  $\Phi_{rq}$  are rotor flux components,  $u_{sd}$ ,  $u_{sq}$  are stator voltage components,  $i_{sd}$ ,  $i_{sq}$  are stator current components,  $\delta$  is leakage factor and p is number of pole pairs.  $R_s$  and  $R_r$  are

stator and rotor resistances,  $L_s$  and  $L_r$  denote stator and rotor inductances, whereas  $M_{sr}$  is mutual inductance.  $T_e$  is the electromagnetic torque,  $T_l$  is the load torque, J is the moment of inertia of the IM,  $\Omega$  is mechanical speed,  $\omega_s$  is stator pulsation, f is damping coefficient,  $T_r$  is rotoric time-constant.

### 3. FLUX ESTIMATOR

For direct field-oriented control of induction machine, accurate knowledge of the magnitude and the position of the rotor flux vector is necessary. In a normal cage motor, as rotor current are not measurable, the rotor flux should be estimated. Various types of estimators and observes have been proposed in the literature. The flux estimator, used in this work is based on the integration of the stator voltage equations in the stationary frame. The flux estimator can be obtained by the following equations [12]:

$$\dot{\hat{\Phi}}_{r\alpha} = \frac{L_r}{M_{sr}} \left( u_{s\alpha} - R_s i_{s\alpha} - \delta L_s i_{s\alpha} \right) \tag{6}$$

$$\dot{\hat{\Phi}}_{r\beta} = \frac{L_r}{M_{cr}} \left( u_{s\beta} - R_s i_{s\beta} - \delta L_s i_{s\beta} \right) \tag{7}$$

 $\theta_s$  is the angle between rotoric vector flux  $\Phi_r$  and the axis of the  $(\alpha, \beta)$  frame

$$\theta_s = \arctan\left(\frac{\hat{\Phi}_{r\beta}}{\hat{\Phi}_{r\alpha}}\right) \tag{8}$$

Where  $\hat{\Phi}_{r\alpha}$ ,  $\hat{\Phi}_{r\beta}$  are the estimated rotor flux components,  $i_{s\alpha}$ ,  $i_{s\beta}$  are the measured stator curent components.

## 4. SLIDING MODE CONTROL DESIGN

Sliding mode control is developed from variable structure control. It is a form of linear control providing a robust means of controlling the nonlinear plants with disturbances and parameters uncertainties.

Sliding mode is the technique to adjust feedback by previously defining a surface so that the system which is controlled will be forced to that surface then the behavior system slides to the desired equilibrium point.

This control consists in two phases:

The first phase is choosing a sliding manifold have a desired dynamics, usually linear and of a lower order.

The second phase is designing a control law, which will drive the state variable to the sliding manifold and will keep them there.

The design of the control system will be demonstrated for a following nonlinear system: [1-7]

$$\dot{\mathbf{x}} = f(\mathbf{x}, t) + B(\mathbf{x}, t) \ u(\mathbf{x}, t)$$
 (9)

Where  $\mathbf{x} \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$  is the control vector,  $f(\mathbf{x},t) \in \mathbb{R}^n$ ,  $B(\mathbf{x},t) \in \mathbb{R}^{nxm}$ . From the system (9), it possible to define a set S of the state trajectories  $\mathbf{x}$  such as:

$$S = \left\{ \mathbf{x}(t) \mid \sigma(\mathbf{x}, t) = 0 \right\}$$
 (10)

Where

$$\sigma(\mathbf{x},t) = [\sigma_1(\mathbf{x},t),\sigma_2(\mathbf{x},t),...,\sigma_m(\mathbf{x},t)]^T$$
(11)

and  $[.]^T$  denotes the transposed vector, S is called the sliding surface.

To bring the state variable to the sliding surfaces, the following two conditions have to be satisfied:

$$\sigma(\mathbf{x},t) = 0$$
,  $\dot{\sigma}(\mathbf{x},t) = 0$  (12)

The control law satisfies the precedent conditions is presented in the following form:

$$u = u^{eq} + u^n. (13)$$

Where u is the control vector,  $u^{eq}$  is the equivalent control vector,  $u^n$  is the switching part of the control (the correction factor)

 $u^{eq}$  can be obtained by considering the condition for the sliding regimen,  $\sigma(\mathbf{x},t)=0$ . The equivalent control keeps the state variable on sliding surface, once they reach it.

 $U^n$  is needed to assure the convergence of the system states to sliding surfaces in finite time.

In order to alleviate the undesirable chattering phenomenon, J. J. Slotine proposed an approach to reduce it, by introducing a boundary layer of width  $\phi$  either side of the switching surface [3].

