ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2007 (315 - 322)
VOLUME : 7
NUMBER : 1

SLIDING MODE CONTROL OF A PERMANENT MAGNET SYNCHRONOUS MACHINE FED AN MLI TENSION INVERTER

A. MEROUFEL A. MASSOUM B. BELABBES MK FELLAH

Engineer Sciences Faculty-Power Department
Intelligent Control & Electrical Power Systems Laboratory (ICEPS)
Djillali Liabes University BP 98 Sidi Bel Abbes 22000 Algérie

E-mail : ameroufel@yahoo.fr

ABSTRACT

In this article, we study the control by sliding mode (CSM) applied to the permanent magnet synchronous machine (PMSM) supplied with a tension inverter with triangulo-sinusoidal -MLI. The control by sliding mode is well adapted to the nonlinear systems. It is characterized by properties of robustness with respect to the external disturbances and parameters variations . The sliding surface is given on the basis of the system and desired performances. While the law of control is selected with an aim of ensuring the convergence conditions and sliding i.e. the attractivity of commutation surfaces. A comparative analysis of all CSM advantages to those of the classical control (PI) was developed in last stage to highlight the performances of this technique. The various results obtained in simulation allow an evaluation of the robustness and performances of this mode of control variable structure.

Keywords: *: PMSM, sliding mode control, MLI tension inverter and classical control.*

1. INTRODUCTION

The progress recorded in the materials field (permanent magnet) allowed the PMSM several industrial applications in particular in robotics. It offers several advantages to know a high specific power, a significant starting torque, a reduction of maintenance etc... [1,2]. In addition the existing coupling between flux and torque makes the PMSM hardly commandable. With the technique of the flux orientation, it is nowadays possible to obtain AC current speed variator as powerful as the DC speed variator [2]. The algorithms of classical control (PI or PID) for the current and speed regulation prove to be insufficient especially if the system is tested under intern or external parametric variations [3,4]. To solve this problem, it is necessary to use the methods of nonlinear control.

Sliding mode control has advantages for the badly identified systems or with variable parameters [3,4,5]. However the discontinuous term of the control algorithm can cause the effect " chattering ". Several solutions were proposed to reduce this phenomenon by improving the function " sgn(s) " [3,4,5]. In this work we present the model of the PMSM in the reference frame of PARK then we develop a control algorithm with variable structure with the objective to regulate the speed and the components of the stator current by the technique of the CSM by using the method of Lyapunov.

Received Date :29.12.2006
Accepted Date: 01.02.2007

The forward current of reference is imposed null to ensuring a maximum Torque. To evaluate the robustness and the performances of the CSM, we finish with a comparative analysis of the results of simulation with those of the classical control.

2. MODEL MATHEMATIQUE OF THE PMSM

With the simplifying conditions [7], the model of the PMSM expressed in the reference frame related to the rotor is written in the form of:

2.1. Nonlinear Model

2.1.1. State Representation

$$\begin{aligned} \dot{X} &= F(X) + GU \\ Y &= H(X) \end{aligned} \quad (1)$$

With

$$X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} I_d \\ I_q \\ \Omega \end{pmatrix} \quad U=\begin{pmatrix} U_d \\ U_q \end{pmatrix} \quad G=\begin{pmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{pmatrix}=\begin{pmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{pmatrix} \quad (2)$$

$$F(X)=\begin{pmatrix} f_1(X) \\ f_2(X) \\ f_3(X) \end{pmatrix}=\begin{pmatrix} a_1x_1+a_2x_2x_3 \\ b_1x_2+b_2x_1x_3+b_3x_3 \\ c_1x_3+c_2x_1x_2+c_3x_2-C_r/J \end{pmatrix} \quad (3)$$

$$a_1=\frac{-R}{L_d} \qquad a_2=\frac{pL_q}{L_d}$$

$$b_1=\frac{-R}{L_q} \qquad b_2=\frac{-pL_d}{L_q} \qquad b_3=\frac{-p\varphi_f}{L_q}$$

$$c_1=\frac{-f}{J} \qquad c_2=\frac{p(L_d-L_q)}{J} \qquad c_3=\frac{p\varphi_f}{J}$$

