# GENERAL PROPERTIES OF AIRBORNE BISTATIC SAR 

Yonghong YANG ${ }^{1}$, Yiming $\mathbf{P I}^{1}$<br>${ }^{1}$ School of E. E. University of Electronic Science and Technology of China, Chengdu, Sichuan, China, 610054<br>E-mail: yangyhxax@sohu.com.cn, ympi@uestc.edu.cn


#### Abstract

A geometry model of bistatic SAR (BiSAR) is provided, which can quantitatively describe the different degree between transmitter and receiver trajectories. The proposed geometry model is derived on the assumption that the earth is local flat and transmitter and receiver antennas are at the side-looking mode. From the point of view of system analysis, the comprehensive properties of airborne, including spatial resolution, baseline range and bistatic angle, are envisaged based on the proposed geometry model. Some characteristics of BiSAR system are exhibited via overall simulation experiments and these appreciated results are helpful to gain further insight into design issues.


Key words: Bistatic synthetic aperture radar; spatial resolutions; bistatic angle; baseline range BiSAR. For spaceborne bistatic SAR system,

## I.INTRODUCTION

With its all-time and all-weather imaging capability and 2-D dimensional high-resolution imagery, synthetic aperture radar (SAR) plays a vital role in earth observations and military reconnaissance missions. In a conventional SAR system, the transmitter and receiver are located at the same flight platform, such as a satellite or aircraft. However, BiSAR are spatially separated between the transmitter and receiver. Compared to conventional SAR, BiSAR with low cost possesses many advantages, for example, improving target scattering coefficients, increasing system survival, and enhancing stealth in military [1-3]. The emergence of BiSAR system indicates an increasing military demand and technique allowance at present. BiSAR system can conduct original scientific experiments, surveillance and reconnaissance as well as remote sensing.
In [2], Loffeld derived the formulae of point target response and point target reference spectrum based on a vectorial model of BiSAR and generalized image formation algorithm for

Moccia et al. have performed many excellent studies [4-6]. The comprehensive performance of spaceborne bistatic SAR system comprise
orbit configuration, signal synchronization, swath overlap, steering of transmitter/receiver antennae, doppler properties and range/azimuth resolution.
Additionally, it is well known that radar cross sections of the interesting targets are as a function of bistatic angle. Moreover, range/azimuth resolution has closely related to the bistatic geometry. As a consequence, analysis of transmitter/receiver trajectories influence to performance of BiSAR system is required.

## 2.RATIONALE

In our case, we consider aircraft as transmitter/receiver platform. The aircraft can be deployed around the interesting area. For the sake of simplicity, the earth is modeled as a local flat. As for airborne case, the assumption is justified. Figure 1a shows the geometry model
of BiSAR. "P" denotes the transmitter, "Q" denotes the receiver and " $T$ " denotes the target within the explored area. $\mathrm{PP}^{\prime}$ is the transmitter height, which denotes $\mathrm{h}_{\mathrm{p}} ; \mathrm{QQ}^{\prime}$ is the receiver height, which denotes $\mathrm{h}_{\mathrm{q}}$. The elevation of transmitter and receiver denotes $\gamma_{p}$ and $\gamma_{\mathrm{q}}$, respectively.

b: top view

Fig. 1 SA-BiSAR geometry model

### 2.1 Baseline range

PQ is the baseline range, which denotes $\mathrm{R} . \mathrm{P}^{\prime} \mathrm{Q}^{\prime}$ is its projection on the ground, which denotes $\mathrm{R}^{\prime}$. The projection of bistatic angle on the ground is the angle $\mathrm{P}^{\prime} \mathrm{TQ} Q^{\prime}$ which denotes $\beta^{\prime}$. The projection of transmitter flight trajectory on the ground is regarded as the reference direction. Via the two parallel line to reference direction through point $\mathrm{P}^{\prime}$ and $\mathrm{Q}^{\prime}$ respectively (see figure 1 b ), we get

$$
\beta^{\prime}=\left\{\begin{array}{l}
\Delta \theta  \tag{1}\\
\pi-\Delta \theta
\end{array}\right.
$$