Then,  $u^n$  is defined by

$$u^{n} = K \ sat(\sigma(\mathbf{x})/\phi) \tag{14}$$

Where  $sat(\sigma(\mathbf{x})/\phi)$  is the proposed saturation function,  $\phi$  is the boundary layer width, K is the controller gain designed from the Lyapunov stability

$$V = \frac{1}{2}\sigma^2 \tag{15}$$

$$\dot{V} = \frac{1}{2} \frac{d}{dt} \sigma^2 = \sigma \dot{\sigma} \le -\eta |\sigma| \tag{16}$$

Where  $\eta$  is a strictly positive constant.

In this work,  $s(\mathbf{x},t)$  is the sliding mode vector proposed by J. J. Slotine [3].

$$\sigma(\mathbf{x}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{17}$$

Where

 $\mathbf{x} = \begin{bmatrix} \mathbf{x}, \dot{\mathbf{x}}, ... \mathbf{x}^{n-1} \end{bmatrix}^T \text{ is the state vector,}$   $\mathbf{x}^d = \begin{bmatrix} \mathbf{x}^d, \dot{\mathbf{x}}^d, \mathbf{x}^d \end{bmatrix}^T \text{ is the desired state vector,}$   $e = \mathbf{x}^d - \mathbf{x} = \begin{bmatrix} e, \dot{e}, ..., e^{n-1} \end{bmatrix} \text{ is the error}$ 

vector, and  $\lambda$  is a vector of slopes of the S.

Commonly, in IM control using sliding mode theory, the surfaces are chosen as functions of the error between the reference input signal and the measured signals [2]

After this step, the objective is to determine a control law which drives the state trajectories along the surface.

#### 4.1 SPEED CONTROL

To control the speed of the induction machine, three surfaces are chosen. Variables of control are the rotation speed  $\varOmega$  and the flux  $\varPhi_r$ . The flux will be maintained at its nominal value to have a maximal torque.

We take n=1, the speed control manifold equations can be obtained as:

(20)

$$\sigma(\Omega) = \Omega_{ref} - \Omega \tag{18}$$

$$\dot{\sigma}(\Omega) = \dot{\Omega}_{ref} - \dot{\Omega} \tag{19}$$

Substituting the expression of  $\Omega$  equation 5 in equation (19), we obtain:

$$\dot{\sigma}(\Omega) = \dot{\Omega}_{ref} - \left(\frac{PM_{sr}}{JL_r}(\Phi_{rd}i_{sq}) - \frac{T_l}{J} - \frac{f}{J}\Omega\right)$$

We take

$$i_{sq} = i_{sq}^{eq} + i_{sq}^{n} (21)$$

During the sliding mode and in permanent regime, we have:

$$\sigma(\Omega) = 0$$
,  $\dot{\sigma}(\Omega) = 0$ ,  $i_{sq}^n = 0$  where the equivalent control is.

$$i_{sq}^{eq} = \frac{JL_r}{PM_{sr}\boldsymbol{\Phi}_{rd}} \left( \dot{\boldsymbol{\Omega}}_{ref} + \frac{f}{J}\boldsymbol{\Omega} + \frac{T_l}{J} \right) \quad (22)$$

During the convergence mode, the condition  $\sigma(\Omega)\dot{\sigma}(\Omega) \leq 0$  must be verified. we obtain :

$$\dot{\sigma}(\Omega) = -\frac{P^2 M_{sr} \Phi_{rd}}{J L_r} i_{sq}^n \tag{23}$$

Therefore, the correction factor is given by:

$$i_{sq}^{n} = Ki_{sq}sat(\sigma(\Omega))$$
 (24)

To verify the system stability condition, the parameter  $Ki_{sq}$  must be positive.

## 4.2 STATOR CURRENT CONTROL AND LIMITATION

In order to limit all possible overshoot of the current  $i_{sq}$ , we add a limiter of current defined by

$$i_{sq}^{\lim} = i_{sq}^{\max} sat(i_{sq}) \tag{25}$$

The current control manifold is

$$\sigma(i_{sq}) = i_{sq}^{\lim} - i_{sq} \tag{26}$$

$$\dot{\sigma}(i_{sq}) = \dot{i}_{sq}^{\lim} - \dot{i}_{sq} \tag{27}$$

Substituting the expression of  $i_{sq}$  equation 2 in equation (27), we obtain:

$$\dot{\sigma}(i_{sq}) = \dot{i}_{sq}^{\lim} - \left( -\omega_s i_{sd} - \gamma i_{sq} - P\Omega K \Phi_{rd} + \frac{1}{\delta L_s} u_{sq} \right)$$
(28)