The variables to be controlled are current Id and mechanical speed Ω.

$$Y(X)=\begin{pmatrix} y_1(X) \\ y_2(X) \end{pmatrix}=\begin{pmatrix} h_1(X) \\ h_2(X) \end{pmatrix}=\begin{pmatrix} x_1 \\ x_3 \end{pmatrix}=\begin{pmatrix} I_d \\ \Omega \end{pmatrix} \quad (4)$$

2.1.2. Relative degree of the output

The relative degree of an output is the number of times that it is necessary to derive to reveal the input U.

a- relative Degree of the current Id

$$\dot{y}_1(X) = L_f h_1(X) + L_g h_1(X) U_d \quad (5)$$

with

$$\begin{aligned} L_f h_1(X) &= f_1(X) \\ L_g h_1(X) &= (g_1 \quad 0) \end{aligned} \quad (6)$$

the relative degree of $y_1(X)$ is r1 = 1.

b- Relative degree of mechanical speedΩ

$$\begin{aligned} \dot{y}_2(X) &= L_f h_2(X) \\ \ddot{y}_2(X) &= L_f^2 h_2(X) + L_g L_f h_2(X) U \end{aligned} \quad (7)$$

with

$$L_f h_2(X) = f_3(X)$$

$$L_f^2 h_2(X) = c_2x_2f_1(X) + f_2(X)(c_3+c_2x_1) + c_1f_3(X)$$

$$L_gL_fh_2(X) = [c_2x_2g_1 \quad g_2(c_2x_1+c_3)]$$

The relative degree of $y_2(X)$ is r2 = 2

2.2. Linear model

2.2.1. State Representation

- Electrical System

$$(\dot{X}_1 = A_1X_1 + B_1U_1)$$

with

$$X_1^T = (I_d \quad I_q)$$

$$A_1=\begin{pmatrix} a_1 & a_2 \\ b_2 & b_1 \end{pmatrix} \; ; \; B_1=\begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix} \quad (8)$$

- Mechanical Equation

$$\begin{aligned} \frac{d\Omega_r}{dt} &= \frac{1}{J}(C_e - C_r - f\Omega_r) \\ C_e &= p[(L_d - L_q)I_d + \varphi_f]I_q \end{aligned} \quad (9)$$

3. VECTORIAL CONTROL OF THE PMSM

3.1. Principle of the control

The principle of the vectorial control consists in comparing the AC current machine with a D.C current machine from the point of view torques [7]. This results by holding component I_d null and the control speed by the component I_q.

Thus the model of the machine is simplified and the equations governing the PMSM are written [7].

$$\dot{X}_S = A_S X_S + B_S U_S$$
$$Y_S = C_S^T X_S \qquad (10)$$

with

$$X_S^T = (I_q \ \ \Omega_r)\,; \quad C_S^T = [0\,1]; \quad U_S = U_q$$
$$A_S = \begin{pmatrix} b_1 & b_3 \\ c_3 & c_1 \end{pmatrix} \quad ; \quad B_S^T = (1/L_q \ \ 0)$$

3.2. Classical control

3.2.1. Compensation Decoupling

To uncouple perfectly the axes d and q, we add on the output of the controllers the fem (e_d, e_q) of compensation. If the compensation is exact, the components of the stator current depend only on their reference.

$$U_q^* - e_q = U_q \ ; \quad U_d^* - e_d = U_d$$
$$U_q = (R - L_q s) I_q \qquad (11)$$
$$U_d = (R - L_d s) I_d$$

with

$$e_q = p\Omega_r L_d I_d + p\Omega_r \varphi_f$$
$$e_d = -p\Omega_r L_q I_q$$

3.2.2. Functional diagram

For the components of the stator current, we choose regulators PI. While for speed, we choose a regulator PI with anti-windup in order to control the variable during the transition phase.