When transmitter and receiver locate at the same side of target, $\beta^{\prime}$ is $\Delta \theta$; when target is between transmitter and receiver, $\beta^{\prime}$ is $\pi-\Delta \theta . \Delta \theta$, which is less than $90^{\circ}$, indicates the different degree
between transmitter and receiver flight trajectory. When $\Delta \theta$ is 0 , the trajectories of transmitter and receiver are identical, i.e., the parallel mode. When $\Delta \theta$ is $90^{\circ}$, the trajectories of transmitter and receiver are perpendicular to each other. In the trapezoid $\mathrm{PP}^{\prime} \mathrm{Q}^{\prime} \mathrm{Q}$, the baseline range is

$$
\begin{equation*}
R^{2}=R^{\prime 2}+\left(h_{p}-h_{q}\right)^{2} \tag{2}
\end{equation*}
$$

Where

$$
R^{\prime 2}=R_{P^{\prime} T}^{2}+R_{T Q^{\prime}}^{2}-2 R_{P^{\prime} T} R_{T Q^{\prime}} \cos \beta^{\prime}
$$

Substituting (1) into (2), we obtain

$$
\begin{aligned}
R^{2} & =R^{\prime 2}+\left(h_{p}-h_{q}\right)^{2} \\
& =\left(h_{p} \tan \gamma_{p}\right)^{2}+\left(h_{q} \tan \gamma_{q}\right)^{2}+\left(h_{p}-h_{q}\right)^{2} \\
& \mp 2 h_{p} h_{q} \tan \gamma_{p} \tan \gamma_{q} \cos \Delta \theta
\end{aligned}
$$

(3)

When target located at the same side of transmitter and receiver, the fourth term of (3) take negative. When target located between transmitter and receiver, the fourth term of (3) take positive. Using equation (3), the baseline range can be evaluated.

### 2.2 Bistatic angle

Via the cosine theorem, the bistatic angle can be obtained in the triangle PTQ,

$$
\begin{equation*}
\cos \beta=\frac{R_{P T}^{2}+R_{Q T}^{2}-R^{2}}{2 R_{P T} R_{Q T}} \tag{4}
\end{equation*}
$$

Substituting (1) and (2) into (4), we obtain

$$
\begin{equation*}
\cos \beta=\frac{h_{p} h_{q} \pm R_{P^{\prime} T} R_{Q^{\prime} T} \cos \Delta \theta}{R_{P T} R_{Q T}} \tag{5}
\end{equation*}
$$

Let both transmitter and receiver are on the ground, i.e., $h_{p}$ and $h_{q}$ are equal to 0 ., the following result, which is consisted with the geometry, can be derived via equation (5)

$$
\begin{align*}
& P \rightarrow P^{\prime}  \tag{6}\\
& Q \rightarrow Q^{\prime} \Rightarrow \begin{array}{l}
R_{P T}=R_{P^{\prime} T} \\
\gamma_{p}=\gamma_{q}
\end{array} \Rightarrow \beta=\beta_{Q T}=R_{Q^{\prime} T}
\end{align*}
$$

Further simplified and rearranged, the bistatic angle becomes
$\beta=\arccos \left(\cos \gamma_{\mathrm{p}} \cos \gamma_{\mathrm{q}} \pm \sin \gamma_{\mathrm{p}} \sin \gamma_{\mathrm{q}} \cos \Delta \theta\right)$