The control voltage is

$$u_{sa}^{ref} = u_{sa}^{eq} + u_{sa}^{n} \tag{29}$$

$$u_{sq}^{eq} = \delta L_s \left( \dot{i}_{sq}^{\lim} + \omega_s i_{sd} + \gamma i_{sq} + P \Omega K \Phi_{rd} \right)$$

$$u_{sq}^{n} = Ku_{sq}sat(\sigma(i_{sq}))$$
(31)

To verify the system stability condition, the parameter  $K_{uqs}$  must be positive

### 4.3 FLUX CONTROL

In the proposed control, we take n=2 to appear control  $u_{sd}$ , the manifold equation can be obtained by:

$$\sigma(\boldsymbol{\Phi}_r) = \lambda_{\boldsymbol{\Phi}} (\boldsymbol{\Phi}_r^{ref} - \boldsymbol{\Phi}_r) + (\dot{\boldsymbol{\Phi}}_r^{ref} - \dot{\boldsymbol{\Phi}}_r)$$
(32)

The control voltage

$$u_{sd} = u_{sd}^{eq} + u_{sd}^n \tag{33}$$

$$u_{sd}^{eq} = \mathcal{I}_{s} \left( \left( \dot{\mathcal{Q}}_{r}^{ef} + \lambda_{tp} \dot{\mathcal{Q}}_{r}^{ef} + \left( \frac{1}{T_{r}} - \lambda_{tp} \right) \dot{\mathcal{Q}}_{r} \right) \frac{T_{r}}{M_{sr}} - \left( -\gamma_{sd} + \omega_{s} i_{sq} + \frac{K}{T_{r}} \mathcal{Q}_{rd} \right) \right)$$

(34)

(30)

$$u_{sd}^{n} = Ku_{sd} \quad sat(\sigma(\Omega))$$
 (35)

To verify the system stability condition, the parameter  $Ku_{sd}$  must be positive.

The selection of coefficients  $Ki_{sq} Ku_{sd}$ ,  $Ku_{sq}$  and  $\lambda$ , must be done in order to satisfy

and  $\lambda_{\Phi}$  must be done in order to satisfy following requirements:

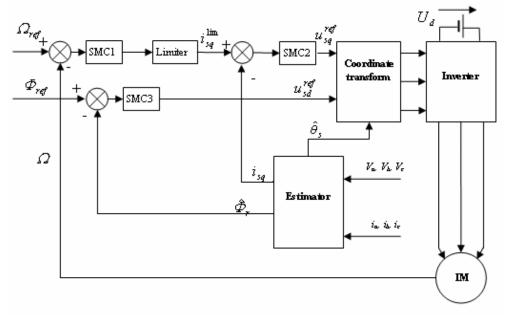
- Existence condition of the sliding mode, which requires that the state trajectories are directed toward the sliding manifold,
- Hitting condition, which requires that the system trajectories encounter the manifold sliding irrespective of their starting point in the state space(insure the rapidity of the convergence),
- Stability of the system trajectories on the sliding manifold,
- Not saturate the control to allow the application of the control discontinuous.

## 5. SIMULATION SCHEME OF THE PROPOSED CONTROL

The bloc diagram of the proposed robust control scheme is presented in figure 1.The blocks SMC1, SMC2, SMC3 represent the proposed sliding mode controllers. The block limiter limits

the current within the limits values. The block 'Coordinate transform' makes the conversion between the synchronously rotating and stationary reference frame. The block 'Inverter' shows that the motor is voltage fed. The block ' $i_{sq}$  and  $\Phi_r$  Estimator' represents respectively the estimated stator current  $i_{sq}$  and the rotor flux  $\Phi_r$ . The block 'IM' represents the induction motor. The IM used in this work is a 1.5 kW,

U=220 V, 50 Hz, P=2,  $I_n$ =6.1 A,  $\Phi_n$ =0.595 Wb. IM parameters:  $R_s$ =1.47  $\Omega$ ,  $R_r$ =0.79  $\Omega$ ,  $L_r$ = $M_{sr}$ =0.094 H,  $L_s$ =0.105H. The system has the following mechanical parameters: J=0.0.0256 Nm/rad/s², f=0.0029 Nm/rad/s. The global system is simulated using MATLAB/SIMULINK software.



**Fig. 1** Block diagram of the proposed control scheme of IM using the sliding mode controllers

#### 5.1. SIMULATION RESULTS

The proposed control has been tested to illustrate its performances, we simulated a loadless starting up mode with the reference speed  $\pm Q_n$  and an application of the load torque ( $T_l = \pm 10$  Nm) at time 1s to 2s and at time 3s to 4s, the reference flux is 0.595Wb (fig.2).