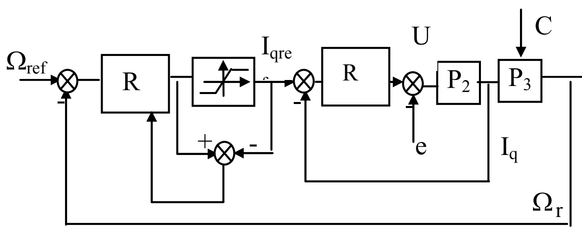

***Fig 1.** The cascade control relating to q axis*

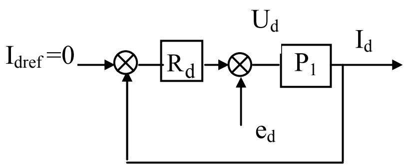

***Fig 2.** Control of Id component*

- The different regulators PI

$$R_\Omega = \frac{1 + T_{n\Omega} s}{T_{i\Omega} s}$$
$$R_q = \frac{1 + T_{nq} s}{T_{iq} s} \qquad (12)$$
$$R_d = \frac{1 + T_{nd} s}{T_{id} s}$$

- The different subsystems

$$P_1 = \frac{1}{R + L_d s}$$
$$P_2 = \frac{1}{R + L_q s} \qquad (13)$$
$$P_3 = \frac{k_t}{f + Js}$$

4. CONTROL BY SLIDING MODE OF PMSM

The design of the algorithm of the sliding mode control consists mainly of determining three stages [6,7].

4.1. Choice of commutation surfaces

J.J. Slotine proposes a form of general equation to determine the sliding surface [6,7].

$$S(X) = \left(\frac{d}{dt} + \lambda \right)^{r-1} e \qquad (14)$$

The error: $e = X_d - X$

λ : Positive coefficient

r: Relative degree:

X_d : Desired value

4.2. Convergence Condition

The convergence condition is defined by the Lyapunov equation [5,6,7].)

$$S(X).\dot{S}(X) < 0 \qquad (15)$$

4.3. The control calculation

The algorithm of control includes two terms, the first for the exact linearization, the second discontinuous for the system stability [6,7].

$$U_c = U_{eq} + U_n \quad (16)$$

- U_{eq} is calculated from the expression $\dot{S}(X)=0$
- U_n is given to guarantee the attractivity of the variable to be controlled towards the commutation surface. The simplest equation has the form of relay

$$U_n = k \operatorname{sgn} S(X), \quad k > 0 \quad (17$$

k high can cause the phenomenon of ' chattering'

4.4. Elimination of the' chattering' phenomenon

This phenomenon of high frequency oscillation can be reduced by replacing the function ' sgn' by a function of saturation [6,7].

$$U_n = \begin{cases} \frac{k}{\varepsilon} S(X) & \text{si } |S(X)| < \varepsilon \\ k \operatorname{sgn}(S(X)) & \text{si } |S(X)| > \varepsilon \\ \varepsilon > 0 \end{cases} \quad (18)$$

5. APPLICATION OF THE SLIDING MODE TO THE PMSM

5.1. Basic structure of the sliding mode regulator

By using the flux orientation principle and by neglecting the converter time-constant, we can represent the model of the PMSM in the d-q reference frame in two independent subsets [6,7]

- Control according to the d axis

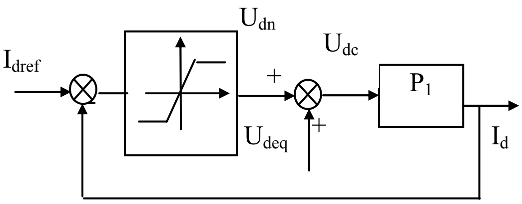

Fig 3. SMC of I_d

- Control according to the q axis

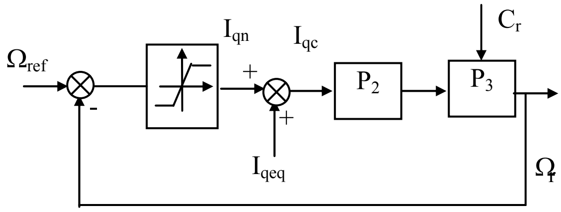

Fig 4. SMC relative à l'axe q

The system P2 is identical to the system relative to the d axis with Iqc=Iqref and the index of the tensions is replaced by q.