## (7)

Equation (7) shows that the bistatic angle depends on $\Delta \theta$ and elevation angle of transmitter and receiver. Furthermore, the effect of the transmitter elevation angle on the bistatic angle is equivalent to that of receiver. Hence, it is convenient to change the receiver elevation angle rather than transmitter elevation angle for cooperative BiSAR. It is worth noting a special situation, i.e., $\Delta \theta$ is 0 . Let $\Delta \theta$ is 0 , equation (7) becomes

$$
\begin{equation*}
\beta=\gamma_{p} \mp \gamma_{q} \tag{8}
\end{equation*}
$$

We remind the reader that equation (7) is derived on the assumption that transmitter and receiver antennae are pointing at the side-looking mode. When $\Delta \theta$ is 0 , the elevation angle of transmitter and receiver must be equal from Figure1a. Equation (8) becomes

$$
\beta=\left\{\begin{array}{c}
0  \tag{9}\\
\gamma_{p}+\gamma_{q}
\end{array}\right.
$$

Equation (9) is the minimum and maximum of bistatic angle. Minimum corresponds to the case of satellite and aircraft located at the same side of target; maximum corresponds to the case of target located between transmitter and receiver. Similarly, we can derive the same conclusion from Figure1. As a consequence, equation (9) is coincident with the geometry model, when $\Delta \theta$ is 0 .

### 2.3 Spatial resolution

From different points of views, spatial resolutions of bistatic SAR have been well investigated [7-9]. Range resolution of bistatic SAR is rewritten (see, for example, [3, 7])

$$
\delta_{r}=\frac{c}{2 B \cos \frac{\beta}{2}}
$$

(10)

Where B is the signal bandwidth and c is the light velocity.Substituting (7) into (10), we obtain

$$
\delta_{r}=\frac{c}{B \sqrt{\frac{1+\left(\cos \gamma_{p} \cos \gamma_{q} \pm \sin \gamma_{p} \sin \gamma_{q} \cos \Delta \theta\right)}{2}}}
$$

Equation (11) shows that range resolution depends on the bistatic angle and has close related to the transmitter and receiver trajectories. And its direction is along the bisector of bistatic angle.
Azimuth resolution of bistatic SAR is rewritten (see, for example, [3, 7])

$$
\delta_{a}=\frac{\lambda}{T_{s}\left(\frac{\left|\left(\mathbf{E}-\boldsymbol{\Psi} \Psi^{\mathbf{T}}\right) \mathbf{V}_{p}\right|}{R_{P T}}+\frac{\left|\left(\mathbf{E}-\boldsymbol{\Phi} \boldsymbol{\Phi}^{\mathbf{T}}\right) \mathbf{V}_{q}\right|}{R_{Q T}}\right)}
$$

## (12)

Where $\mathrm{T}_{\mathrm{s}}$ is the synthetic aperture time and $\lambda$ is the wavelength. $\mathbf{E}$ is the unit matrix, whose dimension is $3 \times 3 . \boldsymbol{\Psi}$ is the direction of transmitter line of sight, i.e., $\mathbf{R}_{\mathbf{P T}} ; \boldsymbol{\Phi}$ is the direction of receiver line of sight, i.e., $\mathbf{R}_{\mathbf{Q T}} . \quad \mathbf{V}_{\mathbf{p}}$ is transmitter velocity vector and $\mathbf{V}_{\mathbf{q}}$ is receiver velocity vector. At the side-looking mode, the projection of sensors velocity vector is velocity itself on the plane, which is perpendicular to sensors line of sight. According to (12), the numerical value of azimuth resolution is

$$
\delta_{a}=\frac{\lambda}{T_{s} \sqrt{\frac{V_{P}^{2}}{R_{P T}^{2}}+\frac{V_{Q}^{2}}{R_{Q T}^{2}}-2 \frac{V_{P}}{R_{P T}} \frac{V_{Q}}{R_{Q T}} \cos (\pi-\Delta \theta)}}
$$