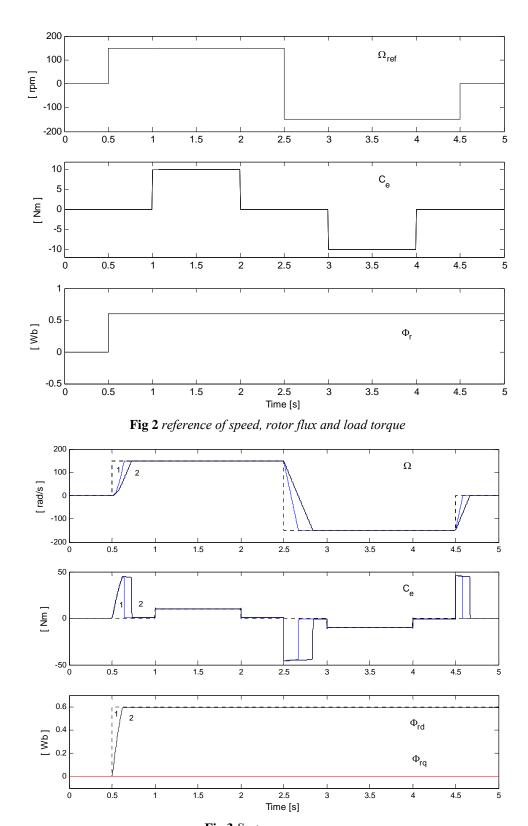
In order to test the robustness of the proposed control, we have studied the speed performances with current limitation. The introduced variations in tests look in practice to work conditions as the magnetic circuit overheating and saturation. Three cases are considered:

- 1. inertia variation,
- 2. Stator and rotor resistance variations
- Stator and rotor inductance and mutual variations.

Figure 3 shows the robustness tests in relation to inertia variations.  $J = m J_n$ ,  $1 \le m \le 2$ .

Figure 4 shows the robustness tests to the stator and rotor resistance variations. Figure 5 shows the robustness tests to with stator and rotor inductance and mutual variations.

The figures 3, 4 and 5 show the parameter variation does not allocate performances of proposed control. The decoupling between the flux and torque is assured in permanent regime in spite of large variations. The flux tracks the desired flux and it is insensitive to parameter variations of the machine. It shows also the limited started torque and the speed response stays insensitive to parameter variations of the machine, without overshoot and without static error, the perturbation reject is instantaneous.



**Fig.3** System responses (Curves 1: Nominal case; curves 2: an increase of Inertia  $2J_n$ )

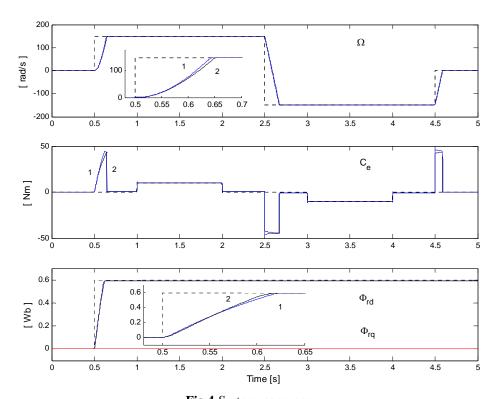


Fig.4 System responses (Curves 1: Nominal case; curves 2: an increase of resistances  $1.5R_s$ ,  $1.5R_r$ )

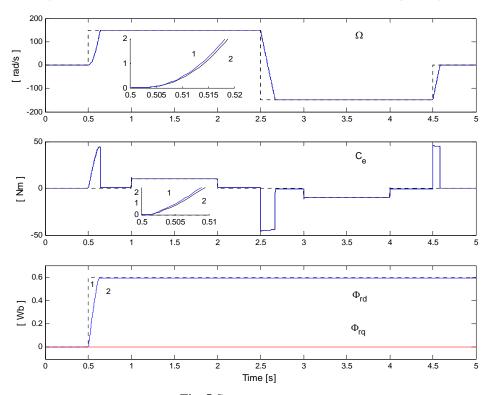


Fig. 5 System responses (Curves 1: Nominal case; curves 2: an increase of inductances  $1.2L_s$ ,  $1.2L_r$ )

### 5. CONCLUSION

A sliding mode control method has been proposed and used for the control of an induction machine using field oriented control. A simple algorithm to estimate the rotor flux is presented. Simulation Results show good performances obtained with proposed control. It has been shown the robustness of proposed control in relation to the presence internal and external perturbations. With a good choice of parameters of control and the smoothing out control discontinuity, the chattering effects are reduced, and the torque fluctuations are decreased. The speed control operates with enough stability and has strong robustness to parameter variations. Furthermore, this regulation presents a simple robust control algorithm that has the advantage to be easily implantable in calculator

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