5.2.The controllers Synthesis

5.2.1. Surfaces Choices

We choose the sliding surface according to the relation of Slotine and the relative degree of the output [7,8,9].

$$\begin{aligned} S_1(I_d) &= I_{dref} - I_d \\ S_2(I_q) &= I_{qref} - I_q \\ S_3(\Omega_r) &= \Omega_{ref} - \Omega_r \end{aligned} \quad (19)$$

For the control in cascade, we distribute the order of surface $S_3(\Omega_r)$ between speed and the component I_q

5.2.2. Control Calculation

- CSM relating to the d axis

$$\dot{S}_1(I_d)=0 \Rightarrow U_{deq}=\frac{1}{g_1}(\dot{I}_{dref}-f_1)$$

$$\dot{S}_1(I_d).S_1(I_d)<0 \Rightarrow \quad (20)$$

$$U_{dn}=\begin{cases} \frac{k_d}{\varepsilon_d} S_1(I_d) & \text{si } |S_1(I_d)| \le \varepsilon_d \\ k_d \operatorname{sgn} S_1(I_d) & \text{si } |S_1(I_d)| > \varepsilon_d \end{cases}$$

Finally the law of control relative to the d axis is written

$$U_{dc}=U_{deq}+U_{dn}=\frac{1}{g_1}(\dot{I}_{dref}-f_1)+k_d \operatorname{sgn} S_1(I_d)$$

(21)

- SMC relative to the q axis

It has two loops of cascades on this axis, that of the current I_q is internal

a- SMC relative to the quadratic current

$$\dot{S}_2(I_q)=0 \Rightarrow U_{qeq}=\frac{1}{g_2}(\dot{I}_{qref}-f_2)$$

$$\dot{S}_2(I_q).S_2(I_q)<0 \Rightarrow \quad (22)$$

$$U_{qn}=\begin{cases}\frac{k_q}{\varepsilon_q}S_2(I_q) & si\ |S_2(I_q)|\le\varepsilon_q \\ k_q sgn S_2(I_q) & si\ |S_2(I_q)|>\varepsilon_q\end{cases}$$

Thus the control law relative to the current I_q is written

$$U_{qc}=U_{qeq}+U_{qn}=\frac{1}{g_2}(\dot{I}_{qref}-f_2)+k_q sgn S_2(I_q) \quad (23)$$

- b SMC relating to speed

$$\dot{S}_3(\Omega_r)=0 \Rightarrow I_{qref}=\frac{1}{g_3}(\dot{\Omega}_{ref}-f_3)$$

$$g_3=\frac{p(L_d-L_q)I_d}{J}+p\frac{\varphi_f}{J}$$

$$\dot{S}_3(\Omega_r).S_3(\Omega_r)<0 \Rightarrow$$

$$I_{qn}=\begin{cases}\frac{k_\Omega}{\varepsilon_\Omega}S_3(\Omega_r) & si\ |S_3(\Omega_r)|\le\varepsilon_\Omega \\ k_\Omega sgn S_3(\Omega_r) & si\ |S_3(\Omega_r)|>\varepsilon_\Omega\end{cases} \quad (24)$$

Thus the algorithm relative to speed is written

$$I_{qc}=I_{qeq}+I_{qn}=\frac{1}{g_3}(\dot{\Omega}_{ref}-f_3)+k_\Omega sgn S_3(\Omega_r) \quad (25)$$

5.2.3. Determination of the stability coefficients

The coefficients of the functions ' sgn(s) ' must be quite selected to ensure the stability of the system and to satisfy the conditions of the sliding mode [6,7,8].

$$k_d<-\max_{I_d,I_q,\omega}|RI_d-L_q\omega I_q|$$

$$k_q<-\max_{I_d,I_q,\omega}|RI_q+L_d\omega I_d+\omega\varphi_f| \quad (26)$$

$$k_\Omega<-\max_{\Omega,C_r}\left|\frac{C_r+f\Omega_r}{p\varphi_f}\right|$$

5.3. Estimate of the load torque

The load torque is hardly measurable what obliges us to use its estimate in the expression of I_{qc}.

The method suggested by lePioufle permits to estimate in real time the load torque [11]. The fig.5 illustrates the estimator principle.