## (13)

## 3.SIMULATIONS

To evaluate the above methods and characteristics of BiSAR, simulation experiments are conducted and simulation parameters are as follows: the height of both transmitter and receiver is 7 km and the velocity of both transmitter and receiver is $100 \mathrm{~m} / \mathrm{s}$; elevation of transmitter $\gamma_{\mathrm{p}}$ is $30^{\circ}$; elevation of receiver $\gamma_{\mathrm{q}}$ is $30^{\circ} 45^{\circ} 60^{\circ}$, respectively; Wavelength is 0.056 m and synthetic aperture time is 1 s . The receiver is deployed between transmitter and the target. And, the angle between transmitter and reciver flight path, i.e., $\Delta \theta$, varies from $10^{\circ}$ to $90^{\circ}$. According to practices issues, the above simulation parameters and geometry configurations are selected.


Fig. 2 Variation of baseline range with $\Delta \theta$
Figure2 shows the negligible variation of baseline range with $\Delta \theta$. When $\Delta \theta$ varies from $10^{\circ}$ to $90^{\circ}$, the baseline range fluctuates within a certain limit.


Fig. 3 Variation of bistatic angle with $\Delta \theta$
Figure3 indicates the variation of bisatic angle with $\Delta \theta$. When $\Delta \theta$ varies from $10^{\circ}$ to $90^{\circ}$, the bistatic angle has obvious change. With the choice of proper geometry configurations, radar cross sections of the interesting target would be obviously increased. Therefore, BiSAR system possesses the potential capability of enhancing target classification and pattern recognition.
Figure 4 indicates the ratio of range resolution
between arbitrary and parallel mode $\left(\delta_{\mathrm{r}} / \delta_{\mathrm{r} 0}\right)$ as a function of $\Delta \theta$, in which $\delta_{\mathrm{r} 0}$ is the range resolution when trajectories of transmitter and receiver are identical, i.e., the parallel mode. When $\Delta \theta$ varies from $10^{\circ}$ to $90^{\circ}$, the ratio of $\delta_{\mathrm{r}} / \delta_{\mathrm{r} 0}$ fluctuates within twenty percent. Furthermore, the numerical value of $\delta_{\mathrm{r}} / \delta_{\mathrm{r} 0}$ is less sensitive to low elevation angle of receiver.


Fig. $4 \quad \delta_{\mathrm{r}} / \delta_{\mathrm{r} 0}$ as a function of $\Delta \theta$
Figure5 indicates the ratio of azimuth resolution between arbitrary and parallel mode $\left(\delta_{\mathrm{a}} / \delta_{\mathrm{a} 0}\right)$ as a function of $\Delta \theta$. $\delta_{\mathrm{a} 0}$ has similar definition of $\delta_{\mathrm{r} 0}$. The numerical value of $\delta_{\mathrm{a}} / \delta_{\mathrm{a} 0}$ is more dynamic when $\Delta \theta$ varies from $10^{\circ}$ to $90^{\circ}$. However, it is less sensitive to elevation angle of receiver when elevation of transmitter is fixed.


Fig. $5 \quad \delta_{a} / \delta_{a 0}$ as a function of $\Delta \theta$

## 4.CONCLUSION

We present the bistatic geometry model, which can quantitatively describe the different degree between satellite and aircraft trajectories. Based on the geometry model, some fundamental properties are envisaged via analytical expressions including spatial resolution, baseline range and bistatic angle. Via simulation experiments, some characteristics of BiSAR system are exhibited: the significant variation of bistatic angle and the negligible variation of baseline range. Furthermore, squint-mode case of general bistatic geometry will be considered in the future.

## ACKNOWLEDGE

This work was supported by Sichuan Provincial Youth Science and Technology of China (06ZQ026-006).

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Yang Yonghong was born in 1971. She is currently a doctoral candidate in University of Electronic Science and Technology of China. Her research interests include radar signal processing and bistatic SAR image Pi Yiming was born in 1968. He is currently a doctoral supervisor in University of Electronic Science and Technology of China. His research interests include GPS, radar signal processing and SAR image formation algorithms.