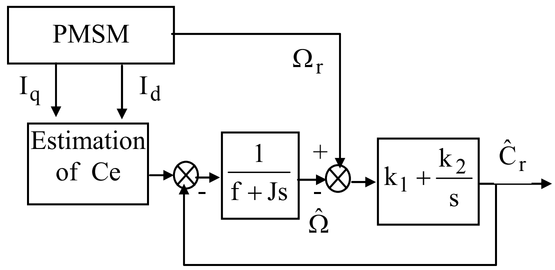

***Fig 5.** The load torque Estimator*

The error between measured speed and estimated one is presented as input of a regulator PI of whose output is :

$$\tilde{C}_r=\frac{1+\frac{k_1}{k_2}s}{1+\frac{1+k_1}{k_2}s+\frac{1}{k_2}s^2}C_r \quad (27)$$

k1 and k2 are determined by poles imposition.

5.4. MLI tension Inverter

The technique of the natural modulation permits to determine the moments and the durations of the switches lighting or extinction by comparison between the tension references and a triangular high frequency carry. The three-phase tensions (with MLI control) provided by the inverter are given according to the states of the switches (Ci, i=1,2,3) and of the direct tension E_d by [10].

$$[V_{in}] = E_d [M][C_i] \quad (28)$$

with

$$[V_{in}]^T = (V_{1n}\, V_{2n}\, V_{3n}) \; ; \; [C_i]^T = (C_1\, C_2\, C_3)$$

$$[M] = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (29)$$

5.5. Performances Comparative study

We make a comparison of the performances between the control with variable structure and the classical one to show the effectiveness and the superiority of the SCM.

6. SIMULATION

The synoptic diagram of the studied control is represented by the fig 6.The fig7 shows the performances of the system under the control of the flux orientation and fem compensation. The obtained results are satisfactory for a invariant linear system .The fig8 shows the starting of the motor with load variation at time 0.1s.

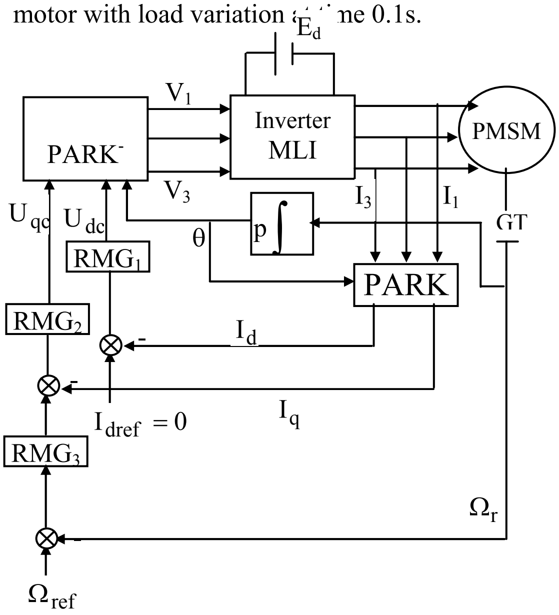

Fig 6.. Global diagram of the Sliding mode control of the PMSM

The analysis of the curves highlights the decoupling of the model (Id=0), the response quickness at starting, the disturbance rejection by the regulation and the system stability without static error. The fig9, fig10 show the SMC robustness characterized by an insensitivity at the internal and external parametric variations. The fig11 shows the comparison of the two algorithms for a tripled inertia . We can notice the superiority of the SMC ver all the performances.

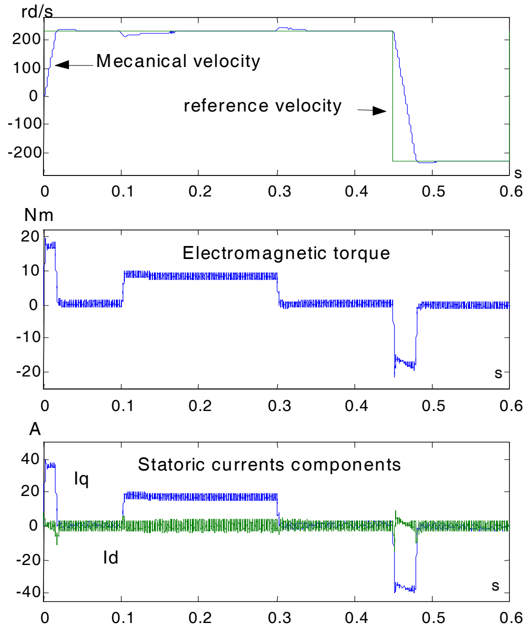

***Fig 7.** PMSM reponses with load variations (t=0.1s, t=0.3s) and velocity inversion (t=0.45s) under the classical control*

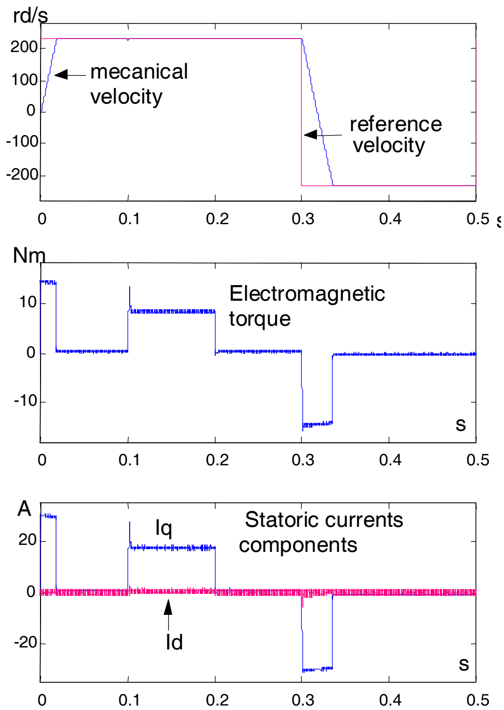

***Fig 8 .** PMSM reponses with load variations (t=0.1s, t=0.3s) and velocity inversion (t=0.45s) under the Sliding mode control*

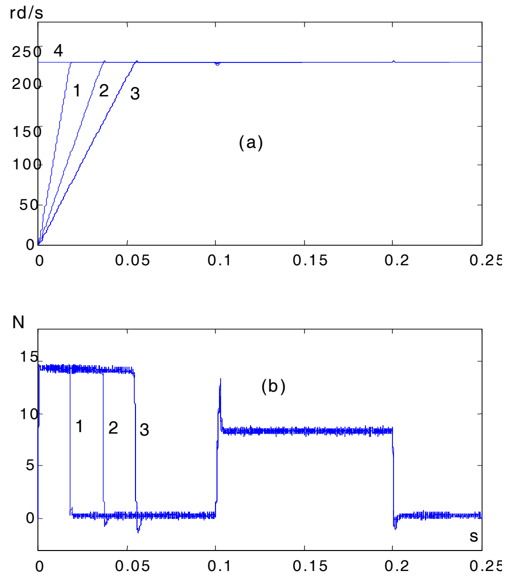

Fig 9. *PMSM reponses with inertia variations J_{nom}, 2-double Jnom, 3- triple J_{nom}) (a) Velocity reponse , (b) Torque response under sliding mode control*

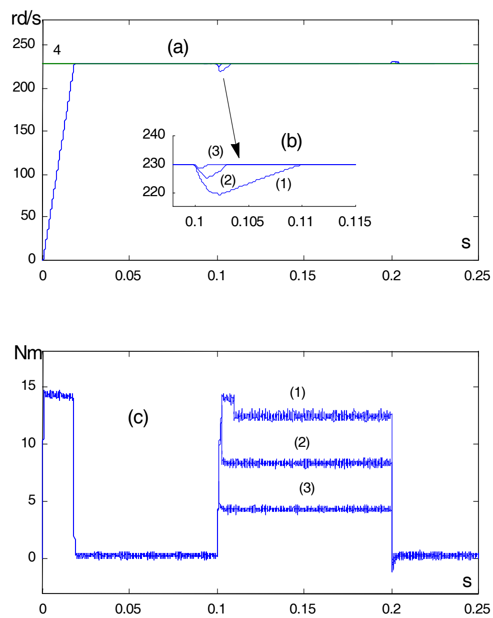

Fig 10. *PMSM responses with load variations (a- velocity response ;(b) perturbation rejection ; (c) torque response [(1) +50%C_e nom, (2)$C_e$nom, (3) –50% $C_e$nom] under the sliding mode control*

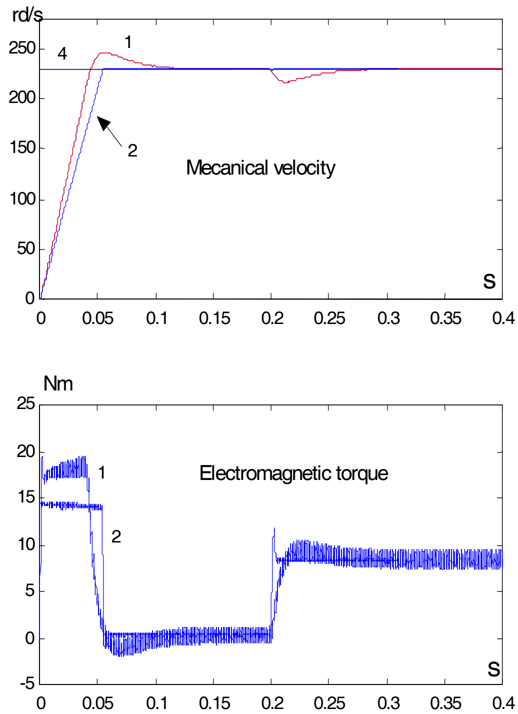

Fig 11. *Velocity responses et torque with inertia $J=3J_{nom}$ and nominal load variation at .2s (1- classical control PI, 2- sliding mode control)*

7 . CONCLUSION

In this article, we developed and applied a method of control to variable structure to the PMSM supplied with an MLI tension inverter. The suitable choice of commutation surfaces permits to obtain very good performances following the nature of the SMC which adapts well to the nonlinear systems. The various results obtained in Simulation show the SMC robustness to the system and load parameters disturbances. In addition the speed follow without overshooting, decoupling, stability and equilibrium convergence are ensured on all the variation interval. The results obtained with this SMC are excellent compared to the classical control. The static inverter and the control nature with variable structure (SMC) introduce high frequency ondulations which appear on the torque level. However, with a choice of a softened control (relay with saturation) and a high modulation index, we can reduce the couple fluctuations considerably. Moreover this control has the advantage of being easily implemented by a program control.

P ARAMETERS OF THE ENGINE USES

L_d=1.4mH L_q=2.8mH φ_f=0.12wb

P=4 J=1.1 10^{-3} kgm^2 f=1.4 10^{-3} Nm/rds^{-1}

C_e=8.5Nm R=0.6Ω I_{qn}=20A Ω_n=230rd/

REFERENCES

[1] P.Pillay, R.Krishnan, " Modeling simulation and analysis of permanent synchronous motor drives " IEEE Trans One Ind Appl flight 25 n02 March/April 89 pp265-273.

[2] P.Pillay, R.Krishnan, " Modeling of permanent motor drives " IEEE Trans One Elect flight 35 n04 Nov88 pp265-273.

[3] E. Bouhasoun, M.O. Mahmoudi, M.S. Boucherit "orders by mode of slip of a synchronous permanent magnet machine with vectorial piloting " ICEL98, 5-7oct USTO pp 11-16.

[4] H. Buhler, *Réglage par mode glissement,* polytechnic Presses Romandes 1986

[5]G.Garrara, D.Casini, A.Landi, and L.Tapenecco " Sliding mode speed controller for trapezoidal brushless motors " Electric Machines and oriented induction drive machine.

[6] M.O.Mahmoudi, N.Madani,M.F.Benkhoris, and F. Boudjema, "Cascade sliding mode control of field oriented induction machine drive" The European Physical Journal. Applied Physics 1999 pp217-225.

[7] B Belabbes, *Commande linearizing of a synchronous magnet permanents machine,* Thesis U.Sidi abbes 2001.

[8] J.J. Slotine, *Applied not linear control*, Englewood Cliffs NJ. Tice Hall 1991.

[9] V.I.Utkin , " Sliding mode control design principles and application to electric drives " IEEE Trans One Ind Elect, flight 40 n01 feb 93 pp23-36.

[10] G Seguy, *The converters of the power electronics: conversion direct-alternative* , vol4 edition Lavoisier 1989.

[11] B.LePioufle, *Comparison of speed not linear control strategies for the servomotor*, electric Machines and power systems, 1993, PP. 151-169.